RBEE

Is there a power law in the Brazilian stock market index? Evidence from 2017-2022

Rafael Jasper Feltrin[1]

Helberte João França Almeida[2]

[1] Programa de Pós-Graduação em Economia.
Universidade Federal de Santa Catarina.
E-mail: rafafeltrin5@outlook.com

[2] Professor do Programa de Pós-Graduação em Economia.
Universidade Federal de Santa Catarina.
E-mail: helberte.almeida@ufsc.br

Resumo: Mercados financeiros são sistemas complexos, que podem ser estudados sob o prisma da econofísica, incluindo o estudo de fenômenos como leis de potência. Este artigo testa a hipótese de uma lei de potência no Ibovespa, o principal índice de mercado financeiro brasileiro, e não conseguimos rejeitar a hipótese de que suas ações estão distribuídas seguindo uma lei de potência. Isso é bastante interessante, já que as ações são ordenadas por volumes de negociação individuais, então um padrão similar sempre emerge organicamente apesar das empresas estarem sempre trocando de posição e sendo excluídas ou adicionadas ao índice.

Palavras-chave: econofísica; lei de potência; volume de negociação; índice de mercado financeiro; expoente de Pareto.

Abstract: *Financial markets are complex systems, which can be studied under the lens of econophysics, including the study of phenomena such as power laws. We test the hypothesis of a power law in Ibovespa, the main Brazilian stock market index, and cannot dismiss the hypothesis that its stocks are distributed following a power law. This is most interesting, as these stocks are ordered by individual trading volumes, thus a similar pattern always emerges organically even though companies are always exchanging positions and being excluded or added to the index.*

Keywords: *econophysics; power law; trading volume; stock market index; Pareto exponent.*

Classificação JEL: C13; G12; G14; G19.

1. Introduction

As of 2022 the Brazilian stock exchange, B3 (Brasil, Bolsa, Balcão), is the 20th largest stock exchange by market cap in USD billions. Since January 1968, the Ibovespa (from now on, IBOV) is its benchmark index. It started with 18 blue chip[1] companies, then 27, similar to the Dow Jones index. Nowadays, it is much more diverse, with an average of 84 companies per portfolio from September 2017 to January 2022, which make up for 14 portfolios – the index changes thrice a year.

However, even though only six companies (VALE3, ITUB4, ABEV3, BBDC4, PETR4 or PETR3, B3SA3) have been present at the top five spots during this period, they are always changing places, depending on stock market trends, international factors and their actual results, of course. The main criterion for ranking IBOV's weights is each stock's trading volume, thus there are many changes on each rebalancing.

As one of the larger stock exchanges, B3 offers potential for many interesting studies utilising its historical data sets. With the rise of econophysics as a branch of economics, finance is the main stage where it has have been applied, even though there have been relevant contributions in other branches like more traditional macroeconomics, such as the works of Acemoglu et al. (2012) or Silva e da Silva (2020) on granular business cycles.

A simple yet remarkable contribution that econophysics has brought into economics and finance is the finding of many power laws (GABAIX, 2009). There is a power law relationship between two quantities when a relative change in one of them equals a proportional relative change in the other, regardless of their initial size – thus, one of them varies as a power of another.

Therefore, we will test the hypothesis that the IBOV is distributed following a power law, by analysing its latest 14 portfolios. Thus, our contribution to literature is on the discussion of whether the stock market is a self-organising system even though it receives inputs from a large amount of individual agents with differing motivations.

Besides this introduction, the article is comprised by a literature review about power laws in finance, a materials and methods section, a results section and the concluding remarks.

2. Power laws in stock markets

Since coming out relatively unscathed of the 2008 crisis, the Brazilian stock exchange has been subject to more attention in the financial world. In the past decade, Brazilian investors have been investing more in stocks, with the lower interest rates being a reason, as remarked by Haas et al. (2021). As a market grows in size, it also grows in complexity and we see the emergence of patterns such as power laws.

