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The Political Game of Impeachment in Brazil and The Case of 2016 Analyzed Using Game Theory¹

O Jogo Político do Impeachment no Brasil e o Caso de 2016 Analisado sob a Ótica da Teoria dos Jogos

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RESUMO

O objetivo deste artigo é analisar o processo de impeachment que ocorreu no Brasil em 2016, contra a ex-presidente Dilma Rousseff. Para tanto, foi aplicada a metodologia da Teoria dos Jogos e apresentados três jogos nos quais participaria a Presidente da República: (i) o primeiro jogo é entre a Presidente da República e a Câmara dos Deputados; (ii) o segundo jogo é entre a Presidente da República e seu Vice; e o (iii) jogo é entre a Presidente da República e o Congresso Nacional. Para cada jogo foram apresentadas a estrutura, as regras e possíveis resultados considerando informações perfeitas, porém incompletas. Este tema é particularmente importante no Brasil em face da atual discussão sobre processos de impeachment e devido à escassez de estudos analisando tal assunto sob a ótica da Teoria dos Jogos. Ademais, este artigo busca mostrar o possível desenrolar dos jogos e os principais aspectos políticos que o envolvem. É demonstrado que na maior parte dos casos foram ajuizados processos, ou seja, o “jogo do impeachment” foi jogado 193 vezes entre 1990 e 2018, a julgar pelo número de reclamações protocoladas, das quais apenas duas foram abertas. Por fim, o artigo apresenta movimentos de partidos entre a eleição de 2014 e o impeachment de 2016, sugerindo que o processo de impeachment não é apenas legal, mas também político.

Palavras-chave: Informações incompletas; Informações completas; Jogo Bayesiano; Jogos Cooperativos e Não-cooperativos.

JEL: K0; K16; D72

ABSTRACT

The objective of this paper is to analyze the impeachment process in Brazil that took place in 2016 against the former President Dilma Rousseff. For that, we applied the Game Theory methodology and presented three games in which the President of the Republic participates in all: (i) a first game is between the President of the Republic and the President of the Chamber of Deputies; (ii) a second game is between the President of the Republic and her deputy; and (iii) a third game is between the President of the Republic and the National Congress. For each game, we presented the structure, rules, and possible outcomes considering perfect but incomplete information. This issue is particularly important in Brazil given the current discussion about impeachment processes and also due to the scarcity of similar studies analyzing this subject using Game Theory. In addition, this article seeks to show the possible unfolding of the games and the main political aspects that involve it. We show that in most cases lawsuits were filed, that is, the “impeachment game” was played, 193 times between 1990 and 2018, judging by the number of complaints filed, of which there were only two openings. Finally, this paper presents the movements of parties between the 2014 election and the impeachment vote in 2016, suggesting that the impeachment process is not merely legal but also political.

Keywords: Incomplete Information; Perfect information; Bayesian game; Cooperative and Non-Cooperative Games.

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1. Introduction

Since 1889 Brazil has been a Republic and lived with formally democratic periods, as in the Old Republic (1889-1930), after the Second World War (1945-1964), and from 1985 onwards, as well as with authoritarian periods, as in the Estado Novo (1937-1945) and during the Military Regime (1964-1985). Since the 1989 presidential elections, the way of ascending to the presidency of the Republic has occurred through direct voting; a second round is held between the two candidates with the most votes if the most voted candidate did not reach an absolute majority.

In such a context, presidential impeachment is a possibility in essentially democratic regimes. Requests of this nature are not rare and can be made by any citizen, and it is up to the President of the Chamber of Deputies to decide whether or not to proceed with the request. At this point, the Federal Constitution of 1988⁶ and the Law no. 1,079 of 1950⁷, which deals specifically with the process, should be highlighted.

A process of impeachment is undoubtedly complex and causes some uncertainties to political agents regarding the future of politics and/or the economy because it involves legal, political, economic, institutional, and public opinion variables, the latter often influenced by the degree of media exposure. Although the process of removal of President Dilma Rousseff may have a relationship with an institutional game between the Executive and the Legislative, variables such as loss of popularity and problems of political coordination apparently are also important. These factors are external to the institutions. In such a context, although external, these factors have become important variables in the establishment of the actors' strategies in the course of events that culminated in Dilma's impeachment, and it should be noted that many congressmen who voted for the removal of the President used the "clamor of the streets" as a justification.

On the other hand, it is necessary to mention an important structural feature of the Brazilian political system. Since the return of direct elections for President of the Republic in 1989, all elected presidents (Collor, FHC1, FHC2, Lula1, Lula2, Dilma1, Dilma2)⁸ have always had to expand their range of support beyond the parties with which they made an alliance during the electoral period. Because of this historical and structural aspect, from the beginning, majority formation strategies in the parliament become more difficult, especially in a scenario of growing fragmentation of party representation in the National Congress, especially in the Chamber of Deputies.

In this context, government coalition is not necessarily composed of all parties that formed the electoral coalition, as it tends to be negotiated on the basis of electoral results. Although there may be party indiscipline, since ministries and other important positions are distributed to parties, congressmen organize their work based on who is in government and who is in opposition. Dispatches are announced and formalized so that bases of support or opposition do not increase or decrease in a fluid manner even though the bases have some degree of volatility. In other words, to maintain governance and block an impeachment process, the President of the Republic and her party needed a great political articulation.

In this perspective, how to explain the decision-making behavior of the agents involved in the impeachment process that took place in Brazil in 2016? Using Game Theory as a

⁶ See Presidency of the Republic (2019a).

⁷ See Presidency of the Republic (2019b).

⁸ The nomenclatures refer to the former presidents and the number of terms they held after 1989: Fernando Collor de Mello (1990-1992), Fernando Henrique Cardoso (1995-1998 and 1999-2002), Luiz Inácio Lula da Silva (2003-2006 and 2007-2010), and Dilma Rousseff (2011-2014 and 2015-2016).

methodology, other questions arise: Who are the players involved? What are the rules? What are the strategies and possible results? How can games be structured?

Answering such specific questions helps to meet the general objective of this article, which is to analyze the 2016 impeachment process of President Dilma Rousseff and offer the possibility of apply the same model to other situations inherent to impeachment processes, that is, when the impeachment request is submitted but without pursuing it. Every time an impeachment request is submitted, the game is played; governments with no impeachment requests are rare. The rarity is in opening the process and effectively impeaching because most requests are barred.

As specific objectives, we seek to expose the game established between the President of the Republic and the President of the Chamber of Deputies, the President of the Republic and her deputy, and the President of the Republic and the National Congress; they are players directly involved in the game. In addition, this study aims to expose the circumstances that could lead a vice-president to betray or not the President in an eventual impeachment process. Finally, we intend to highlight the reasons that may lead the President of the Republic to propose or not an agreement with other players and the reasons that would lead the Speaker of the House to start the impeachment process and the National Congress to accept or not the request for president removal.

In this background, the subject is justified by the following points: (i) this is a current, relevant, and possible theme to be clarified through a formal methodology (Game Theory); (ii) there are no works that analyze the impeachment process in Brazil using Game Theory and that take into account the country's political conditions; and (iii) there is an international lack of work on this topic because formal processes of removal of presidents, in addition to being rare, only occur in essentially democratic regimes. Therefore, even if requests for impeachment are natural in a democracy, and it should be considered that few are successful, there is no way to minimize the importance of the issue.

To this end, in addition to this introduction and the final considerations, we divided this article into four sections. In the first part, we present a theoretical reference on Game Theory and a literature review on impeachment. In the second part, we highlight the rules of the game as provided for in the constitutional legal regime, especially those inherent to impeachment. Then, in the third part, we explain the proposed methodology and the possible results. Finally, in the fourth part, we analyze the Brazilian case of impeachment of 2016.

2. Theoretical Reference and Literature Review

2.1 Theoretical Reference

Game Theory and its application in the economic sphere has given rise to numerous publications over the years. Although the work of Von Neumann and Morgenstern (1944) is a milestone in this area, other previous contributions should be highlighted, such as those of Cournot (1838), Edgeworth (1881), and Zermelo (1913). Later, Nash (1950a; 1950b; 1951) offered relevant contributions to the so-called non-cooperative games, combining the Nash Equilibrium concept with Game Theory. As a result, John Forbes Nash, together with Reinhard Selten and John Harsanyi, the latter two were awarded with the Nobel Prize in Economics in 1994 for other contributions (HANEKE; SADDI, 1995).

In the 1960s and 1970s, Reinhard Selten introduced the concept of perfect subgame into Game Theory, as well as refinements to it. Still in the 1960s, the contributions of John Harsanyi stood out. He introduced the idea of solving games via mixed strategies, that is, finding the result for a game, by assigning probabilities for the occurrence of events when the game cannot be solved with pure strategies. As a result, the understanding of possibilities and uncertainties seemed to bring Game Theory closer to economic sciences (HANEKE; SADDI, 1995).

In addition to the contributions mentioned, those of Harsanyi (1973), Aumann (1964), Aumann (1974), and Holloway (1979) also stand out. Later, Thomas Schelling's work "The Strategy of Conflict" gained notoriety⁹. In 2005, as a result of his contributions, Thomas Schelling shared the Nobel Prize in economics with Robert Aumann. Recently, new studies related to Game Theory have been awarded with the Nobel Prize in Economics. In 2012, Lloyd Shapley and Alvin Roth were awarded this prize for the theme "Allocation and Market." In addition, there is "The Theory of Contracts" attributed to Oliver Hart and Bengt Holmström in 2016¹⁰.

