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International power trade as a sequence of repeated bargaining games

Abstract: International electricity spot trade between two countries is modeled as a sequence of bargaining games, one for each trade opportunity. Each game has a benefit per time unit and random duration. There is a correspondence between this trade game and the bargaining game for the partition of a pie studied by Rubinstein. If the game admits the possibility of "money burning", actions by a player to destroy surplus to punish the other player's rejection of an offer, each game admits multiple inefficient perfect Nash equilibria, with delay in the partition. Conditions are found for the existence of predefined agreements between the players, sustaining efficient immediate partition in each game, as an alternative to inefficient Nash equilibria in the sequence of games.

Keywords: International power trade; Bargaining games

Palavras-chave: Comércio internacional de energia elétrica; Jogos de barganha.

JEL: C73, C78, L94

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1 Introduction

This paper addresses the problem of negotiation of prices between countries in international power spot trade, in the institutional context prevailing for such trade in South America and particularly in the Mercosur. We will understand by international power spot trade, the cross-border wholesale transactions of electricity between countries or firms on a spot basis, without any long run contractual obligation for the seller to supply.

International spot power trade has two singularities when compared to the trade of almost any other good:

- Electricity production costs in each trading country and therefore the direction of the trade and its potential benefit are random, as a consequence of the randomness of hydraulic and wind generation, the availability of thermal plants, and in the long run of the unpredictable cycles of under and overinvestment in power generation capacity. Consequently a model of international spot trade should be based on the existence of a sequence of trade opportunities with random duration and benefits. This randomness has been an essential feature of international electricity trade in the Mercosur region in the past ten years.
- For technical reasons power trade always requires some kind of regulation.

In some regions of the world, as the European Union, regulations for international power trade aim at creating a single spot market unifying the national markets of the countries involved in trade. A vast literature describes the institutional and economic problems of designing and implementing this single market for electricity in the EU, for instance Boucher and Smeers (2001), Glachant and Lévêque (2005), ERGEG (2006), and Meeus, Belmans and Glachant(2006).

The main issue in this case is to ensure equal rights and non-discriminatory participation in trade in an open market for firms from every interconnected country. Among the theoretical economic problems that arise, perhaps the most important are the analysis of the effect on market power of a limited capacity interconnection between two markets (for instance Borenstein, Bushnell and Stoft (1999), Parisio and Bosco (2006)) and the assignation among competitors of the scarce transmission capacity (Joskow and Jean Tirole (2000)).

By contrast, in South America and specially in the Mercosur countries, regulation has given place most frequently to bilateral trade regimes, where countries explicitly determine prices for the energy traded between them. A description of such regimes can be found in CIER (2004). In this case it is essential to find long run agreements with rules to determine prices, to avoid a case by case bilateral bargain, each time a new trade opportunity appears. International spot trade in our region then raises two interesting problems. First the design, description and discussion of institutionally feasible trade price agreements for spot international trade. Second, the formal analysis of bargaining between countries to determine trade prices, as in Moitre and Rudnick (2000) who tried to determine prices in the framework of the Nash bargaining solution

The present paper is in the second line of work.

The objectives of the paper are: i) explaining formally the inefficiencies of case by case bargaining in spot power international trade, in a region like Mercosur, where the direction of the trade and its potential benefits are random, and ii) finding conditions to be fulfilled by long run price setting rules, to be preferable to case by case bargaining.