Gleria et al. (2004) point out a growing interest in understanding the dynamics of complex systems, mostly in biology and physics, which is home to the field of statistical mechanics. Amongst the phenomena studied by this branch of physics are power laws. Clauset et al. (2009) remark that power law distributions occur in many situations, natural or man-made and there are several ways to identify power laws, one of them being a visual test along with linear regression.

Equation (1) defines a power law (also called a scaling law), mathematically:

$$Y = aX^{\beta}, \qquad (1)$$

where Y and X are variables of interest, β is the power law exponent or Pareto exponent and *a* is a constant.

Due to a large wealth of available data, financial markets are subject to many power

[1] Inspired by poker games, where they are the most valuable, "blue chips" are stocks very sought out by investors, due to their market power in their line of business and high liquidity in financial markets.

law estimations leading to some interesting results, e.g. Mantegna e Stanley (1995) finding that returns from the S&P 500 are distributed according to a power law. Also, Yan et al. (2005) and Bai e Zhu (2010) find power law properties in Chinese stock market returns, but correlations are markedly stronger for the Shenzhen exchange than for the Shanghai one.

Even though most of research is focused on returns distribution, more recently we have seen some studies about power laws in trading volumes, such as Kristoufek (2015) finds out that Google searches for the Dow Jones component stocks are power law distributed and there are some cross-correlations between these searches and trading volumes.

Zhao et al. (2016) test for power laws in daily cross-sections of different cohorts from the S&P 500, finding out relationships between stocks that can help in predicting the implied volatility index (VIX). Also, Qian et al. (2010) study 30 stock market indices and find evidence of scaling behaviour in most of them that does not depend on the specific stocks, current market movements or the lenght of the index.

This suggests that trading volumes might in follow a power law, which Gabaix et al. (2003) also point out. The authors guess that a reason for that is that big market operators, when trying to optimise their gains, will reach individually the conclusion that there are no more short term gains on certain stocks and slow down on trading these stocks after a certain point.

3. Materials and methods

In 2014, B3 released the latest update on IBOV's methodology. As described in the B3 Index Policies and Procedures, it is a total return index which purports to indicate the mean performance of prices from the most traded and representative stocks in Brazilian stock market.

It excludes BDRs, companies that are in administration and companies with penny stocks (that is, below 1 BRL). The weights are chosen based on the Tradability Index which is, simply put, the most traded stocks in B3 over the last rolling year.

Besides, they must compose at least 0.1% of the exchange's total financial volume and have been present in at least 95% of the sessions over the last rolling year. The only other restriction (which has not been needed at least on this study's period) is that a single company cannot compose more than 20% of IBOV.

B3 does not allow free viewing of past IBOV portfolios, however we have been able to retrieve past compositions (with each company's quantity of stocks and weight in the theoretical portfolio) by web scraping old news from financial newspapers[2], gathering data since the September 2017 press release until January 2022 which gives us a sample of 14 portfolios to analyse.

Equation (2) shows how we rebuilt the old portfolios by looking up the old stock prices:

$$NW_{IBOV} = \sum_{i=1}^{n} price_i * quantity_i, \tag{2}$$

with *NW* being the portfolio's total net worth, which is the sum of *n* stocks with each one's individual value given by multiplying their closing price at the press release day by the quantity in the theoretical portfolio. To do so, we fetched the adjusted close prices of the stocks composing the index during the release of each IBOV portfolio.

For instance, to rebuild the September 2017 IBOV portfolio, we fetched prices from 04/09/2017, when the press release with the new index composition went online on B3's

[2] Past compositions that were removed from the B3 website were mostly found in Valor Econômico, Época Negócios and Estadão articles, while prices were fetched from the Yahoo Finance API.

website. Comparing the stock weights from the B3 press releases with our reconstructed portfolios, they clearly check out, with only small variations because B3 does not clarity the exact date and time used to choose the weights, but keeping the order and proportions between them completely intact.

This is shown in Table 1 which shows our effort to reenact the January 2022 portfolio, with the calculated the weight of each stock and the error between our reconstruction and the actual portfolio. The top 5 stocks have an absolute error in the vicinity of -0.20 percentual points, but it tapers off to 0 in the lower weighted stocks, as we can see in the 92nd and last stock in the portfolio, POSI3.