In view of this brief history, numerous works deal with Game Theory. For a mathematical analysis, in addition to the original texts, we can mention some others: Kreps (1990), Fudenberg and Tirole (1991), Gibbons (1992), Osborne and Rubinstein (1994), Mas-Colell, Whinston and Green (1995), Dixit and Skeath (2004), Osborne (2004), Rasmusen (2006), Jehle and Reny (2011), Bierman and Fernandez (2011), and Owen (2013). On the other hand, Varian (2012) and Fiani (2015) stand out as introductory works in this respect.

In view of this, based on Mas-Colell, Whinston and Green (1995), a game means a situation in which a certain number of individuals (players) interact in a scenario of strategic interdependence to achieve a certain result and enjoy the reward. Faced with this, there are:

- Players: those involved in the game;
- Rules: what players may or may not do;
- Strategies: possible decisions within the game;
- Results: results from the actions taken by players based on the decision-making of others involved; and
- Payoffs: the rewards.

According to Osborne (2004), there is a search for the players' ideal payoff since knowing the game and the interests the rationality will always act to choose the strategy that maximizes its usefulness. Thus, Mas-Colell, Whinston and Green (1995) structured the presentation of the game. Equation (1) shows the formalization of the game via pure strategies¹¹. Formally:

$$\Gamma_N = [I, \{S_i\}, \{U_i(s_i; s_{-i})\}] \quad (1)$$

Where:

Γ_N : Normal way of representing the game;

I: Players;

S_i :: Possible game strategies;

$U_i(s_i; s_{-i})$: Payoff functions of the game, where the level of utility is related to the outcome of the strategies s_1, \dots, s_i .

⁹ See Schelling (1980).

¹⁰ See Shapley (1953) and Hart (1995).

¹¹ Games with pure strategies are those in which players make their strategic decisions not based on random strategies or by attributing probability to the occurrence of events.

Considering Equation (1), a finite set of strategies for the player i , the same will have a payoff. Therefore, strategies can also be demonstrated via payoff matrices (Frame 1).

Frame 1: Representation of the game in the payoff matrix

		Player 2	
		Strategy_{s_i}	Strategy_{s_{i-1}}
Player 1	Strategy_{s_i}	Payoff (1; 2)	Payoff (1; 2)
	Strategy_{s_{i-1}}	Payoff (1; 2)	Payoff (1; 2)

Source: Own elaboration based on Mas-Colell, Whinston and Green (1995).

On the other hand, there is the extensive form (game tree); it is best method of visualization when it comes to games of the sequential type and will be used for the purpose of this work. For Mas-Colell, Whinston and Green (1995), a game in the extensive form can be described mathematically and specified as follows:

$$\Gamma_N = \{\mathcal{E}, A, I, p(\cdot), \alpha(\cdot), \mathcal{H}, H(\cdot), i(\cdot), \rho(\cdot), \mu\} \quad (2)$$

Where:

\mathcal{E} is the finite set of knots;

A is the finite set of possibilities for action;

I is the finite number of players (1,..., I);

$p(\cdot)$ is a function indicating a single immediate predecessor of each \mathcal{E} node; $\alpha(\cdot)$ is a function that leads to some non-initial node x from $p(x)$;

$\alpha(\cdot)$ is a collection of information;

$H(\cdot)$ is a function assigned to each decision node x the information set $H(x) \in \mathcal{H}$;

$i(\cdot)$ is a function that assigns each set of information in \mathcal{H} to the player who moves with the decision nodes in that set;

$\rho(\cdot)$ is a function that assigns probabilities to the information set;

μ is a payoff function.

The usefulness of the players (\cdot) for every possible game to be played means what the players know when they move in every Situation of the game. In an extensive game, a function $p(\cdot)$ specifies a single immediate predecessor of each node, where a function $\alpha(\cdot)$ leads to some non-initial node x from its immediate predecessor $p(\cdot)$ and a collection of information set (D) and a function $D(\cdot)$, assigning each decision node x to information set $(D) \in D$.

The strategic equilibrium, or Nash equilibrium, is the strategy that no player can deviate from unilaterally to get the best result. In this same game, there can be different Nash equilibria, both in pure and mixed strategies. In pure strategies, there is not always a Nash equilibrium but every finite game necessarily has at least one Nash equilibrium in mixed strategies. In a pure strategy game, the Nash equilibrium is as follows:

$$\Gamma_N = [I, \{S_i\}, \{U_i(s_i; s_{-i})\}] \quad (3)$$

Where S_i is a list of strategies $s = (s_1, \dots, s_I)$ for all $i = 1, \dots, I$, $U_i(s_i, s_{-i}) \geq U_i(s'_i, s_{-i})$ on the whole $s'_i \in S_i$. In static games with incomplete information, the concept of the solution is the Bayesian Nash equilibrium. Formally, in a Bayesian game, each player i has a payoff function $U_i(s_i, -s_i, \theta_i)$, whereby $\theta \in \Theta_i$ is a random variable chosen by nature and observed only by the

player i . The joint probability distribution of θ_i 's is given by $F(\theta_1, \dots, \theta_z)$, which is widely known by players. Thus, a Bayesian game can be represented by:

$$[I, \{S_i\}, \{U_i(s_i, -s_i)\}, \theta, F(\cdot)] \quad (4)$$

In a Bayesian Nash equilibrium, each player must make the best strategic decision given the conditional distribution of the opponents' strategies. For Mas-Colell, Whinston and Green (1995), there are four types of games and each type has its own type of equilibrium: (i) static with complete information and the Nash equilibrium as a solution; (ii) static with incomplete information (Bayesian Nash equilibrium); (iii) dynamic with complete information (perfect equilibrium in sub-games); and (iv) dynamic with incomplete information (perfect Bayesian equilibrium).

For the purpose of this study, as it is a dynamic game, the best form of explanation is the extended form or "game tree". It is important to highlight that games involving impeachment are dynamic games, with perfect but incomplete information. There is knowledge about the game structure, the strategies of each player, and the decision node they are in (perfect information), but there is not a complete knowledge about possible results (incomplete information).

In the words of Gale and Stewart (1953), "the notion of a game with perfect information was defined by von Neumann and Morgenstern. The concept intuitively corresponds to games like checkers and chess in which the player's movements and positions are known to his opponents all the time." On the other hand, games with incomplete information are games in which players are unsure about some important parameters of the game situation, such as the payment function, strategies available to multiple players, information that other players have about the game etc. (HARSANNYI, 1967).

The results are known only regarding the President of the Republic, whether or not she can be removed from the presidency, given that she knows exactly which decision node she is in and the possible game results for her. Thus, following the same reasoning as Kim (2006), "a theoretical model of Bayesian game (incomplete information), seems appropriate." This is because possible results are subjective, especially for people who decide whether or not to remove the President of the Republic.

2.2 Literature Review

The second round of the presidential election in Brazil in 2014 was one of the most disputed since the re-democratization in 1985. The candidate Dilma Rousseff (51.64% of votes), of the Workers' Party (PT), beat the candidate Aécio Neves (48.36%), of the Brazilian Social Democracy Party (PSDB) (BECKER, 2017; VAN DIJK, 2017; REBELLO, GIORA, DAMIN, 2019). In the words of Giambiagi (2016), after 16 years of mandate of two very remarkable former presidents (1995-2010), it was natural that any politician who took over the presidency would have a smaller personality than the predecessors. The author conducts an economic analysis of the first Rousseff administration (2011-2014) and shows that political factors, combined with economic mistakes, were points of intense debate but that even so they did not represent a defeat in the 2014 election. Although uncontrolled public accounts were a feature of the 2011/2014 phase, the beginning of her second term was characterized, in the words of Giambiagi (2016), as a sort of "turnaround." The change in the economic team, especially the Ministry of Finance, showed that fiscal adjustment was a necessity, in some points contradicting proposals of the electoral campaign.

However, based on the words of Becker (2017), Van Dijk (2017) and Rebello, Giorna and Damin (2019), beyond issues of an economic nature, there was an aggravation of the post-2014 political crisis, especially because of the "Operation Car Wash." The work of Kim (2006), which analyzes the impeachment process in South Korea, argued that economic crisis, political crisis, and tight election results, as well as political polarization, are factors that stimulate demands for impeachment, motivating the losing side not to accept the defeat. Apparently, *mutatis mutandis*, the political and economic factors of South Korea and Brazil, when their impeachment processes occurred, have important similarities, even though the South Korean case occurred for apparently more banal issues (the non-acceptance of the President to apologize for campaigning in advance to politicians of his choice).¹²

Regarding the Brazilian case, the President of the Chamber of Deputies accepted a formal accusation in December 2015; it was followed by several public demonstrations, both against and in favor of the impeachment, fomenting an intense debate on the limits of the political crisis in Brazil (BECKER, 2017). The political wear and tear of the Workers' Party (PT) became increasingly evident in March 2016, when the Party of the Brazilian Democratic Movement (PMDB) left its allied base. This was the party of the Vice President Michel Temer. Other parties, until then belonging to the government's support base, also adopted the same strategic decision of the Brazilian Democratic Movement Party (PMDB) probably because they knew that in order to remain in government at that moment would be the same as being in the opposition the next moment (REBELLO, GIORA and DAMIN, 2019).

Before that, the worsening of the economic situation in Brazil under Rousseff's administration, combined with a political destabilization fostered by the corruption scandal triggered by the Operation Car Wash from mid-2014, had already generated a great wear and tear of public figures of the Workers' Party (PT). However, it should be noted that this situation reached not only the top of the Workers' Party (PT), but also contractors and members of other allied parties, including the vice president's party (PASSOS; BAPTISTA, 2018).

