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A REVISION ON THE WELFARE COSTS OF INFLATION AND SEIGNIORAGE IN BRAZIL: A MODEL WITH LEISURE

Uma Revisão sobre os Custos Sociais da Inflação e Senhoriagem no Brasil: Um Modelo com Lazer

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Abstract

This is a revised version of DIAS (1993) on the implications of inflation for the social-welfare of the Brazilian economy. It estimates the model with a new data set and measurements to the welfare loss overall society has due to high inflation rates, from 1973 to 1993 for Brazil. The main focus of analysis starts with the measurement of seigniorage. What happens to seigniorage at high inflation rates, and why might governments be tempted to over-inflate? The model presents a utility function with leisure depending on real money balances and consumption as in McCallum (1989). The theoretical propositions are useful to study economies with high inflation rates.

Keywords: Brazilian economy; Welfare costs of inflation and seigniorage; Leisure.

JEL Classifications: E31; E5; I3.

Resumo

Esta é uma versão revisada de DIAS (1993) sobre as implicações da inflação para o bem-estar social da economia brasileira. Este trabalho estima o modelo com um novo conjunto de dados, incorporando novas medições para os custos sociais da inflação para o Brasil no período de inflação alta, 1973 a 1994. A análise destaca a perda de bem-estar geral da sociedade em termos de redução do consumo agregado como percentual do PIB devido a altas taxas de inflação. O foco principal da análise começa com a medida da senhoriagem. O que acontece com a senhoriagem com altas taxas de inflação, e por que os governos podem ser tentados a inflacionar demais? O modelo apresenta uma função de utilidade com lazer dependendo dos saldos monetários reais e do consumo como em McCallum (1989). As proposições teóricas são úteis para estudar economias com altas taxas de inflação.

Palavras-chave: Economia brasileira; Custo social da inflação e senhoriagem; Lazer.

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1. Introduction

This paper captures the notion that an important motive for inflation is the revenue raising potential of the inflation tax. Particularly for developing economies, the inflation tax is an important revenue source for their governments. The additional advantage of the inflation tax as a source of revenue is that it is not necessary to legislate it. Our objective is to analyze the implications of inflation to seigniorage and to the social welfare. We concentrate in the Brazilian economy because of its high inflation rates over the past decades. The vehicle for this analysis is the money demand equation. This is one of the reasons for having real money balances in the utility function. In this context, the model describes an economy in which the typical household has in his utility function leisure depending on consumption and real money balances. After deriving the first-order conditions, we can find the equation of demand for real money balances. The steady-state demand for real money balances and the definition of seigniorage allow us to analyze the seigniorage ratio. In addition, the demand for money and the utility in the steady state provide us tools to derive the loss function of social welfare measured as the decrease in consumption per GNP when inflation goes from zero to a positive number.

Since the known theory for economies with hyperinflation was presented by CAGAN (1956), the link between money and prices has received considerable attention. Equally interesting is the study of those ingredients necessary to terminate inflation. As pointed out by CAGAN, another explanation for the large issue of money, and consequently for the large increase in inflation, in the postwar periods, is the response of the tax base (real money balances) to the tax rate (growth rate of prices). CAGAN's findings describe two characteristics that are summarized by the pattern of time series for money and prices:

... (1) the ratio of the quantity of money to the price level - real cash balances - tended to fall during hyperinflation as a whole but fluctuated drastically from month to month and (2) the rates at which money and prices rose tended to increase and in the final months preceding currency reform reached tremendous heights. This second pattern supplies the identifying characteristic of hyperinflation, but the explanation of the first holds the key to an explanation of the second and logically comes first in order of presentation.

According to BARRO (1979), optimal fiscal policy requires that the business cycle effects of a temporary tax change be isolated. That is, it is necessary to smooth tax rates over time. As he pointed out, an important proposition is that deficits are varied to maintain expected constancy in tax rates. Also, he added that

This behavior implies a positive effect on debt issue of temporary increases in governments spending (as in wartime), a countercyclical response of debt to temporary income movements, and a one-to-one effect of expected inflation on nominal debt growth.

He tested the hypotheses with time-series data on the public debt issue in the United States since World War I. His result was that the historical data provides no evidence about the impact of temporary changes in federal taxes. The model assumed that government finances its expenditures through two methods: current taxation and public debt issue. Although he did not consider currency issue, he proposed that the model could be modified to account for this variable. This is what MANKIW (1987) presented.

MANKIW (1987) extended BARRO's framework to account for the use of the inflation tax as a source of revenue by governments. In line with a classical model and given exogenous expenditure and an intertemporal budget constraint, MANKIW found that government chooses

the rate of taxation and inflation to minimize the present value of the social cost of raising revenue. According to his model, government raises revenue from two sources, such as: a tax on output (income tax or sales tax) and seigniorage. The tax smoothing of optimal fiscal policy investigated by BARRO was represented by a first-order condition that equated the marginal social cost of taxation today and in the future in MANKIWI's framework. His objective was to show that the optimal collection of seigniorage over time implies that inflation and nominal interest rates are approximately random walks. This purpose was achieved. Also, the theory implied that nominal interest rates and inflation move with tax rates. To investigating these results empirically, he used time-series data from 1952 to 1985 for the U.S. economy. Accordingly, an increase in federal government revenue of 1 percent of GNP raises the nominal interest rate by 1.1 to 1.4 percentage points. Furthermore, the suggestion is that the revenue requirement can explain approximately one third of the variation in the nominal interest rate. Finally, the theory of optimal seigniorage supplies a partial explanation for fluctuations in money growth, inflation, and nominal interest rates.

Other researchers give different views to the analysis of seigniorage. According to CUKIERMAN, EDWARDS, AND TABELLINI (1992), seigniorage is an inexpensive source of government revenue. Also, its importance concerning other sources of government revenue differs across countries. With a political model of tax reform, they tested cross-sectional data for 79 countries. The prediction was that countries with more unstable and polarized political systems have more inefficient tax structure and rely more heavily on seigniorage. They defined the degree of political instability as the probability of a government change as perceived by the current government. And, polarization is the frequency of government changes. When polarization takes the form of a coup, it has greater effects on the reliance of seigniorage than when polarization takes the form of regular (more democratic) changes.