Table 1 - Portfolio reconstruction example, January 2022.

Ticker	Price (BRL)	Quantity	Volume (BRL)	Weight (%)	Error
VALE3	78.00	3.84b	300.00b	15.27	-0.25
PETR4	29.09	4.57b	133.00b	6.76	-0.26
ITUB4	21.51	4.78b	1.03b	5.24	-0.22
BBDC4	19.69	4.69b	92.4b	4.70	-0.19
PETR3	31.52	2.70b	85.2b	4.34	-0.18
...	...	...	...	...	...
POSI3	10.5	78.1m	820m	0.04	0.00

Equation (3) presents a simple but very useful method for approaching power laws described by Gabaix (2016):

$$\ln(Rank) = \alpha - \beta \ln(Size), \tag{3}$$

where *Rank* is the variable's position in the ranking and *Size* is the variable in whatever measure it is. Gabaix e Ibragimov (2011) remark that even though there are more sophisticated methods, the log-log rank-size regression is still very popular because it is very robust for its simplicity, making for a very "cost-efficient" methodology.

In our case, the regression's dependent variable will be the natural logarithm of the i_{th} stock's rank and the independent variable, the natural logarithm of its participation in the IBOV (in millions of BRL) as seen in Equation (4). The specification is formally defined as:

$$\ln(Rank_i) = \alpha - \beta \ln(Value_i), \tag{4}$$

In such a regression, the Pareto exponent is the β, thus we expect it to be statistically significant with a very high confidence level – ideally 99%. Also, while most regressions with a very high R-squared are worrisome for various reasons, here we expect it be as high as 0.80 or more, since the independent variable's size alone is supposed to be a great predictor of its rank. Also, the log-log dispersion plot looking like a straight line is a necessary but not sufficient condition, and an easy way to dispute the existence of a power law.

While some power law estimations with multiple periods of data employ panel regressions, we will not organise our data in a panel because due to a plethora of companies getting excluded or included in each IBOV change. This would result in a very unbalanced panel and we would rather run a cross-section regression for each one of the 14 IBOV portfolios available. Next section presents their results.

4. Results

As mentioned beforehand, if there is a power law relationship whatsoever between two sets of data, then a dispersion plot of them is expected to somewhat resemble a straight line at list. Figures 1 through 4 show dispersion plots pitting stock's ranks versus their values in IBOV. The Y-axis is the logarithm of the stock's value in the IBOV portfolio and the X-axis is the logarithm of the stock's rank in it.

Figure 1 - Fitted power laws from May 2021 to Jan 2022.

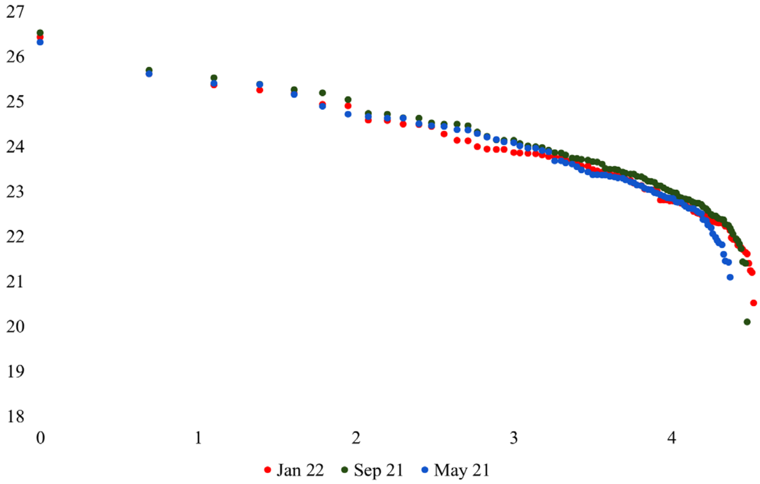
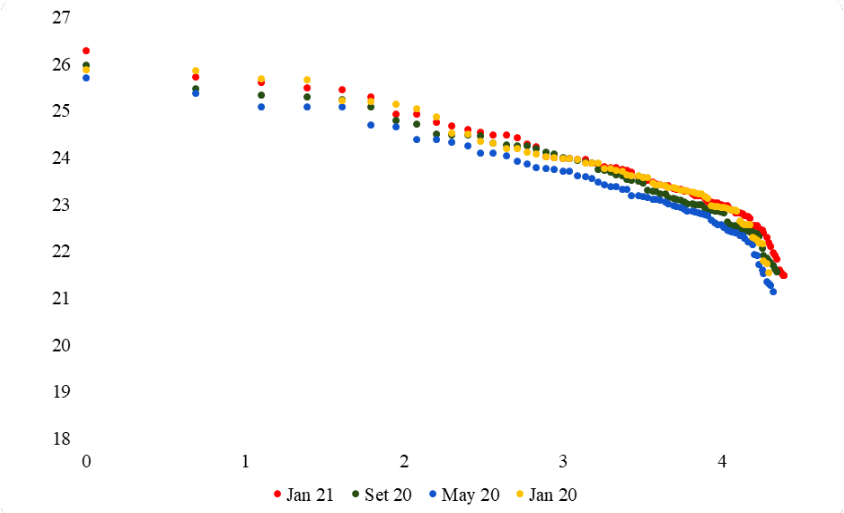