Based on Rebello, Giora and Damin (2019), the rupture between the parties of the President of the Republic and the Vice President was essential for the continuity of the impeachment process initiated by the President of the Chamber of Deputies at that time, Eduardo Cunha (PMDB-RJ). Moreover, the process was accepted by the President of the Chamber of Deputies at that time as a retaliation, as he was being tried for corruption in the House of Ethics Council.¹³ Once the impeachment process was accepted, the political movements began to organize themselves to choose the best strategic decision for what would come next. The Chamber of Deputies accepted the impeachment process on April 17, 2016, by 367 votes in favor of the process admissibility, with 137 votes against it. With this result, the President Dilma Rousseff was removed from office for 180 days. The definitive impeachment happened after the vote in the Senate on August 31, 2016, with 60 votes in favor and 21 against it (BECKER, 2007).

According to Villaverde (2019), the impeachment process can be formulated using Games Theory considering the institutional balance between the government branches, constitutional and legal provisions for impeachment, party political structure, the President's popularity, and other observable factors such as economic conditions. Analyses of impeachment processes in other countries have already used Game Theory, as was the case of South Korea in 2004.

¹² See Kim (2006).

¹³ See also Martins (2019).

To verify the reasons for the political disagreements between President Roh Moo-hyun and the National Assembly, Kim (2006) applied a Bayesian model of incomplete information. The results showed that Game Theory has a great potential to analyze political decisions with real implications. Thus, the importance of Game Theory to understand the relationship between political agents is essential to evaluate an impeachment process since a legislative force can be used for both the fall of a government and for its permanence (PÉREZ-LIÑÁN, 2014).

Specifically for the Brazilian case, the lack of parliamentary support was endogenous to the drop in popularity. On the other hand, the drop in popularity was endogenous to the economic and political crisis. On the other hand, the fall in popularity is endogenous to economic crises. According to Rebello, Giora and Damin (2019), the lack of political support from party leaders was one of the factors that contributed the most to the impeachment. The very legal argument used to justify the impeachment of the presidential mandate, the "tax rides," was legalized after the impeachment (VAN DIJK, 2017). In addition, subsequent events such as the failure of the Supreme Electoral Court to annul the slate after the definitive withdrawal of Dilma Rousseff and the "protection" of the House of Representatives to President Temer, with respect to the votes on the admissibility of accusations of corruption charges, were signs of behavior changes by the legislature.

For Corrêa, Santos and Nunes (2018) and Tatagiba (2018), there are numerous findings that point out that the lack of political organization was the main factor leading to the end of Rousseff's administration. One such observation appeared in speeches in the impeachment vote in the House, marked by little consideration of the crime of fiscal responsibility, with speeches that presented a lack of objectivity and justification.

In the observations of Martins (2016), the political game between the President of the Chamber at that time (Eduardo Cunha) and the President of the Republic (Dilma Rousseff) showed that the dominant strategy was "to continue with the impeachment process" for the President of the Chamber, and "do not protect Cunha" for Dilma. As a result, both were removed from their positions. The other strategies show that, although it was best for both of them to remain in their positions, this did not happen due to the lack of coordination.

Although this is a recent subject, some studies have already studied this issue, but not necessarily using Game Theory as a method, which once again justifies the unprecedented analysis this study conducts. Among studies, we can cite Almeida and Lima (2016), Tavares and Berger (2016), as well as Feres Júnior and Sassara (2016) and Oliveira (2016), Alonso (2017), Prandi and Carneiro (2017) and Queiroz (2017), Kasahara and Marsteintredet (2018), and Braga (2018).

The authors Almeida and Lima (2016) commented on the importance of the media in the dissemination of this theme and how it can interfere with the formation of opinion in society, highlighting the need for a process of media democratization, thus allowing a greater plurality of positions. The authors suggest that the concentration of communication power interferes in the political positions of society taking as an example of Dilma Rousseff's removal process. For Tavares and Berger (2016), along the same lines of Almeida and Lima (2016), there are doubts about the behavior of the press in the coverage of the 2016 impeachment, suggesting that in the case of Dilma Rousseff it was a "announced coup d'état."

Feres Júnior and Sassara (2016) are more emphatic and attest that there was an increase in negative coverage of the re-elected President in 2014, suggesting that the media behaved in a highly politicized manner by apparently hindering the strengthening of democratic processes. In a probably ironic tone, the authors mention the President's impeachment process as a type of third election round.

Alonso (2017) offers a comparative analysis of repertoires for and against the removal of the President and the opportunities for protests, as well as an analysis of three cycles that led to the impeachment: the mosaic cycle (June 2013), the patriot cycle (March to April 2015), and the impeachment cycle (December 2015 to March 2016). For that author, the wave of global protests between 2011 and 2013 and sports events in Brazil, such as the Confederations Cup (2013), World Cup (2014), and the Olympics (2016), served as the basis for possible mass protests.

On the other hand, from a political point of view and anchored on a high popularity, the author highlights that there has been a change in the treatment of the ruling party in relation to social movements (i.e., a lack of political coordination). Furthermore, Alonso (2017) highlights that the political polarization in Brazil has contributed to the expansion of discourses both for and against the impeachment. In short, a series of events contributed to the "social unrest," the unfolding of which led to the impeachment, suggesting that political and economic crises favor the removal of the President.

Prandi and Carneiro (2017) offer an analysis of Members' justifications for opening the procedure. For them, the "tax rides" or legal merits for the approval of the impeachment had little importance in the analysis of the president's removal, as more emphasis was placed on moral issues relating to family, religion, and the electoral basis of each parliamentarian. In addition, the authors expounded on the influence of religion in politics, emphasizing how much the "evangelical benches" contributed to the impeachment of President Dilma Rousseff given that 93.8% of the aforementioned group of parliamentarians voted in favor of her.

For Queiroz (2017), "crimes of presidential responsibility have the legal nature of eminently political offences, although they may sometimes present their own traits of criminal offences, like passive corruption." The author, therefore, highlights those cases of President removal, in addition to traumatic, do not contribute to the political and economic stability of the country.

Kasahara and Marsteintredet (2018), for their part, offer an interesting discussion about the interruptions of presidential mandates that occurred in Latin America after the 1980s. For them, although Latin America has recently experienced essentially democratic regimes, they have not prevented institutional ruptures, as there have been 16 falls of presidents, only two of them by military means. Specifically, via impeachment, there were six president removals, of which two were in Brazil (1992 and 2016). The other institutional ruptures happened either through resignation/election preparations (eight) or military action (two); ruptures through military means occurred in Ecuador (2000) and Honduras (2009). In this sense, the authors emphasize that political and/or institutional crises, combined with problems of legitimacy and the presence of political polarization, have contributed to the progress of impeachment processes. Furthermore, Kasahara and Marsteintredet (2018) detailed their analysis by questioning the political stability in presidential regimes and suggested that the parliamentary regime, even if there is resistance to it, apparently has a greater political stability as breaks happen in a less traumatic way.

Braga (2018) compared both impeachment processes that took place in Brazil and reported a similarity between both: the lack of support from central parties, which damaged governability and made it possible for the presidents to be removed. However, one difference is important: while Collor was elected in 1989 without much support of Congress, this cannot be said for President Dilma Rousseff in 2014. Specifically in relation to Dilma Rousseff, Braga (2018) points out that the disembarkation from the government by the Brazilian Democratic Movement Party (PMDB), the party of the vice-president, was decisive to interfere in the decision of other parties to support the impeachment.

Other authors, however, have used Game Theory applied to political issues although not necessarily focused on the process of the impeachment, among which we can cite: Portugal (2006), Bugarin and Meneguín (2016), Kluppel (2007), Montagner (2017), and Morais and Koprik (2018). Publications that applied Game Theory to impeachment processes referenced in this research are based only on Kim's work (2006).

The analysis of literature review and the justifications of parliamentarians when expressing their votes, many of which influenced by the "clamor of streets," show that it did not stand out regarding the possible crimes of responsibility of President Dilma Rousseff. This corroborates, therefore, the idea that the impeachment process also has a political character, perhaps more relevant than the legal one. The literature review, therefore, shows that political, economic, and institutional aspects are relevant for the analysis of institutional ruptures, especially in Latin America. This was not necessarily the case in Korea. A research carried out on the Scopus platform using "Impeachment" and "Game Theory" as key search words showed that this relationship is not much studied; only the work of Kim (2006) was the research result.

3. The Rules of the Game

For Mas-Colell, Whinston and Green (1995) and Osborne (2004), the rules of a game refer to what players can and cannot do. For the specific case of the impeachment, there is basically the Federal Constitution of 1988 and Law no. 1,079 of April 10, 1950, which deals specifically with the impeachment of the President and the vice president.

Since the choice of representatives does not please everyone, the elected representatives can be removed from their positions according to previous findings of political-administrative infractions and/or acts contrary to the functional duties established in the Federal Constitution of 1988, such as crimes of responsibility. For that, there must be an admission of impeachment processes by the Chamber of Deputies and a judgment by the Senate, as provided for in art. 52, I, and arts. 85 and 86 of the Federal Constitution of 1988 (PRESIDENCY OF THE REPUBLIC, 2019a)¹⁴.