In contrast to the work discussed above, COOLEY AND HANSEN (1991) demonstrated that the revenue from the inflation tax can become an important part of total revenues also in a politically stable system as United States. The reason is that there is no legislation involving this source of revenue. The model they analyze is very different from CUKIERMAN, EDWARDS, AND TABELLINI. The objective of COOLEY AND HANSEN is to address the issue of the revenue and welfare costs of policies to reduce moderate inflation rates to zero. The vehicle for analysis is an equilibrium growth model with money. Money is introduced by a cash-in-advance constraint. That is, consumption is a "cash good" while leisure and investment are "credit goods". They are related to each other by a log-linear utility function. In addition, there are no uncertainty and no government debt. The idea was to address the policy issue corresponding to the proposal that Federal Reserve should be required to reduce inflation to zero. They used a calibration procedure that chooses values based on observed features of the data for the U.S. during the postwar period. They showed that policies designed to reduce moderate inflation supply only small improvements in welfare. Also, the presence of other distorting taxes doubles the estimated welfare costs of the inflation taxes and decreases the revenue potential of the tax. In addition, they demonstrate that the welfare costs of inflation in percent of GNP with and without taxes is an increasing function of the annual inflation rate in the U.S. economy. Finally, they

...show that policies that reduce the inflation rate to zero only temporarily actually make the economy worse off because of the intertemporal substitution that takes place.

In general, reducing the cost of being subject to inflation rates is one of the objectives of monetary policy. CARRASQUILLA, GALINDO, AND PATRÓN (1994) conducted a study in Colombia, the authors found a loss of social welfare of 7% of GDP when the inflation rate increased from 5% to 20%.

POSADA (1995) and RIASCOS (1997), based on the Colombia dataset and considered the inflation rate of 20%, their estimations were 3.9% and 1.5% of GDP, respectively.

LÓPEZ (2000) calculates the seigniorage rate and the welfare cost of the inflation rate in Colombia using the Sidrauski's model. The estimated parameters imply a considerable cost of welfare for inflation and seigniorage. When there is low inflation, seigniorage increases with increasing inflation and, when inflation is high, it reaches an asymptote.

For the Brazilian economy, the results by PASTORE (1997) estimating the welfare cost of moderate inflation rates approached the results by SIMONSEN AND CYSNE (1994) - the inflationary average close to 40% per year. These were not far from those by ROSSI (2003). ROSSI (2003) estimated the "shoe-leather" costs, using quarterly data from 1966 to 1985, comparing with other results in the literature. PASTORE (1997), using the partial equilibrium analysis, indicated the welfare cost of inflation around 8%.

POLATO AND FAVA (2002), using Sidrauski's (1967) general equilibrium model, with quarterly data from 1975 to 1996, implied the welfare cost of inflation rate of 10% per year close to 2% of GDP for Brazil. This result was higher than those by ROSSI (2003), around 1.3% of GDP. For high inflation rates (between 300% and 400% annually), POLATO AND FAVA (2002) pointed out the welfare cost of inflation from 4% to 5% of GDP in Brazil.

YOSHINO (2002) estimated higher welfare costs to high inflation rates. Throughout a general equilibrium model, he estimated the costs of the annual inflation rate of 2.590% being from 16% to 39% of GDP. Note that his estimations included other distortions of inflation, such as "shoe-leather" costs and from the banking sector.

POLATO AND FAVA, AND ROCHA (2003) estimated the welfare cost of the inflationary tax for the Brazilian economy using the general equilibrium approach and comparing the results with those obtained with the partial equilibrium approach. They used quarterly data, beginning in 1975 and ending in 1996. The result was that, with 10% annual inflation, the loss of welfare was around 2% of GDP. Moreover, this paper is a revised version of DIAS (1993) on the implications of inflation for the welfare of the Brazilian economy. The analysis here is concerned with the welfare loss overall society has due to high inflation rates. The main focus of analysis start with the measurement of seigniorage. What happens to seigniorage at high inflation rates, and why might governments be tempted to over-inflate? The model presents a utility function with leisure depending on real money balances and consumption. The theoretical propositions are useful to study economies with high inflation rates.

This study is divided into five sections, in addition to this introduction and conclusion. The paper begins with the presentation of the basic model; then we have Seigniorage and Welfare Costs of Inflation; then, Empirical Analysis; right after the Results for the Basic Model; followed by Implications for Seigniorage and Welfare Costs of Inflation.

2. The Basic Model

In this model leisure is assumed to be a positive function of real money balances and inversely related to consumption expenditures. This is one way to introduce the role of real money balances in utility functions. Therefore, the utility function representing households' preferences is

$$E_t \sum_{t=0}^{\infty} \beta^t U(l_t, c_t) \quad (1)$$

where E_t represents expectations conditional on information available at time t , β is a subjective discount factor, m is real money balances, c is consumption services, and $U(.)$ is a concave utility function that is increasing with respect to real money balances and consumption. Thus, the consumer maximizes the present discounted value of expected utility, conditional on information at time zero. l_t represents leisure in period t and is equal to $l(m_t, c_t)$ and $l_1 > 0$ and $l_2 < 0$. As real money balances increase, the shopping time spent to purchase any commodity decreases and more time is left to enjoy leisure. However, as consumption expenditures increase, more shopping time is spent if the number of transactions increases, and then the time left to enjoy leisure decreases. Besides, as consumption expenditures increase, more time has to be spent working to replace part of the income spent with such expenditure and then to guarantee some consumption tomorrow. This again implies less time left to enjoy leisure.

The definition of c_t as consumption services is a way of avoiding the influence of business cycle fluctuations in this model. Empirically, consumption of services changes by less than the full changes in GNP. That is, services purchases do not depend strongly on variations of disposable income.³ Consumption of nondurables fluctuates more than services purchases but less than consumption of durables. Therefore, over time, total consumption purchases fluctuate less than GNP. The same is true for consumption expenditures. Furthermore, consumption expenditures vary more than consumption purchases because of the presence of durable goods.⁴ Consequently, we use services and total consumption purchases instead of expenditures.