Figure 2 - Fitted power laws from Jan 2020 to Jan 2021.



A visual test does not offer enough to reject the hypothesis of a power law throughout the entire sample studied, even though from 2019 backwards the fit is clearly worse by looking at the dispersion lines. Noticeably, the largest and smallest weights present much worse behaviour on Figures 3 and 4. This may be caused by the quality of data.

As mentioned on section 3, B3 does not divulge the exact date and time of the theoretical portfolio's buildup. As we try to remain unbiased instead of overfitting, by choosing the closing prices from the day the portfolio was announced, we have created small gaps between the divulged weights and the actual weights. It is easy to see that an error of 0.01% is much more impactful when applied to a weight of 0.11% than to a weight of 10%, and any 0.01% translates into hundreds of BRL millions, which can have noticeable effects on a log-log plot.

Figure 3 - Fitted power laws from Sep 2018 to Sep 2019.

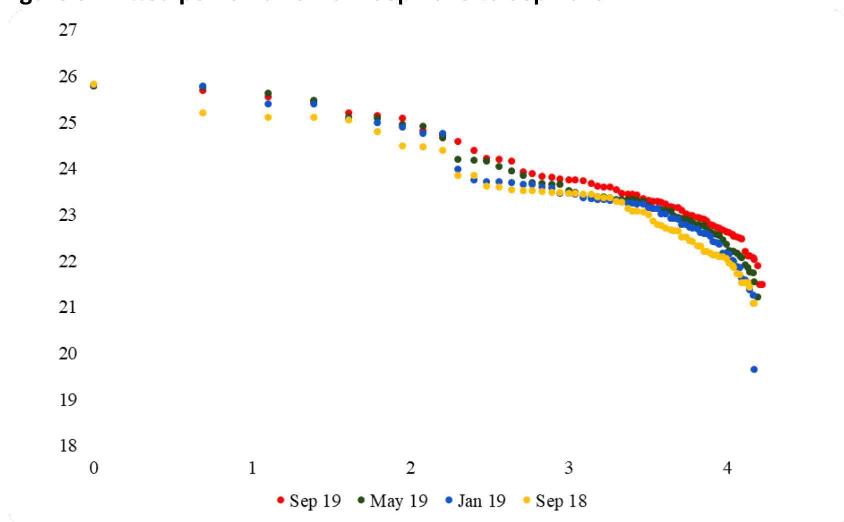
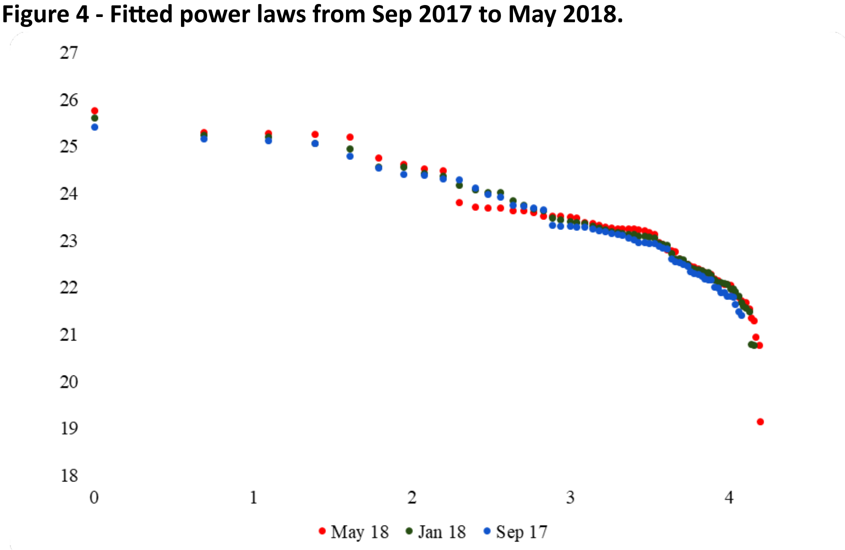