The so-called crimes of responsibility are political-administrative violations committed by political agents holding public offices, as provided for in the Law no. 1079 of April 10, 1950, art. 4 (PRESIDENCY OF THE REPUBLIC, 2019b). In the Brazilian legal system, the impeachment is provided for in the 1988 Constitution with the purpose of controlling the acts practiced by the President of the Republic. Taking into account that the impeachment begins by assuming the removal of the person who has not honored his or her functional duties and the commitment with formal legal regulations, there is a procedure for impeachment process in the National Congress which begins with the authorization in the House of Representatives, according to art. 51, I, of the

¹⁴ The crimes of responsibility of the President of the Republic are provided for in article 85 of the Constitution of the Federative Republic of Brazil of 1988; in verbis:

"Article 85. Those acts of the President of the Republic which attempt on the Federal Constitution and especially on the following, are crimes of malversation:

I – the existence of the Union;

II – the free exercise of the Legislative Power, the Judicial Power, the Public Prosecution and the constitutional Powers of the units of the Federation;

III – the exercise of political, individual and social rights;

IV – the internal security of the country;

V – probity in the administration;

VI – the budgetary law;

VII – compliance with the laws and with court decisions."

1988 Constitution; any citizen can make a regular denunciation, as provided for in arts. 14 to 18 of the Law no. 1.079, of 1950 (PRESIDENCY OF THE REPUBLIC, 2019b).

The initial examination of the complaint made by any citizen is the responsibility of the President of the Chamber of Deputies, who may either reject it or accept it. A special committee is then designated to examine the request in the event of acceptance, in accordance with the articles 19 to 23 of the Law no. 1,079 of 1950. The examination procedure of the impeachment results in an opinion by the plaintiff. This is the stage of pronouncement of parties and the roll-call vote of each Federal Deputy to decide on the initiation of the process, requiring a quorum of two-thirds of the 513 Federal Deputies, that is, 342 favorable votes. After that, the process is referred to the Senate, which, after all proceedings, finally decides whether or not to impeach the president in a session conducted by the President of the Supreme Court.

4. The Proposed Methodology and Possible Outcomes

In this section, we present three games to explain the impeachment process in Brazil. In them, the President of the Republic is a player who establishes a game with the President of the House, another game with the vice-president, and a third game with the National Congress, which in the Brazilian case is bicameral (House and Senate). The president's probable strategic decisions were laid out for each player, as well as the possible results. It is worth pointing out that although these are games of perfect information, they are of incomplete information, following the same idea of Kim (2006).

For the purpose of this work, we assumed that a process of impeachment is composed of numerous elements that may or may not be interrelated, such as political, economic, or institutional crises. Popular support, reflected in mass and diverse manifestations, and a strong media coverage contribute to the progress and the success of a process of removal of the President of the Republic. Thus, the more elements there are (political, economic, institutional crises, mass demonstrations or popular dissatisfactions, and media coverage), the greater the chances of success of the impeachment.

Thus, although important and necessary, the procedures of an impeachment process are anchored not only in technical or legal aspects. Political reasons, such as disputes for space, positions, and weakening of an opposing party, although they should not be preponderant elements, are accompanied by popular support and reflected in the media.

For the games proposed, we assumed the possibility of existence of two distinct and mutually exclusive situational situations in a process of impeachment: the presence of a crisis or its absence. For the purpose of the games presented, we assumed that nature defines or accomplishes the president's strategic choice due to crises or not. When discussing crises this term has generalities, although it is natural to think, at first, only of economic aspects such as a reduction in economic activity, increase in unemployment and informality, capital flight, poverty increase, and an upsurge in inflation or, in the case of deeper crises, characterized by a fall in aggregate demand, generating deflation. In any case, although different and complex, economic crises have something in common: the reduction in the population well-being.

Nevertheless, the term "crisis" can also be used for the political conditions that, among many other aspects, are characterized by (i) a lack of support in Congress for the President's party, reflected in the low number of congressmen belonging to the government's support base; (ii) escalation of political and/or ideological disputes, reflected in polarization and/or in highly

contested elections; (iii) systematic denunciations of corruption against the ruling party or parties belonging to the government's support base, contributing to its weakening in face of the public opinion; and (iv) popular dissatisfaction, reflected in demonstrations of a different nature.

On the other hand, a crisis may also be of an institutional nature, such as the systematic attack of one power to another, threats of institutional ruptures or coups d'état, as well as disregard for the constitutional limits of each power. Modern Constitutions, such as the Brazilian Constitution of 1988, have systems of checks and balances in all three powers so that none of them has absolute or sovereign powers over the others. The aim is self-control between them. Institutional crises occur, for example, when there is no respect for the limits of the powers, usurpation of competencies, or when there is hypertrophy of some power over the others, such as (i) executive or legislative or judicial strongly over the others; (ii) the judiciary replacing or entering into the responsibilities of the legislative in the role of legislating; or (iii) the judiciary exercising the typical role of the executive in the application of situations.

Moreover, a crisis of the institutional type happens when, for example, the executive tries to corrupt the judiciary and/or legislatures (i) through corruption or admission of cadres, (ii) by appointing allies to the Supreme Court (STF), or (iii) by politically manipulating elections to favor allied parties. In this context, an institutional crisis may lead to institutional ruptures/coups d'état in extreme situations or, perhaps less aggressively, one power refusing to accept the decision of another power.

Comparing the various nuances inherent in the removal of President Dilma Rousseff in 2016, as well as the removal of former President Collor in 1992, considering the aforementioned characteristics belonging to each type of crisis, one can observe to a greater or lesser extent the presence of each of them. Specifically for the case of 2016, some points such as the resurgence of an economic crisis after 2015, the increase in unemployment and informality, the intensified political polarization after the 2014 election, the election of a President of the Chamber of Deputies not allied to the party of the President of the Republic, although belonging to the party of the Vice President, the fall in the President's popularity, and the various accusations of corruption raised by Operation Car Wash, reported by the media, contributed to the success of the impeachment process. Therefore, the legal reason for removing the President, i.e., a possible crime of responsibility, was not the only determining element, so that very little exists in literature reviews or in arguments of parliamentarians when they justify their vote.

Finally, it is necessary to mention that requests for impeachment are common in the Brazilian political history after 1989. Most of the times, even though the requests have preponderant elements for the removal, they are filed and do not advance. Therefore, there is a need for further elements in addition to crimes of responsibility with a view to strengthening support for the removal of the President. An exception to the rule may have occurred in South Korea in 2004, in which the removal of the President occurred for apparently banal reasons (KIM, 2006). In Brazil, perhaps, this was more tolerable.

4.1 The Match Between the President of the Chamber of Deputies and the President of the Republic

For the purposes of this game, we assumed that nature makes the first strategic decision, which can be "With Crisis (CC)" or "Without Crisis (SC)" (Figure 1). The President of the Republic then chooses the best strategy and the president and its party can propose an agreement to the President of the Chamber of Deputies (PA) for the process not to prosper, or the president may not propose the agreement (NPA), with the risk that the process advances. In turn, the Speaker of the House, faced with the strategic choice of the President of the Republic, may start the process (AP) or not start the process (NAP); in the event of not starting the process, it is filed.

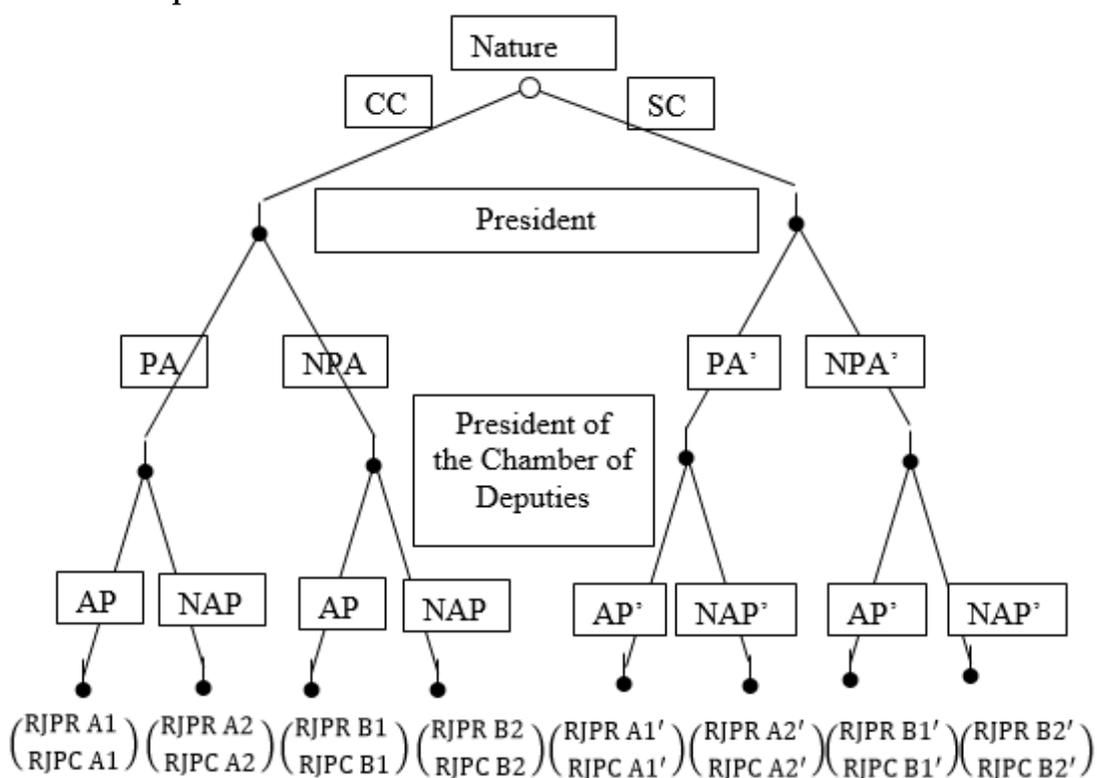
The possible results of this game, which involves the President of the Republic and President of the Chamber of Deputies, however initiated by nature, are only possible to be measured in the case of the President of the Republic (possibility of being removed or not); this is not clear to the President of the Chamber of Deputies. This situation characterizes the game as having perfect but incomplete information. In other words, it is a game of perfect information because each player knows exactly which decision nodes they are in, but incomplete information because of the difficulty of measuring the possible payoffs of all players, with eight possible end nodes (Figure 1). In a generic way, the possible results consider the presence of a crisis of some nature, as follows:

- i. RJPR A1: Match Result for the President of the Republic in Situation A1 (With Crisis).
- ii. RJPC A1: Match Result for the President of the Chamber of Deputies in Situation A1 (With crisis).
- iii. RJPR A2: Match Result for the President of the Republic in situation A2 (With crisis);
- iv. RJPC A2: Match Result for the President of the Chamber of Deputies in situation A2 (With crisis).
- v. RJPR B1: Match Result for the President of the Republic in situation B1 (With crisis).
- vi. RJPC B1: Match Result for the President of the Chamber of Deputies in situation B1 (With crisis).
- vii. RJPR B2: Match Result for the President of the Republic in situation B2 (With crisis).
- viii. RJPC B2: Match Result for the President of the Chamber of Deputies in situation B2 (With crisis).

On the other hand, we used the following nomenclatures for the presentation of the game considering the absence of a crisis:

- i. RJPR A1': Match Result for the President of the Republic in Situation A1' (No Crisis);
- ii. RJPC A1': Match Result for the President of the Chamber of Deputies in Situation A1' (No Crisis);
- iii. RJPR A2': Match Result for the President of the Republic in situation A2' (No Crisis);
- iv. RJPC A2': Match Result for the President of the Chamber of Deputies in situation A2' (No Crisis).
- v. RJPR B1': Match Result for the President of the Republic in Situation B1' (No Crisis).
- vi. RJPC B1': Match Result for the President of the Chamber of Deputies in situation B1' (No Crisis).
- vii. RJPR B2': Result of the Game for the President of the Republic in the B2' (No Crisis) situation.
- viii. RJPC B2': Match Result for the President of the Chamber of Deputies in B2' (No Crisis) situation.

Figure 1: Impeachment game - President of the Republic *versus* President of the Chamber of Deputies



Source: Prepared by the authors based on Kim (2006).

In a context of crisis represented by the left side of the "game tree" (Figure 1), what would be the reasons for the President of the Republic to be willing to propose an agreement to the President of the Chamber of Deputies to bar the process? Undoubtedly, such a question has numerous answers; some are (i) great possibility of federal deputies and later senators to vote for the president's removal; (ii) clear loss of governability by the President of the Republic and its party, being represented, among other points, by a large number of parties/parliamentarians willing to support the impeachment; (iii) existence of illegalities or crimes of responsibility capable of contributing to the removal of the President; (iv) low popularity of the government that could be an incentive to parliament to support the impeachment request; (v) interest of the President of the Chamber of Deputies in the removal of the President for various reasons, such as belonging to an opposition group, even if affiliated to the same party as the vice-president¹⁵; and (vi), perhaps to a lesser extent, the willingness of the vice-president to betray the President by supporting the impeachment.

The same reasons, however, could be given for the President of the Republic not being willing to propose an agreement with the President of the Chamber of Deputies: (i) little chance of federal deputies or later senators to vote for the president's removal because of the political articulation; (ii) governability on the part of the President of the Republic; (iii) absence of illegalities capable of contributing to the removal of the President; (iv) high popularity of the government, which could serve as a disincentive to parliament to support the president's removal; (v) little

¹⁵ For the Brazilian case in 2016, the president of the chamber of deputies belonged to the same political party as the vice president of the Republic.

interest of the President of the Chamber of Deputies in the removal of the President of the Republic for various reasons, such as belonging to the support base of the government or to the same party as the President; and (vi) low willingness of the Vice President to betray the President, perhaps explained by belonging to the same party¹⁶.

Once the President of the Republic's strategy has been signaled, the President of the Chamber of Deputies may decide whether or not to start the process for various reasons, such as (i) possibility of success in the president's removal due to facts presented and political, economic, and institutional conditions; (ii) interest in the president's removal resulting from political disputes; (iii) response to popular pressure and/or significant parliamentary appeal for openness; (iv) availability of collaboration of the Vice President of the Republic after the impeachment; and (v) defense of the public interest purely anchored in the accusations made in the process. The South Korea case in 2004, however, seemed out of touch with reality. The process was supported by more trivial issues, which, due to the possible hypotheses raised, can be classified as various political disputes.

If the president of the Chamber of Deputies is allied with the government, especially when it also belongs to the party of the President of the Republic, it is suggested that the difficulties for starting an impeachment process and/or its success would be greater. Because of this, but not only because of this, if the President of the Chamber of Deputies' office is a deputy allied to the government or, preferably, belonging to the same party as the President of the Republic, it fundamentally hinders the opening of any impeachment process. However, the most important thing is governability, reflected in parliamentary support and often directly related to the popularity of the government.

Therefore, in starting the impeachment process, the possible end results for the President of the Republic are being removed or not being removed from office. In the case of the President of the Chamber of Deputies, there is no way of affirming possible results, given that this is an incomplete game of information, whose results depend on the political conjuncture of each moment (Figure 1).

However, for the Brazilian case of 2016, the possible payoffs for the President of the Chamber of Deputies seemed clearer. Although the President of the Chamber of Deputies belonged to the vice president's party, he clearly assumed an oppositional role in relation to the President of the Republic, corroborating with the opening of the impeachment process, which occurred on the same day (02/12/2015), when there was the signaling of the party of the President of the Republic in the Ethics Council of the House, supporting the continuity of the process of annulment of the President of the Chamber of Deputies. Such events suggest a kind of retaliation by the President of the Chamber of Deputies. On the other hand, the party of the President of the Republic may have believed that it could have sufficient support to bar the process in the House or the Senate. The result was that both the President of the Republic and the Speaker of the House were removed from office, each in his own time, manner, and for different reasons.

Thus, an analysis of the political and economic situation at that time shows that regardless of whether the President of the Republic tried to propose an agreement to the President of the Chamber of Deputies or not, it was expected that the dominant strategy of the President of the Chamber of Deputies was to start the process regardless of what had happened to him or to the President of the Republic. However, it should be noted that generally, when the President of the Chamber of Deputies chooses to start the process and has an interest in the removal of the

¹⁶ In no direct elections in Brazil to the presidency (1989, 1994, 1998, 2002, 2006, 2010, 2014, and 2018) did the candidate or the victorious party have a vice president belonging to the same party.

President of the Republic, it is inferred that a more detailed observation of the political game takes place so that the opening of the process in fact results in the removal of the President of the Republic. In short, it is suggested that a process is hardly started when the chances of its prospering are small.

For the Brazilian case in 2016, the existence of a crisis has strengthened the impeachment process. In addition, the party of the President of the Republic signaled, through support for the inquiry in the House Ethics Council against the President of the Chamber of Deputies, that it would not propose an agreement, which led the President of the Chamber of Deputies to start the process on the very same day (02/12/2015).

There is, therefore, no way to confirm whether the impeachment process would start had the government proposed an agreement; this is more in the field of speculation. However, it is possible to affirm possible paths, as described in the "tree of the game" (Figure 1), that is, this is a sequential game of perfect but incomplete information. For the Brazilian case, referring to the post-re-democratization period and up to the time of writing this article, all impeachment processes accepted by the President of the Chamber of Deputies (1992 and 2016) resulted in the removal of the President of the Republic. Coincidentally or not, they were the most turbulent political and economic periods of the Brazilian economy after the 1985 re-democratization.

The same reasons could explain the development of the game on the right side of the "game tree" (Figure 1), considering however the absence of a crisis. In such context, the political, economic, and institutional conditions would not be favorable for the start and success of an impeachment process, and the path of the game is quite uncertain, especially since it hardly involves low government popularity. Moreover, the existence of parliamentary support, the popularity of the government, among other factors, hinders the success of the process. However, most impeachment processes in Brazil are requested; if political, economic, and institutional environments are not favorable for the president's removal, the President of the Chamber of Deputies files the process, not knowing for sure if there was a proposal of agreement by the President of the Republic.

At this point, it is worth commenting on the work of Kim (2006) on the impeachment process in South Korea in 2004. In this case, the game was directed to a side without a significant economic or institutional crisis, with very little popular support for the impeachment. There was therefore no significant crisis that could justify an impeachment process of the South Korean President, but an apparently banal and end-of-term issue. However, even though the President refused to apologize (did not propose an agreement), the process started, and the President was removed.

Perhaps the South Korean case is an example to explain that an impeachment process is possible even when there are no significant political, economic, or institutional crises; this is because the National Congress ultimately decides on the process viability: in the South Korean case, the National Assembly. However, this is not the most common situation, mainly because parliamentarians may not obtain popular support in the process, risking losing the next election, as pointed out by Kim (2006).

4.2 The Match Between the President of the Republic and the Vice President

This same game structure can be applied to the President of the Republic and its vice president (Figure 2). Nature is assumed to make the first strategic decision, providing the environment: With Crisis (CC) or Without Crisis (SC). In both opportunities or "strategic choices of Nature," there is the possibility of an impeachment request. Once the strategic decision of nature has been defined, the President of the Republic is the first to signal a strategy to the vice-president, and, as in the previous game, can propose an agreement (PA) or not propose an agreement (NPA).