The household's budget constraint for period t is given by

$$b_{t-1}(1 + r_{t-1}) + m_{t-1}(1 + \pi_t)^{-1} + y_t = b_t + m_t + c_t \quad (2)$$

where the sources of funds include the real values of bonds and money that were held last period adjusted for interest and inflation, $b_{t-1}(1 + r_{t-1}) + m_{t-1}(1 + \pi_t)^{-1}$, summed with the real income from other sources, y_t . π_t is the inflation rate from period $t-1$ to t . Therefore, the inflation rate (π_t) is equal to $\left[\left(\frac{p_t}{p_{t-1}}\right)^{-1}\right]$. The real interest factor $(1 + r_{t-1})$ is equal to $\frac{(1+R_{t-1})}{(1+\pi_t)}$, where R_{t-1} is the nominal interest rate on assets held from time $t-1$ to t . The uses of funds in real terms are the real values of one-period financial assets, b_t , real money balances, m_t , and the consumption chosen by household in period t , c_t .

The maximization of this model is carried out with dynamic programming that reduces multiperiod problems to a sequence of simpler two-period problems. Our objective function is equation (1) subject to the budget constraint (2). Solving the budget constraint for c_t and substituting the result into equation (1), differentiating with respect to b_t and m_t , and rearranging the resulting equations, we have the following first-order conditions:

$$-\frac{U_{12}(t)}{U_2(t)} + \beta E_t \left\{ \left[\frac{U_2(t+1)}{U_2(t)} + \frac{U_{12}(t+1)}{U_2(t)} \right] (1 + r_t) \right\} - 1 = 0 \quad (3)$$

³For example, consumers will not purchase more health services because their disposable income is higher. They will do so when they are sick or need it.

⁴It takes more time to actually consume a durable good than to spend the money to buy it.

$$\frac{U_{11}(t) - U_{12}(t)}{U_2(t)} + \beta E_t \left\{ \left[\frac{U_2(t+1)}{U_2(t)} + \frac{U_{12}(t+1)}{U_2(t)} \right] (1 + \pi_{t+1})^{-1} \right\} - 1 = 0 \quad (4)$$

where $U_{ij}(t+s)$ is the marginal utility of the i 'th argument ($i = 1,2$) with respect to the j 'th argument ($j = 1,2$) evaluated at time $t+s$ ($s = 0,1$). For example, $U_{11}(t+1)$ is the marginal utility of leisure with respect to real money balances evaluated at time $t+1$. Put another way, this is the utility from extra leisure provided by an incremental unit of real money balances held. Equation (3) is the first-order condition for maximization with respect to one-period financial assets b_t . It represents the relationship between current consumption expenditures and future consumption, as they affect utility direct or indirectly through leisure. It shows that if the household decides to give up the direct and indirect marginal utility of *current* consumption, he then will gain the expected marginal utility of *future* consumption *net* of expected marginal cost (in utility units) from leisure sacrificed by an incremental unit of consumption. Equation (4) is Euler's equation with respect to real money balances (m_t). It implies that the cost of reducing direct and indirect marginal utility of *current* consumption is equal to the gain of utility from additional leisure. This additional leisure is provided by an extra unit of money held plus the expected marginal utility of *future* consumption *net* of the expected marginal utility cost from leisure sacrificed by an additional unit of consumption.

Inspection of the first-order conditions reveals that the necessary conditions for household optimization involve five variables: c_t , c_{t+1} , m_t , R_t , and π_{t+1} . If the two equations above can be solved for real money balances, we can use them to derive the portfolio balance relationship. In the special case in which the nominal interest rate is known, the Euler equations above can yield a portfolio balance relation. Solving equation (3) for $E_t[U_2(t+1) + U_{12}(t+1)(1 + \pi_{t+1})^{-1}]$, and substituting it into equation (4), we have

$$\frac{U_{11}(t)}{U_2(t) + U_{12}(t)} = \frac{R_t}{(1 + R_t)} \quad (5)$$

The equation above shows that the marginal utility of leisure provided by an extra unit of real money held divided by the marginal utility of consumption plus net marginal utility of indirect consumption must be equal to the interest earnings foregone per unit of real money held. If consumption expenditures are a relevant measure of expenditure volume, this equation can be a representation of money demand function.

In line with this framework, consumer preferences are assumed to be

$$U(l_t, c_t) = \frac{[l_t^\gamma c_t^{1-\gamma}]^\theta - 1}{\theta} = \frac{\left[\left(\frac{m_t}{c_t}\right)^\gamma c_t^{1-\gamma}\right]^\theta - 1}{\theta} \quad (6)$$

where γ and θ are preference parameters, $0 < \gamma < 1$ and $\theta < 1$. This has the basic economic properties of simple constant relative risk aversion utility functions. The elasticity of substitution between consumption at any two points in time, t and $t+s$, is constant and equal to $\left[\frac{1}{(1-\theta)}\right]$. Therefore, the parameter $1 - \theta$ represents the inverse of the elasticity of intertemporal substitution. Because of the presence of uncertainty in this model, there is an alternative interpretation for θ . $1 - \theta$ is also a coefficient that reflects the attitude of the consumer toward risk. It is the coefficient of relative risk aversion. Recall that we are looking for the optimal decisions

of the household, given his budget constraint and uncertainty. Under uncertainty and considering labor income risk diversity⁵, the consumption behavior of the household is similar to that under certainty. Consumption is a linear function of wealth (labor income, financial assets, and other sources of income). The household's marginal propensity to consume out of wealth depends on the expected rate of return (the expected interest rate) on his portfolio. The sign of the effect of changes in the expected rate of return depends on whether the degree of risk aversion is greater or less than one. Notice that the analysis of risk aversion is equivalent to the analysis of the elasticity of substitution between consumption today (period one) and consumption tomorrow (period two). If the interest rate increases, the price of second-period consumption decreases. If the elasticity of substitution between consumption in both periods is greater than one, the substitution effect of this increase in the interest rate dominates the income effect. Hence, the sign of the effect of this change on the marginal propensity to consume today is negative. Thus, savings will increase in period one.