Figure 4 - Fitted power laws from Sep 2017 to May 2018.



We cannot say assertively that these portfolios follow a power law only with a log-log plot, but they do behave well enough that it makes sense to prob further. Indeed, in our log-log rank-size regression we find all the results are statistically significant with very small p-values that allow us to accept their significance at a confidence level of 99%. All of them present high R-squared values, with only three being lower than 0.90. Results are displayed in Table 2. The average Pareto exponent is -0.83 and the average R-squared is 90%.

This is stronger evidence that in fact the IBOV portfolios are distributed ac- cording to a power law. Even with the aforementioned doubts over the quality of data in the lower end of the older sets, all results are very strongly significant and very similar in all periods, down to the standard errors of both α and β.

Thus, based on Gabaix (2016), we can affirm with a certain safety that the IBOV portfolios have been following a power law for the last 56 months. In fact, the result is similar to what was guessed by previous studies such as Gabaix et al. (2003) and Kristoufek (2015), even though they did not focus on this hypothesis. This is impressive, as there were many heavy changes in IBOV's lower ranks, particularly after the COVID-19 pandemic.

Table 2 - Log-log rank-size regression results.

Date	Sample size	Coefficient	Intercept	R-squared
Jan 2022	92	-0.82*** (0.03)	22.62***(0.64)	0.91
Sep 2021	88	-0.81*** (0.03)	22.42***(0.75)	0.88
May 2021	79	-0.84*** (0.03)	23.03*** (0.74)	0.90
Jan 2021	81	-0.85*** (0.03)	23.45*** (0.70)	0.91
Sep 2020	77	-0.88*** (0.03)	23.84*** (0.71)	0.92
May 2020	75	-0.87*** (0.03)	23.44*** (0.78)	0.90
Jan 2020	73	-0.91*** (0.03)	24.66*** (0.78)	0.91
Sep 2019	68	-0.88*** (0.03)	23.88*** (0.72)	0.92
May 2019	66	-0.84*** (0.03)	22.80*** (0.68)	0.93
Jan 2019	65	-0.74*** (0.04)	20.33*** (0.88)	0.85
Sep 2018	65	-0.82*** (0.03)	21.98*** (0.70)	0.92
May 2018	67	-0.70*** (0.04)	19.29*** (0.88)	0.83
Jan 2018	64	-0.80*** (0.03)	21.71*** (0.78)	0.90
Sep 2017	59	-0.86*** (0.03)	23.05*** (0.76)	0.92
Average	73	-0.83*** (0.03)	22.60*** (0.75)	0.90

5. Concluding remarks

This study was motivated by our curiosity about a rather unexplored question: whether the most traded stocks follow a power law, using the Brazilian stock market index to test our hypothesis. This is a very interesting problem because if there is such a power law, that means a repeating pattern is emerging even though companies change positions and drop in and out of the index.