In turn, the vice president may accept the agreement and not betray the President (NT) or not accept the agreement and betray the President (I); the payoffs for both are unknown since it is a game of perfect but incomplete information (Figure 2).¹⁷ In other words, the possible results, although they can be inferred, are not certain as they depend on the decision of the National Congress (House of Representatives and Senate) when the impeachment is voted.

According to the 1988 Federal Constitution, in the event of a vacancy of the President of the Republic, the impeachment is one way for the Vice President to take over the presidency. In such a context of gambling, what could lead the President of the Republic not to propose an agreement with the vice president to avoid treason?

Although it should be understood that in politics there are many variables, some can be pointed out: (i) high popularity of the President of the Republic, who is capable of making the impeachment process impossible and, consequently, betray the Vice President;¹⁸ (ii) the Vice President is from the same party as the President, which in a first analysis could make the process in the Congress meaningless;¹⁹ (iii) a President of the Republic's agreement with the Speaker of the House not to open the process; and (iv) sufficient parliamentary support to stop the impeachment either in the House and/or in the Senate.

In such a context, there would be no reason for the Vice President to betray the President of the Republic, as such an act could only lower its reputation with the President and perhaps also with the public opinion. Therefore, if there are no conditions for impeachment, the best the Vice President can do or its best dominant strategy is not to betray the President of the Republic regardless of any strategic decision of the President of the Republic: to propose an agreement or not to propose an agreement.

In the same sense, the betrayal of the President of the Republic only favors the vice president when the atmosphere of support for the impeachment is very favorable, i.e., there is a high probability that the removal of the elected President will occur. Thus, betraying the President of the Republic in such a situation could mean support from the public opinion and a sign of collaboration with the National Congress.

Therefore, for the President of the Republic, observing the games presented, the best thing the president can do is to propose an agreement to the President of the Chamber of Deputies to collaborate and not start the impeachment process. With this, the vice president will have no other better alternative but to continue collaborating and not betraying the President of the Republic.

¹⁷ In Figure 2, the acronym RJVP stands for Result of the Game for the Vice President. The acronym RJPR stands for Result of the Game for the President of the Republic, the same meaning as shown in Figure 1.

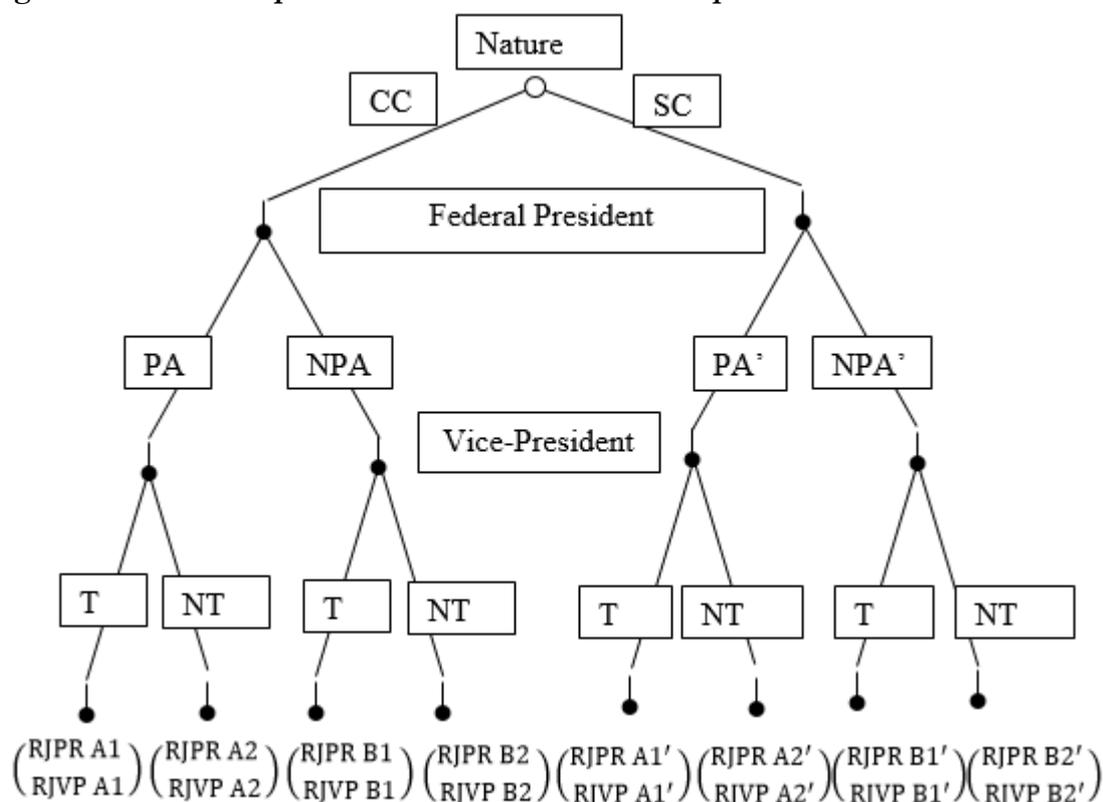
¹⁸ In other words, the vice president is a politically smaller figure than the president is.

¹⁹ That is, why depose the president if the vice is from the same party? Apparently, there would not be enough reasons for such a process.

On the other hand, once the impeachment process starts, there is no longer a game between the President of the Republic and the President of the Chamber of Deputies, but only with the vice president and, above all, with the National Congress, which is ultimately the one who decides the impeachment.

Observing the game between the President of the Republic and its deputy (Figure 2) and considering the start of the process in the Chamber, the deputy may choose two positions: betray (T) or not betray (NT). However, the president may maintain a position of neutrality since at least the vice presidency is guaranteed, and it is prudent to adopt a more conservative strategy. In such a situation, betrayal only becomes relevant when the game points towards the impeachment. The behavior of the game between the President of the Republic and the National Congress should thus be observed.

Figure 2: Game of impeachment - President of the Republic *versus* the Vice President



Source: Prepared by the authors based on Kim (2006).

Perhaps the vice president's position in the impeachment game is the most comfortable because the vice president either becomes the President with the removal of the president elect or remains as vice president with the permanence of the president elect. However, since the impeachment decision is of the slate, i.e., this decision can only be made by the Supreme Electoral Court, then the game's possible results may be less comfortable even when the president-elect has already been removed.

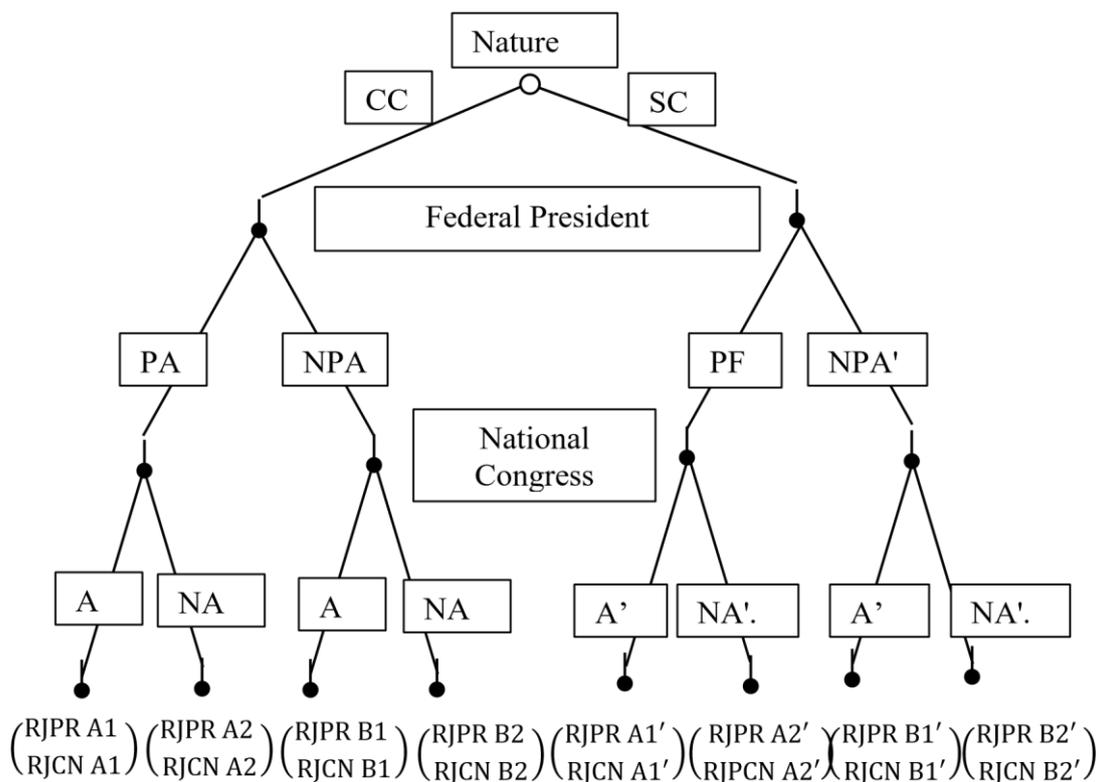
4.3 The match between the President of the Republic and the National Congress

The structure of the game between the President of the Republic and the National Congress is the same as previously presented. There are initially two scenarios: With Crisis (CC) and Without Crisis (SC). The game between the President of the Republic and the National Congress (Figure 3) only begins after the end of the game between the President of the Republic and the Speaker of the House and only if the Speaker of the House decides to start the impeachment process.²⁰

As before, the scenarios With Crisis (CC) and Without Crisis (SC), provided by nature, are a major factor in the game. Once the impeachment process starts, the President of the Republic has two possible strategies: (i) proposing an agreement with the National Congress (PA) to block the impeachment process either in the Chamber or the Senate; or (ii) not proposing an agreement (NPA).