The marginal utilities presented in the first-order conditions (3) and (4) are shown in terms of parameters by

$$U_{11}(t) = \gamma(m_t)^{\gamma\theta-1}(c_t)^{\theta-2\gamma\theta} \quad (7)$$

$$U_{12}(t) = -\gamma(m_t)^{\gamma\theta}(c_t)^{\theta-2\gamma\theta-1} \quad (8)$$

$$U_2(t) = (1-\gamma)(m_t)^{\gamma\theta}(c_t)^{\theta-2\gamma\theta-1} \quad (9)$$

Here, if θ equals zero, $U_{11}(t) = \gamma(m_t)^{-1}$, $U_{12}(t) = -\gamma(c_t)^{-1}$, and $U_2(t) = (1-\gamma)(c_t)^{-1}$, respectively. Therefore, this would correspond to a log-utility representation, such as: $(.) = (1-\gamma)\log c_t + \gamma\log m_t - \gamma\log c_t$.

3. Seigniorage and Welfare Costs of Inflation

If we compare this model's steady states for different rates of inflation, we can derive the implications for seigniorage revenue and welfare costs of inflation. To see this, let us start by assuming that consumption and real money balances grow at the same constant rate $\phi > 0$. In addition, assume that all real variables are invariant with respect to steady-state changes in the rate of inflation. Using this information and equation (4), solving for the steady-state value of real money balances, and rearranging the result, the steady-state portfolio balance relation is⁶

$$m = \frac{\gamma c}{(1-2\gamma) \left[1 - \frac{\alpha_3}{(1+\pi)} \right]} \quad (10)$$

where $\alpha_3 = \beta(1+\phi)^{\theta(1-\gamma)-1}$, and c and π are the steady-state values of consumption expenditures and inflation, respectively. Recall that γ is restricted to be in the open interval $(0,1)$.

⁵According to BLANCHARD, O. J. AND FISCHER, S. (1989, p. 283), labor income risk diversity means that individuals are always able to diversify part of their labor income risk by holding a portfolio of financial assets whose returns are negatively correlated with labor income.

⁶The information above implies that $c_{t+1} = (1+\phi)c$ and $c_{t-1} = \left[\frac{1}{(1+\phi)} \right] c$ in the steady state.

If the parameters γ , β , ϕ , and θ are invariant to steady-state changes in the inflation rate (π), the absolute value of the elasticity of the demand for money is

$$\eta \equiv \left| \frac{\partial m}{\partial \pi} \frac{\pi}{m} \right| = \left| -[(1 + \pi)\alpha_3^{-1} - 1]^{-1} \left(\frac{\pi}{1 + \pi} \right) \right| \quad (11)$$

The equation above shows that the elasticity of demand for real money balances depends on the steady-state inflation rate and the parameters β , ϕ , θ and γ .

In what follows, we define the necessary framework to analyze the consequences for seigniorage and the welfare costs of inflation. Seigniorage (S_t) is the government's revenue from the creation of money. When government prints money to purchase real resources, it is the same as a transfer that the private sector has given to the government. This transfer is an imposed tax. We call this kind of tax of seigniorage. Furthermore, the inflation resulting from new money imposes a tax on real money balances by depreciating the value of money. The revenue in real terms is the product of the growth rate of prices (the tax rate) and real money balances (the tax base). Government determines the rate of the tax through the process implied by the demand for real money balances function by setting the growth rate of the quantity of money.

In developing and unstable economies, government frequently chooses seigniorage as a source of revenue. This is so because it is too costly to modify the fiscal system and rearrange the general economic system to increase government revenue. In addition, to operate any economic reform, government needs to spend resources that are short in those types of economies. Furthermore, there is no incentive to do so because the instability of the system may not help the current government to be reelected.⁷

Inflation reduces the real value of the currency and individuals will try to minimize the amount of money in their possession, in order to mitigate the loss of purchasing power. With this, individuals need to go to banks more often to apply their income to investment funds or even to search for lower prices when they go shopping, and thus their leisure time decreases - since the time of purchases is discounted from leisure. The more visits to banks, reduce the amount of time available for productive activities. That is, it can be said that the shoe-leather costs is related to all the effort that the individual accomplishes in order to mitigate losses with inflation, that is, costs of time and energy (MANKIW, 2008).

Seigniorage is an endogenous variable related to the inflation rate and the demand for money balances. We can represent seigniorage (S_t) by

$$S_t = \frac{M_t^s - M_{t-1}^s}{P_t} \quad (12)$$

Multiplying equation above by $\frac{M_t^s}{M_t^s}$ and arranging the result, we have

$$S_t = \frac{M_t^s - M_{t-1}^s}{M_t^s} \frac{M_t^s}{P_t} \quad (13)$$

⁷More on this issue is stressed in CUKIERMAN, A., EDWARDS, S., AND TABELLINI, G. (1992). They show that seigniorage is an inexpensive source of government revenue relative to others for 79 countries. They also find that the equilibrium efficiency of the tax system and, therefore, of seigniorage also depends on political stability and polarization.

where M_t^s is the monetary base of period t , and $\frac{M_t^s}{P_t}$ is equivalent to m_t^s , which is its real value. The s superscript is to remind us this is a measure of money supply. Recall that real money balances grow at the constant rate ϕ in the steady state. Also recall that the steady-state value of the inflation rate $\left[\left(\frac{P_t}{P_{t-1}}\right) - 1\right]$ is π . In addition, we know that demand and supply of money grow at the constant rate μ . The gross rate of change of the monetary base $\left(\frac{M_t^s}{M_{t-1}^s}\right)$ is therefore equal to $(1 + \phi)(1 + \pi)$.