Based on available data, we rebuilt the last 14 IBOV portfolios and ran re- gressions for each one of them, as there is too much variation between portfolio compositions for a panel approach. With a log-log rank-size regression, which is very easy to calculate and understand, we can find the Pareto coefficient for the supposed power law, pending on it also passing a visual test and the results being statistically significant.

If the dispersion plots are not rather close to a straight line, it is not needed to investigate further, but fortunately they were, in particular the most recent ones. Afterwards, all the regressions presented statistically significant results at a confidence level of 99% and large R-squared values, which are also required for our hypothesis of the existence of a power law not to be rejected. The average results from the 14 regressions were -0.83 for the Pareto exponent and 0.90 for the R-squared.

Even with triannual changes to IBOV, we have seen a pattern surfacing across all portfolios. This is a novel result, with very few similar studies in recent literature searching for it, even though it was suggested by Gabaix et al. (2003) and Kristoufek

(2015). Some suggestions for future variations on this theme could be checking if there are similar phenomena in other stock exchanges around the world or trying to find cross-correlations to better understand the emergence of these patterns in Brazilian stock market.

References

ACEMOGLU, D. et al. The network origins of aggregate fluctuations. Econometrica, Wiley Online Library, v. 80, n. 5, p. 1977–2016, 2012.

BAI, M.-Y.; ZHU, H.-B. Power law and multiscaling properties of the chinese stock market. Physica A: Statistical Mechanics and its Applications, Elsevier, v. 389, n. 9, p. 1883–1890, 2010.

CLAUSET, A.; SHALIZI, C. R.; NEWMAN, M. E. Power-law distributions in empirical data. SIAM review, SIAM, v. 51, n. 4, p. 661–703, 2009.

GABAIX, X. Power laws in economics and finance. Annu. Rev. Econ., Annual Reviews, v. 1, n. 1, p. 255–294, 2009.

. Power laws in economics: An introduction. Journal of Economic Perspectives, v. 30, n. 1, p. 185–206, 2016.

GABAIX, X. et al. A theory of power-law distributions in financial market fluctuations. Nature, Nature Publishing Group, v. 423, n. 6937, p. 267–270, 2003.

GABAIX, X.; IBRAGIMOV, R. Rank- 1/2: a simple way to improve the ols estimation of tail exponents. Journal of Business & Economic Statistics, Taylor & Francis, v. 29, n. 1, p. 24–39, 2011.

GLERIA, I.; MATSUSHITA, R.; SILVA, S. D. Sistemas complexos, criticalidade e leis de potência. Revista Brasileira de Ensino de Física, SciELO Brasil, v. 26, p. 99–108, 2004.

HAAS, G. P. et al. Existe value premium para os fundos imobiliários brasileiros?: uma análise para o período 2013 a 2018. Revista brasileira de economia de empresas, v. 21, n. 1, p. 117–130, 2021.

KRISTOUFEK, L. Power-law correlations in finance-related google searches, and their cross-correlations with volatility and traded volume: Evidence from the dow jones industrial components. Physica A: Statistical Mechanics and its Applications, Elsevier, v. 428, p. 194–205, 2015.

MANTEGNA, R. N.; STANLEY, H. E. Scaling behaviour in the dynamics of an economic index. Nature, Nature Publishing Group, v. 376, n. 6535, p. 46–49, 1995.

QIAN, M.-C.; JIANG, Z.-Q.; ZHOU, W.-X. Universal and nonuniversal allometric scaling behaviors in the visibility graphs of world stock market indices. Journal of Physics A: Mathematical and Theoretical, IOP Publishing, v. 43, n. 33, p. 335002, 2010.

SILVA, M.; DA SILVA, S. The brazilian granular business cycle. Economics Bulletin, AccessEcon, v. 40, n. 1, p. 463–472, 2020.

YAN, C. et al. Power–law properties of chinese stock market. Physica A: Statistical Mechanics and its Applications, Elsevier, v. 353, p. 425–432, 2005.

ZHAO, L.; LI, W.; CAI, X. Structure and dynamics of stock market in times of crisis. Physics Letters A, Elsevier, v. 380, n. 5-6, p. 654–666, 2016.