Considering the composition of the Brazilian Congress, in which it is difficult for the party of the President of the Republic to obtain a majority in the Chamber or Senate in elections, it is natural for the President of the Republic and the president's party to adopt a dominant strategy of trying to reach an agreement with the National Congress to block the process (PA) either in the Chamber or the Senate (Figure 3). Once the agreement is proposed, Congress may choose to remove (A) the President of the Republic or not remove her (NA). In such a context, the payoffs of the President of the Republic are evident: she is removed from office if the Congress (House and Senate) approves her removal or, if not, she is not removed.

Figure 3: Game of impeachment - President of the Republic *versus* National Congress



Source: Prepared by the authors based on Kim (2006).

²⁰ In the game shown in Figure 3, the acronym RJCN stands for Result of the Game for the National Congress. The acronym RJPR stands for Result of the Game for the President of the Republic, the same meaning shown in Figure 1.

In order to prevent impeachment, it is important for the President's party to have a majority of allies in at least one of the legislative houses (the House or the Senate). If the House approves the impeachment, the Senate can stop it. On the other hand, if the House does not approve the impeachment, the Senate will not have to vote. For example, in 2020 there was a request for impeachment in the United States, where the process was approved in the House by a Democratic majority (opposition to the President). However, it was rejected in the Senate, with a Republican majority (political group supporting the President). Because of this, the game (Figure 3) takes into account the decision of the National Congress (House and Senate). If any of these houses do not approve the impeachment, the National Congress did not step down from government (NA).

Thus, for the President of the Republic, there are two games in which she can act to bar the impeachment. The first is established with the President of the Chamber of Deputies and the second with the National Congress. Among many aspects, the impeachment process can be considered a real possibility when (i) the mayor belongs to an opposition group, even though he belongs to the vice president's party; (ii) there are not enough parliamentarians to block the process in Congress; and (iii) popular support is small.

Regarding the relationship between the President of the Republic and the Vice President, the game is apparently not so relevant to the President of the Republic, as the Vice President alone does not have the authority to remove the President of the Republic. However, the idea of proposing an agreement could be strongly related to political issues, as the figure of the vice-president could signal to the National Congress an intention to remain faithful to the President of the Republic, which could perhaps lead the Congress to rethink the request for removal.

In such a context, the possible betrayal of the Vice-President is a sign of the continuity and success of the impeachment process. Perhaps a suggestion to make the Vice President's interest in betraying the President unfeasible would be a change in the impeachment law, pointing out that the President's impeachment also extends to the Vice President. However, the legal discussion on this point is deeper, in that possible crimes of responsibility can be committed by the President and not jointly by the vice president.

5. Brazilian History and Analysis of the 2016 Case

Requests for impeachment, especially in Brazil after the 1989 elections, are common independently from the ruling party (Table 1). The unusual is the acceptance of requests by the President of the Chamber of Deputies, which results in the removal of the President. Between 1990 and 2018, the number of impeachment requests was 193 (Table 1). Against the former President Fernando Collor de Mello, there were 29 requests, only one of which was accepted and culminated in the removal of the President for 180 days from 02/10/1992. Former President Fernando Collor de Mello resigned on 12/29/1992 (DEPUTY CHAMBER, 2020).

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Against former President Dilma Rousseff, there were 68 impeachment requests, of which only one (1) was accepted, triggering the definitive removal of the President by the Senate on 08/31/2016. Most requests against Dilma Rousseff were made proposing or not an agreement with the President of the Chamber of Deputies, showing that of the 68 times the game was played, the game resulted in the non-start of the process in 67 times, the result of which was the non-removal of the President (Table 1).²¹

Table 1: President impeachment requests in Brazil (1990-2018)

President	Period in Office	Number of Requests
Fernando Collor De Mello	03/15/1990 until 12/29/1992	29
Itamar Franco	12/29/1992 until 01/01/1995	4
Fernando Henrique Cardoso	01/01/1995 until 01/01/2003	24
Luiz Inácio Lula Da Silva	01/01/2003 until 01/01/2011	37
Dilma Rousseff	01/01/2011 until 08/31/2016	68
Michel Temer	08/31/2016 until 01/01/2019	31
Total requests		193

Source: Prepared by the authors based on House of Representatives (2020).

Therefore, in most cases, whether or not the President of the Republic has proposed an agreement, the Speaker of the House ordered filing the requests even in an economic, political, and institutional crisis, which shows that the assembled structure of the game has a significant capacity for explanation.

Considering that out of 193 requests only two have prospered, if such events apply to the proposed game, the strategic decision of the President of the Chamber of Deputies normally is not to start the process, with or without the President of the Republic's proposal of a deal. However, every time the process started, the National Congress removed the President. Therefore, it is always feasible for the President of the Republic to propose an agreement so that the process does not start, avoiding the subsequent game with the National Congress. In addition, the removal of the President is rarely not traumatic for the country, as it interferes with political and economic variables. The acceptance of any impeachment request is not recommended since many are based on political disputes.

Observing the game between the President of the Republic and the National Congress, every time the impeachment process started, regardless of whether or not an agreement was proposed, the President was impeached, which suggests that the strategic decision of the Congress is always to remove the president. However, it is important to point out that the processes that started took place in a political and economic conjuncture that was very favorable to the impeachment. Because of this, the impeachment process is not only influenced by legal issues but also, and perhaps to a greater extent, by political issues. In other words, the question of governability, the performance of the economy, and the popularity of the President are very important variables.

²¹ Information obtained from the General Secretariat of the Board of the Chamber of Deputies by e-mail: sgm@camara.leg.br and telephone (0xx61) 3216-1000 / 3216-1001. Detailed data on impeachment requests are not available on the website of the Chamber of Deputies. Such information is passed on when requested to the Bureau of the Chamber of Deputies. Any citizen can access information upon request based on the Access to Information Act. For further details, see Presidency of the Republic (2020).

O Jogo Político do Impeachment no Brasil e o Caso de 2016 Analisado sob a Ótica da Teoria dos Jogos

In this context, the analysis of Brazilian politics, especially from the 1989 presidential elections, shows that it is possible to win a presidential election without a great political support in coalitions. However, it is unlikely that the government will succeed without the support of other parties, especially when there is not much popular support. In the case of the 2014 election, although the party of the President of the Republic elected the largest chamber bench (69 deputies), it corresponded to only 13.5% of the Chamber of Deputies (Table 2).

If we include the political parties that belong to the winning coalition, the total reached 59.3% of the Chamber seats (304 out of 513). Therefore, this number is insufficient to ensure governability, the approval of government projects, or even to block a process of impeachment. In this sense, it is natural for the ruling party to seek support after the elections in order to broaden its support base. By way of simplification, all other parties that did not belong to the winning plate of the presidential election would assume the role of opposition. This, however, varies depending on the political negotiations.

The winning coalition of the 2014 presidential election also included the following parties: PMDB, PP, PSD, PR, PDT, PROS, and PC do B. All these parties, together with the Workers' Party (PT), elected 304 parliamentarians out of a possible 513 (59.3%) (Table 1). The Workers' Party (PT) and the Brazilian Democratic Movement Party (PMDB), the parties of the elected President and the vice president, respectively, formed the two largest benches in the Chamber (69 deputies of the PT and 65 of the PMDB).

However, for various reasons, but more especially because of the unfolding of Operation Car Wash,²² the Workers' Party (PT) and the then elected President in 2014, suffered severe political wear and tear. This led many parties, especially on the eve of the impeachment vote, to step off from the government and migrate from the situation to the opposition. With the decrease in support, it became increasingly difficult to keep the Workers' Party (PT) in power. If the ruling party eventually managed to maintain its support base, the impeachment process was very difficult unless PT had a member of its party in the chair of the House of Representatives.

The point here is that most parties that were part of the situation, realizing that the fall of President Dilma Rousseff was practically certain, concluded that to stay in the government meant being part of the opposition at the next moment. Soon, the strategy of leaving the government and then returning to it, but with another party in the presidency, seems to have been dominant in the parties with significant numbers of elected congressmen. On the other hand, there are examples of parties that did not adopt this strategy; they were left-wing parties such as PSOL and PC do B. These parties remained against the impeachment process until the end.

²² The name of the operation ("Car Wash") was due to the use of a commercial car wash company, whose service was washing vehicles, for money laundering.

Table 2: Composition of the Chamber of Deputies - 2014 Election - Number of Federal Deputies elected per party

Party	Federal Members Elected	Share % of the total	Cumulative %	Post-election 2014
PT	69	13.5%	13.5%	Situation
PMDB	65	12.7%	26.1%	Situation
PP	38	7.4%	33.5%	Situation
PSD	36	7.0%	40.5%	Situation
PR	34	6.6%	47.2%	Situation
PRB	21	4.1%	51.3%	Situation
PDT	20	3.9%	55.2%	Situation
PROS	11	2.1%	57.3%	Situation
PC do B	10	1.9%	59.3%	Situation
PSDB	54	10.5%	69.8%	Opposition
PSB	34	6.6%	76.4%	Opposition
PTB	25	4.9%	81.3%	Opposition
DEM	21	4.1%	85.4%	Opposition
SD	15	2.9%	88.3%	Opposition
PSC	13	2.5%	90.8%	Opposition
PPS	10	1.9%	92.8%	Opposition
PV	8	1.6%	94.3%	Opposition
PHS	5	1.0%	95.3%	Opposition
PSOL	5	1.0%	96.3%	Opposition
PTN	4	0.8%	97.1%	Opposition
PMN	3	0.6%	97.7%	Opposition
PRP	3	0.6%	98.2%	Opposition
PEN	2	0.4%	98.6%	Opposition
PSDC	2	0.4%	99.0%	Opposition
PTC	2	0.4%	99.4%	Opposition
PRTB	1	0.2%	99.6%	Opposition
PSL	1	0.2%	99.8%	Opposition
PT do B	1	0.2%	100.0%	Opposition
Total	513	100.0%		

Source: Own elaboration based on High Electoral Court (2019a; 2019b; 2019c).