Transforming equation (10) into the steady-state supply of money requires the multiplication of the volume of transactions (c) to the inverse of the money supply multiplier, v . The procedure to get the seigniorage ratio ($\Re$) follows. First, solve equation (13) for its steady state. Second, divide the result by gross national product. Third, substitute for m the supply of the real monetary base in the steady state obtained from equation (10). Thus, the ratio of seigniorage to GNP is

$$\Re = \left[1 - \frac{1}{(1 + \phi)(1 + \pi)}\right] \left[\frac{\gamma \vartheta v}{(1 - 2\gamma) \left(1 - \frac{\alpha_3}{1 + \pi}\right)} \right] \quad (14)$$

According to equation above, when steady-state inflation (π) increases, the inflation-tax rate raises the seigniorage ratio. The effect of the base tax rate in the seigniorage ratio will depend on whether γ is close to one or close to zero. For example, if γ is equal to 0.5, the base tax rate is equal to $\left(\frac{\gamma \vartheta v}{\phi}\right)$. For this base tax rate, the seigniorage ratio can not be determined. If $0.5 < \gamma < 1$, any increase in the steady-state level of inflation will increase the seigniorage ratio ($\Re$). On the other hand, if $0 < \gamma < 0.5$, increases in π cause the seigniorage ratio described by equation (14) to decrease. These examples considered that α_3 is positive. This condition is met when $\beta > 0$ and $-1 < \phi < +\infty$.

Let us use the following welfare loss function ($\Im$) to calculate the welfare costs of various steady-state levels of inflation:

$$\Im = \left(\frac{U_{\pi=0} - U_{\pi>0}}{U_{\pi>0}} \right) \frac{c}{y} \quad (15)$$

The welfare loss function above incorporates the percentage decrease in utility translated into consumption per GNP. In the above equation, $\frac{c}{y}$ is the ratio of consumption to GNP. U is obtained in the equation above by first solving the utility function (6) for its steady state. Second, we substitute the steady-state portfolio balance relation (10) into the resulting utility function. Finally, we use the resulting equation to get the welfare loss function expressed by the percentage decrease in consumption when the steady-state level of inflation goes from zero to a positive number. Notice that consumption with positive inflation is lower than consumption with zero inflation rates in the steady state. The resulting full-fledged social welfare loss function for this framework is

$$\Im = \vartheta \left\{ \left[\frac{1 - \alpha_3(1 + \pi)^{-1}}{1 - \alpha_3} \right]^{\gamma \vartheta} - 1 \right\} \quad (16)$$

where ϑ is the ratio of consumption per GNP, $\frac{c}{y}$. When the steady-state inflation rate increases, the welfare loss function increases. Thus, the loss is higher to social welfare measured as the percentage decrease in utility translated into consumption per GNP.

4. Empirical Analysis

The model presented in last section was estimated using Brazilian quarterly data. We tested two time periods, from 1973:I to 1994:IV and from 1980:I to 1994:IV. The Brazilian economy is the object of study because of its high inflation rate in the 1980s. The causes and implications of high inflation have complicated economic growth in that economy.⁸ Brazilian authorities' interventions with the political instability have contributed to the aggravation of many economic problems in the country. The estimates of the parameters are useful to analyze the consequences for seigniorage and the welfare costs of inflation to the social welfare in the Brazilian economy.

As in the first version, the econometrics methodology is the Generalized Methods of Moments (GMM). It is based on HANSEN (1982) and HANSEN AND SINGLETON (1982).⁹

As pointed out by HALL (1987), to test a theory, one does not need to derive an explicit solution for consumption or saving choices under uncertainty. All someone needs is to test the first-order conditions. If the first-order conditions fail the test, we can reject the theory.

The estimation procedure through GMM allows us to estimate the model directly from first-order conditions derived from maximization procedures of a representative household. Through this method, a chosen set of n instruments can form n orthogonality conditions for each of the first-order conditions. That is, we use instrumental variables to minimize the errors when estimating the first-order conditions through the GMM procedure. The estimators of this method coincide with three stages least squares (3SLS) estimators when the errors are serially independent and the same instruments are used for each equation. The nonlinear estimates obtained through this procedure are consistent and asymptotically efficient. When there is no serially correlation among the errors and we use the same instruments to estimate each equation, the estimates of GMM are also numerically identical to the 3SLS ones.

Following HANSEN (1982), we choose b_T belonging to the same space to minimize the following objective function:

$$J_T(b) = g_T(b)'W_T g_T(b) \quad (17)$$

where W_T is a weighting matrix that is symmetric, has dimension r by r , and is positive definite. b is a parameter vector that is unknown. $g_T(b)$ is the estimator evaluated at $b = b_0$, $g_T(b_0)$, which should be close to zero for large values of T . It can depend on the sample information. This objective function converges in distribution to a stochastic variable distributed as chi-square with degrees of freedom equal to $n-1$, which can test the validity of the overidentifying restrictions. Notice that n is the number of instrumental variables.

⁸WACHTER, S. M. (1976, p. 2). Accordingly, Latin American monetarists argue that inflation can bring economic uncertainty with the result that investment and economic growth decline.

⁹ For more details on the method see Dias (1993).

The computer program used to estimate this model was Time Series Processor (TSP). This package can perform the General Methods of Moments estimation, with standard errors robust to heteroscedasticity and autocorrelation.

Estimating this model we used the data series as follows. The variable price level (P_t) is general price index (IGP-DI) from Conjuntura Economica magazine of Getulio Vargas Foundation. Two series account for nominal money (M_t), monetary base and $M1$, in dollars terms by the exchange rate, from Central Bank of Brazil. Consumption is households' total consumption from IPEA. To the Gross National Product we used Gross Domestic Product (GDP) also from IPEA. A continuum of data for the nominal interest rate series came from Central Bank of Brazil Bulletins with the one paid to overnight operations.

The vectors of instruments used to estimate the first-order conditions (3) and (4) are the following: $z_{t1} = \left[1, \frac{c_t}{c_{t-1}}, \frac{M_t}{M_{t-1}}, (1 + r_t)\right]$ and $z_{t2} = \left[1, \frac{y_t}{y_{t-1}}, \frac{c_t}{c_{t-1}}, \frac{M_t}{M_{t-1}}\right]$. In this framework, the parameters to be estimated are θ , γ and β . The estimates of θ , γ and β and the value of the objective function J_T are displayed in Table 1.