The vote for the impeachment of President Dilma Rousseff, held in the Chamber of Deputies on April 17, 2016, was perhaps the best example of the shrinking support to her second term. There were 367 votes in favor of the process (71.5% of the 513 deputies) and 137 against it (26.7%); seven deputies abstained from voting (1.4%) and only two were absent (0.4%) (Table 2).

In the House, there were eight parties all of whose representatives voted for the impeachment: (i) PSDB, the largest opposition party with 52 votes; (ii) DEM, with 28 votes; (iii) PRB, with 22 votes; (iv) SD, with 14 votes; (v) PSC, with ten votes; (vi) PV, with six votes; (vii) PSL, with two votes; and (viii) PMB, with one vote. Such unanimous behavior by parliamentarians belonging to the aforementioned parties was expected, since most were not part of the coalition that elected Dilma Rousseff as President, and many did not compose the base of the government afterwards (Table 3). The exception was the PRB, which was part of the government's support base and was elected in the same coalition, but unanimously decided to support the impeachment. This is a clear case of a party that migrated from the government base to the opposition (Table 3).

Table 3: Political position of the political parties in the 2014 election and the 2016 impeachment

Party	Number of Federal Deputies Elected in 2014 (DFE)	Political Position after election 2014	Number of Federal Members on the day of the Impeachment (DFI) vote	Bench Variation = DFI - DFE	Voted for the impeachment	Voted against the Impeachment	Political position of to the party in the Impeachment
PT	69	ST	60	-9	0	60	ST
PMDB	65	ST	67	2	59	8*	OP
PP	38	ST	45	7	38	7*	OP
PSD	36	ST	37	1	29	8	OP
PR	34	ST	40	6	26	14*	OP
PRB	21	ST	22	1	22	0	OP
PDT	20	ST	19	-1	6	13*	ST
PROS	11	ST	6	-5	4	2	OP
B's PC	10	ST	10	0	0	10	ST
PSDB	54	OP	52	-2	52	0	OP
PSB	34	OP	32	-2	29	3	OP
PTB	25	OP	20	-5	14	6	OP
DEM	21	OP	28	7	28	0	OP
SD	15	OP	14	-1	14	0	OP
PSC	13	OP	10	-3	10	0	OP
PPS	10	OP	8	-2	8	0	OP
PV	8	OP	6	-2	6	0	OP
PHS	5	OP	7	2	6	1	OP
PSOL	5	OP	6	1	0	6	ST
PTN	4	OP	12	8	8	4	OP
PMN	3	OP	0	-3	-	-	-
PRP	3	OP	0	-3	-	-	-
PEN	2	OP	2	0	1	1	OP/ST
PSDC	2	OP	0	-2	-	-	-
PTC	2	OP	0	-2	-	-	-
PRTB	1	OP	0	-1	-	-	-
PSL	1	OP	2	1	2	0	OP
EN of B	1	OP	3	2	2	1	OP
PMB	0	-	1	1	1	0	OP
REDE	0	-	4	4	2	2	OP/ST
Total	513		513	0	367	146	

Source: Prepared by the authors based on House of Representatives (2020a).

Note: OP stands for "opposition" and ST stands for "situation."

On the other hand, every parliamentarian of only three parties voted against the impeachment process: (i) PT, with 60 votes, party of the President of the Republic; (ii) PSOL, with six votes, which was not part of the government but probably for ideological reasons did not support the impeachment, raising the "coup d'état" argument, as did PT; and (iii) PC do B, with ten votes, a former ally of PT who certainly would not be part of an eventual government of Michel Temer (PMDB), as in fact it did not (Table 4).

In the Federal Senate, in a voting session on August 31, 2016, President Dilma Rousseff was definitively removed from office with 61 votes in favor of the impeachment and 20 against it (Table 4). When analyzing the number of senators on the day of the final impeachment vote, the party position in the 2014 elections and its behavior in the vote on the removal process, one can conclude that 34 senators migrated from the situation to the opposition, five migrated from the opposition to the situation, and 42 kept their positions either in the situation or the opposition.

Table 4: Political position of parties in the 2014 election and the 2016 impeachment - Senate

Party	Number of Senators on the day of the final impeachment voting	Political position in the 2014 election	Number of Senators Who Voted for the Impeachment	Number of Senators Voting Against the Impeachment	Political Position of the party in the impeachment
PMDB	19	ST	17	2	OP
PSDB	11	OP	11	0	OP
PT	10	ST	0	10	ST
PP	7	ST	6	1	OP
PSB	7	OP	5	2	OP
PSD	4	ST	3	1	OP
PR	4	ST	4	0	OP
DEM	4	OP	4	0	OP
PDT	3	ST	3	0	OP
PTB	3	OP	1	2	ST
PSC	2	OP	2	0	OP
PRB	1	ST	1	0	OP
PC do B	1	ST	0	1	ST
PPS	1	OP	1	0	OP
PV	1	OP	1	0	OP
PTC	1	OP	1	0	OP
Without Party	1	OP	1	0	OP
REDE	1	OP	0	1	ST
Total	81		61	20	

Source: Own elaboration based on Federal Senate (2020).

Note: OP stands for "opposition" and ST stands for "situation".

Table 4 shows that proposing an agreement to avoid the impeachment process is a reasonable strategic decision on the part of the President of the Republic and the party in a scenario in which the number of senators belonging to the party, and therefore with real possibilities of blocking the process, is low. In the case of the Workers' Party (PT), there were only ten senators out of 81 in a scenario in which at least 28 senators were needed to block the process. Only 20 senators were opposed to the process filing. Apparently, it is assumed that the government has sought some sort of agreement to try to block the impeachment process both in the House and the Senate. In both houses and for various political reasons, the agreement was not accepted or was politically not advantageous. The result was the permanent removal of the President of the Republic, and the real payoffs of the other player, the National Congress, could not be attested.

6. Final Considerations

Because it involves political positions and not only legal aspects, the impeachment process, especially that of 2016 in Brazil, divides opinions. The existing publications are journalistic and unscientific, or they adopt some formal methodology to explain the process of removal of President Dilma Rousseff. Specifically in relation to the use of Game Theory to analyze the impeachment processes, Kim (2006) studied the case of South Korea in 2004, and that study was perhaps the only one whose main guidelines served to create the game presented in this work.

Requests for removal of a president in Brazil are not rare; there were 193 requests filed between 1990 and 2018. The rarity lies only in the acceptance and effective president removal.

However, the principle of democratic normality presupposes political stability and the fulfillment of the elected President's mandate, and the rarity of removal should be regarded as the rule and not the exception. Nevertheless, the presence of essentially democratic regimes in both Brazil and Latin America has not prevented the interruption of presidential mandates whether through legal means, such as impeachments or President resignation, or through military means, such as coups d'état or depositions. On this occasion, based on Kasahara and Marsteintredet (2018), between 1990 and 2018 Brazil was one of the Latin American countries that most often deposed an elected President via impeachment (1992 and 2016), together with Paraguay (1999 and 2012).

Thus, although the main contribution of this work is to offer a formal analysis of the 2016 impeachment using Game Theory as a methodology, the game presented here also explains the strategic behavior of the players every time they are called to play the "political game of impeachment." Between 1990 and 2018, the "political game of impeachment" between the President of the Chamber of Deputies and the President of the Republic was played 193 times, and in only two of them the President of the Chamber of Deputies adopted a strategic decision to start the process; this same number of times the National Congress decided to withdraw the request. In other words, in the vast majority of times that there was a denunciation against the President of the Republic the game was cooperative between the President of the Republic and the President of the House of Representatives, being non-cooperative on only two occasions. On the other hand, the game between the President of the Republic and the National Congress was always non-cooperative for the period under analysis and with the game considered here.

Therefore, one of the conclusions of this article is that governability is an essential factor for the President of the Republic, attesting that it is possible to win elections without political coalitions but that, without them, it is unlikely to be successful in governing. Further investigations on this issue are needed. Moreover, an analysis of Brazilian politics allows us to infer that there are three distinct groups of parties: opposition, situation, and adhesions to political parties. In such a context, the removal of parliamentarians from the government base was a largely dominant strategy of parties that could have the possibility of holding political positions in the next government, even though many of them already belonged to the winning coalition.

In addition, once the president was impeached, a protectionist behavior of the National Congress towards a "fearful government" emerged, which became visible in several votes incriminating it. Even the process of casting the winning ballot, as judged in the Superior Electoral Court (TSE), lost its force, probably influenced by the impeachment process of President Dilma. Finally, the importance of this method of analysis for various political situations should be highlighted, especially regarding the reasons that led the Congress to support an impeachment or not, as well as analyses regarding governability. These points are suggestions for future works.

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