Table 1: Estimates for the Brazilian Economy from 1973 to 1994

Parameters	Mon. Base	M1	Mon.Base & M1	Mon. Base & M1
	Real Int.Rate	Real Int.Rate	Real Int. Rate	Log Int. Rate
	A	B	C	D
β	0.99 (0.0076)	0.995 (0.0072)	0.99 (0.0063)	0.97 (0.0015)
θ	0.77 (0.27)	0.70 (0.27)	1.09 (0.23)	1.08 (0.055)
γ	0.0068 (0.0004)	0.014 (0.0005)	0.015 (0.0005)	0.017 (0.0006)
J	4.33	3.85	5.68	5.57
$\rho(d1, d2)$	(0.003,0.15)	(0.003,0.013)	(0.003,0.013)	(0.0002,0.015)

Source: Research Data.

Note: J_T is the value of the objective quadratic function. The degree of freedom to test J_T is equal to the number of instruments minus one. The set of instruments is the one reported in the text. $\rho(d1, d2)$ is the correlation between the estimated residuals. $d1$ is the residual of the first-order equation with respect to b_t and $d2$ is the residual of the first-order equation with respect to m_t . Student t-statistics of the estimates are given in parentheses.

All estimates are statistically significant. The degrees of freedom to test the over identifying restrictions in this model is three. For 5 percent level of significance J_T must lie in the interval (0.352, 7.815). Then, the model's overidentifying restrictions were not rejected by the sample information. Other vectors of instruments were also tested. They all gave similar results. The values and significance of γ and β are very stable. The oscillation of θ has the size of its standard deviation. We chose the result reported in column A of Table 1 because it implies a concave utility function. In fact, if we use standard deviation of all results we got, the estimate of θ would be close to unity. Yet, we make some comments and calculations using all columns of Table 1.

5. Results for the Basic Model

As indicated above and using the results of Table 1, the utility function described by equation (6) with some arrangements is now equal to

$$E_t \sum_{t=0}^{\infty} \beta^t U(l_t, c_t) = E_t \sum_{t=0}^{\infty} 0.99^t \left(\frac{m_t^{0.01} c_t^{0.98}}{0.77} - 1.3 \right) \quad (18)$$

Notice that the estimate of θ , 0.77 implies that the elasticity of substitution between consumption in period t and period $t+1$ is about 4.35, therefore elastic. In addition, the degree of relative risk aversion $(1 - \theta)$ is 0.23. This implies that the typical household is a risk taker. These results may be a consequence of incorporating leisure in this framework. If this household increases leisure in this period, the time left to work is lower. Consequently, less will be left to the household to enjoy as consumption next period. Furthermore, equation (18) also shows that consumption purchases represent higher direct utility than real money balances. The reason may be that, in this new framework, people do not want more leisure but more consumption. The estimate of β implies a quarterly subjective discount rate of 1.01 percent.

6. Implications for Seigniorage and Welfare Costs of Inflation

In the light of equations (10) to (16), we can analyze the implications for seigniorage from changing steady-state values of inflation. They also allow us to analyze the costs of inflation for social welfare. This analysis is enhanced with the parameter estimates of the last section. We had the quarterly sample means of final consumption of households and GDP in dollars terms. These variables came from the same source and used the same calculation technique. We computed the sample mean of the share of final household consumption of GDP. The result was that consumption accounts for 81 percent of GDP (9). We assumed that the steady-state growth of consumption (ϕ) was equal to the quarterly sample mean of the rate of change in the consumption. This was equal to 0.011. For the inverse of the money supply multiplier $\left(\frac{1}{V=v}\right)$, we also used the sample mean, which was equal to 0.51.

Given the above information and Table 1, we can compute equations (11), (14), and (16). Following that, we can find the results displayed in Table 2.

Table 2 shows the results for the elasticity of the demand for real money balances (η), the seigniorage as a percentage of GDP ($\mathfrak{R}$), and the welfare loss as a percentage of GDP ($\mathfrak{S}$) when steady-state inflation goes from zero to 500 percent, quarterly. It displays the resulting simulations to three set of parameters estimates, those from columns A, C, and D of Table 1.

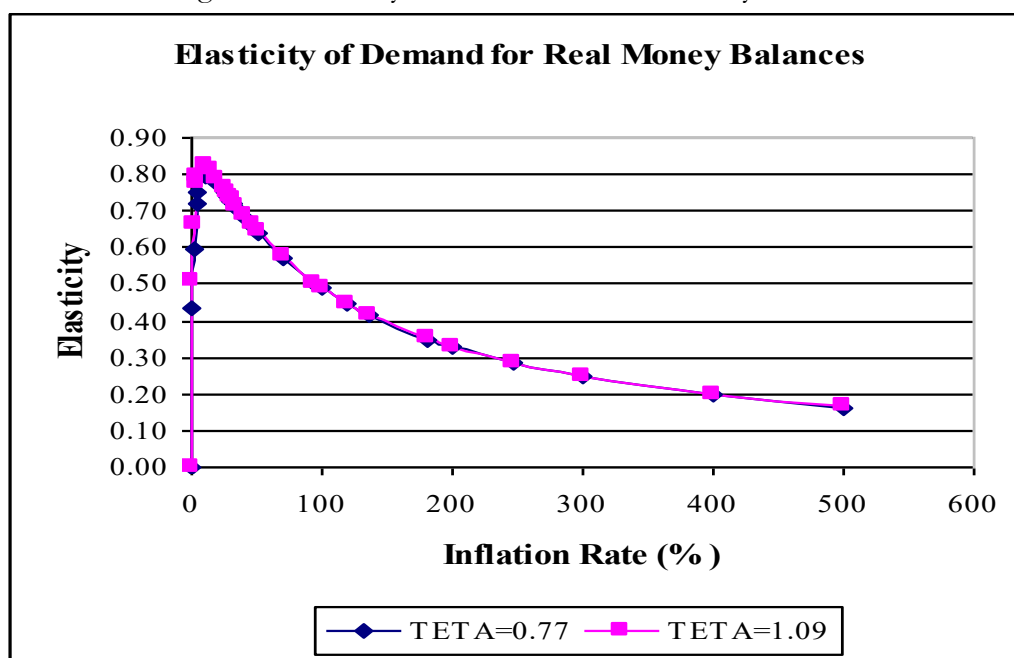
According to the second column of Table 2 and Figure 1, the demand for real money balances was always inelastic in the Brazilian economy in the period analyzed. The elasticity curve of demand for real money balances increases, reaches a maximum, then decreases as inflation rises. It reaches its maximum point at an inflation rate equal to 10 percent per quarter. That represents an average of approximately 3.3 percent a month.

Table 2: Money Demand Elasticity, Seigniorage Ratio, and Welfare Cost of Inflation in Brazil

Series 1				Series 3			Series 3		
π (%)	η_A	$\mathcal{R}_A$	$\mathcal{S}_A$	η_C	$\mathcal{R}_C$	$\mathcal{S}_C$	η_D	$\mathcal{R}_D$	$\mathcal{S}_D$
1	2	3	4	5	6	7	8	9	10
0	0.00	0.2470	0.00	0.00	0.7553	0.00	0.00	0.2695	0.00
5	0.75	0.2772	0.66	0.80	0.6569	2.44	0.58	0.5578	1.42
10	0.80	0.2806	0.90	0.82	0.6486	3.21	0.68	0.6232	2.09
15	0.79	0.2819	1.03	0.81	0.6455	3.67	0.71	0.6521	2.52
20	0.77	0.2826	1.13	0.79	0.6439	3.99	0.71	0.6685	2.84
25	0.75	0.2830	1.20	0.76	0.6430	4.23	0.69	0.6789	3.08
28	0.74	0.2832	1.24	0.75	0.6425	4.35	0.69	0.6836	3.20
30	0.73	0.2833	1.26	0.74	0.6423	4.42	0.68	0.6862	3.27
32	0.72	0.2834	1.28	0.73	0.6421	4.49	0.67	0.6886	3.34
35	0.71	0.2835	1.31	0.72	0.6418	4.58	0.66	0.6916	3.43
40	0.68	0.2836	1.35	0.69	0.6414	4.71	0.65	0.6957	3.57
46	0.66	0.2838	1.38	0.67	0.6411	4.82	0.62	0.6996	3.68
50	0.64	0.2839	1.41	0.65	0.6409	4.92	0.61	0.7016	3.78
70	0.57	0.2841	1.50	0.58	0.6403	5.21	0.55	0.7086	4.09
94	0.50	0.2843	1.57	0.51	0.6399	5.44	0.49	0.7131	4.33
100	0.49	0.2843	1.58	0.49	0.6399	5.48	0.47	0.7139	4.37
120	0.44	0.2844	1.62	0.45	0.6397	5.60	0.43	0.7161	4.50
137	0.41	0.2844	1.64	0.42	0.6396	5.68	0.40	0.7174	4.59
180	0.35	0.2845	1.69	0.35	0.6394	5.83	0.34	0.7196	4.75
200	0.33	0.2845	1.71	0.33	0.6393	5.88	0.32	0.7204	4.80
247	0.28	0.2846	1.73	0.28	0.6392	5.97	0.28	0.7216	4.90
300	0.25	0.2846	1.76	0.25	0.6392	6.05	0.24	0.7226	4.98
400	0.20	0.2847	1.78	0.20	0.6391	6.14	0.19	0.7237	5.08
500	0.16	0.2847	1.80	0.16	0.6390	6.20	0.16	0.7243	5.14

Source: Research Data. Note: The results above refer to equations (8), (10), and (12) of the model. As we explained in the text, $\vartheta = 0.81$, $\phi = 0.011$, and $\nu = 0.51$.

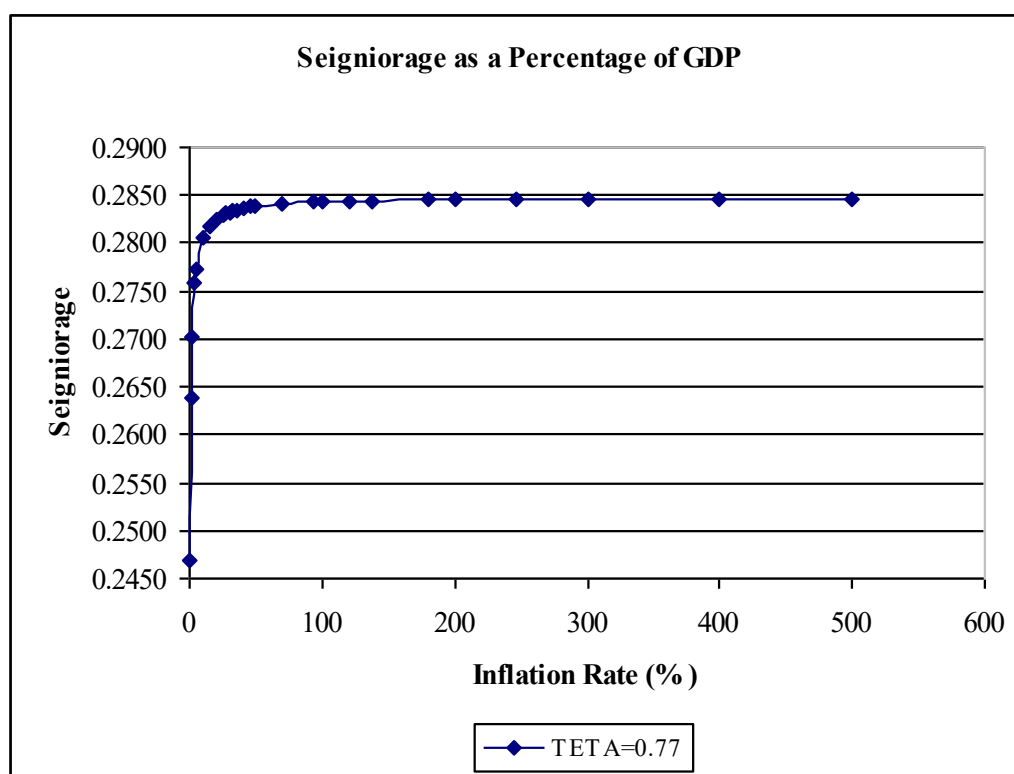
Figure 1: Elasticity of Demand for Real Money Balances



Source: Research Data.

The simulations from the third column of Table 2 and Figure 2 show that the seigniorage as a percentage of GDP is a positive function of inflation rate. As inflation increases, $\mathfrak{R}$ increases at a decreasing rate. Furthermore, the magnitude of the seigniorage as a percentage of GDP in the steady state is small. In order to achieve a government's revenue from printing money of approximately 0.29 percent of GDP, the inflation rate should be steadily high, close to 500 percent or more. This means that if the only objective is to increase the seigniorage ratio, the inflation rate will tend to infinity. Notice that after an inflation rate equals to 10 percent per quarter, the seigniorage ratio is increasing as the demand for real money balances is decreasing. The reason is that the inflation rate is increasing faster enough to decrease the demand for real money balances.

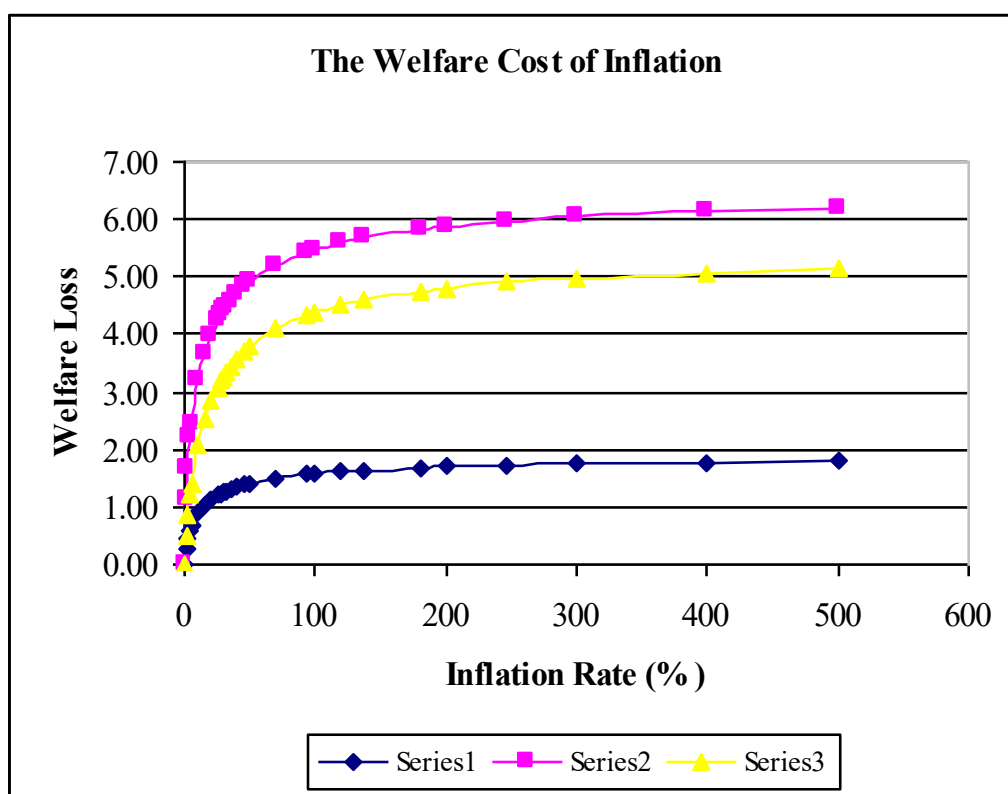
Figure 2: Seigniorage as a Percentage of GDP



Source: Research Data.

When we analyze the percentage decrease in utility translated in consumption per GDP for various levels of inflation, we use the fourth column of Table 2 and Figure 3. They show that the welfare loss increases as inflation increases.

Figure 3: The Welfare Cost of Inflation



Source: Research Data.

The behavior of the welfare loss curve is equal to the seigniorage ratio but the magnitude changes. Let us compare both curves for the quarterly sample mean inflation rate of 137 percent. For this value of steady-state inflation, the seigniorage is about 0.284 percent of GDP and the welfare loss is about 1.64% of GDP. Since the loss in welfare is higher than the revenue achieved through seigniorage, *ceteris paribus*, society will decrease its consumption by 1.64% of GDP if government decides to print money, inflate, in order increase its revenue possibilities. Throughout the time period analyzed, the Brazilian government created money continuously. This probably happened because the seigniorage rate is an increasing function of inflation even in the steady state.

7. Conclusion

Using a model in which the typical household's utility function has leisure, depending on consumption and real money balances, and consumption subject to a typical budget constraint, we derived the first-order conditions. We then found the equation of demand for real money balances. The steady-state demand for real money balances and the definition of seigniorage allowed us to analyze the seigniorage ratio. In addition, through the demand for money and the utility in the steady state we derived the loss function of social welfare measured as the decrease in consumption per GDP when inflation goes from zero to a positive number. The parameters of the model were then estimated by the generalized methods of moments. In general, we found for the Brazilian the following:

1. The elasticity of substitution between consumption in period t and period $t+1$ is about 4.35.
2. The degree of relative risk aversion $(1 - \theta)$ is low, 0.23. This implies that the typical household is a risk taker.
3. Consumption purchases represent higher direct utility than real money balances. Therefore, people do not want more leisure but more consumption.
4. The estimate of β implies a quarterly subjective discount rate of 1.01 percent.
5. The welfare costs of inflation, measured as the decrease in consumption as a percentage of GDP, is always positive and increases as the inflation rate increases.
6. In contrast, seigniorage is an increasing function of the inflation rate. This may be a reason why government in Brazil may be tempted to over-inflate.
7. The elasticity of demand for real money balances reaches its maximum point at the quarterly inflation rate of 10 percent. During the period interval of 1973-1994, the sample mean of the inflation rate was relatively high, 46%. This is in accordance with a tendency of Brazilian households to substitute real money for another asset.

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