The paper is organized as follows. International spot power trade is modeled as an infinite sequence of bargaining games, each one of potentially infinite duration, one for every trade opportunity. In section 2, the formal model for each trade opportunity is presented. In each trade opportunity, the buyer and the seller, make in turn a proposal as to how the benefit of trade should be divided. Each trade opportunity lasts for a random number of time periods. Time periods of equal length are denoted by $t=1,\ldots$ In the nth trade opportunity there is an assignment of the roles of buyer and seller, a benefit per time unit b_n to split between the two players (equal to the difference between the avoided cost of the buyer and the incremental cost of the seller if trade takes place) and a probability p_n of the game surviving to the next period of time. Therefore the duration of trade opportunity n is a random variable with geometric distribution with parameter $(1\text{-}p_n)$. The set of these parameters is a random variable denoted by v_n. When the nth opportunity ends, nature generates a new n+1-th with parameters resulting from a new random variable v_{n+1}. The main result of section 2 is that there is a correspondence between the game of negotiation between countries in each trade opportunity, and the classic model of bargaining over a single amount or 'pie' presented in Rubinstein (1982).

In section 3, the correspondence is extended to the case described by Avery and Zemsky (1994), when in a Rubinstein game, a player can worsen the result of possible agreements each time his offer is rejected (an action of "money burning"), to improve his/her bargaining position. We will consider one form of money burning consisting of a player delaying the game when his/her offer is rejected. In the context of international spot power trade, this means refusing to trade and delaying new negotiations for a period of time. Avery and Zemsky show that such games have infinite SPNE with partitions belonging to a segment $[s_{min}, s_{max}]$ and with agreement in any period t between 0 and a higher bound depending on the parameters of the problem. Therefore there is a theoretical support for the evidence that real bilateral negotiations to split the benefits of international energy trade will probably produce inefficient (delayed) outcomes.

Then the following question of prime practical importance arises: would it be possible for both countries, knowing in advance they will face a practically infinite series of trade opportunities, to settle a permanent rule to set prices and split benefits without delay, and fully exploit the potential benefits of trade? Such a rule should determine right away the energy price as a function of the observed parameters of the trade opportunity. In section 4, conditions are found for such rules to be sustainable. The main results of the paper stem from the application of the formal results to the case of energy trade, which lead to an intuitive interpretation.

2 Correpondence of a trade game with a Rubinstein game

Let us call trade game (TG) the bargaining game corresponding to one single trade opportunity.

Each trade game can be put in correspondence with another game widely studied by the literature: the bilateral bargaining for the partition of a fixed amount B, in which each player has to make in turn a proposal as to how it should be divided. After one player has made an offer the other must decide to accept it or reject it and continue the bargaining with the roles reversed. The sequence of offers and rejections can last infinitely if agreement is never reached. Both players have discounting rates so the delay to achieve an agreement is inefficient. Let us call this game Rubinstein game (RG) since it was first studied in Rubinstein (1982).

In this section we prove that given a TG it is possible to put it in correspondence with a particular RG where subgames, strategies and equilibria in both games are in biunivocal correspondence. It is proved that the expected benefits for the players in a TG, discounted to the beginning of the game when they play strategies σ_1 and σ_2 are respectively equal to the sure benefits in the corresponding RG, discounted to the beginning of the game, when they play the corresponding strategies. Therefore the subgame perfect Nash equilibria (SPNE) of both games can also be put in a biunivocal correspondence. This allows us to employ the abundant results found in the literature for RGs to the analysis of TGs.

Rubinstein obtained a very interesting result: there is a single SPNE in the RG consisting of the players reaching immediate agreement in the first period of time, with the partition close to one half for each player if discount rates are equal, with a small advantage to the player who makes the first proposal.

In the RG there is an amount B to split between players 1 and 2, with discounting rates per time unit ρ_1 y ρ_2 respectively. Bargaining to divide B develops through a sequence of equal duration periods of time, t = 0,1,….. Let us call instant t the beginning of period t. As long as no agreement is reached, at the beginning of each time period, one player i makes an offer s to player j, that j can either accept or reject. If the offer is accepted, player 1 has a benefit sB and player 2 a benefit (1-s)B, dated at the beginning of period t. If the offer is rejected, in the following period t+1 the roles are reversed and j makes an offer to i. Let us convene that the variable offered by both players is player 1's fraction of the benefit.

Panel A) from Diagram 1 in the next page represents schematically the Rubinstein game RG. Panel B) represents a trade game TG.

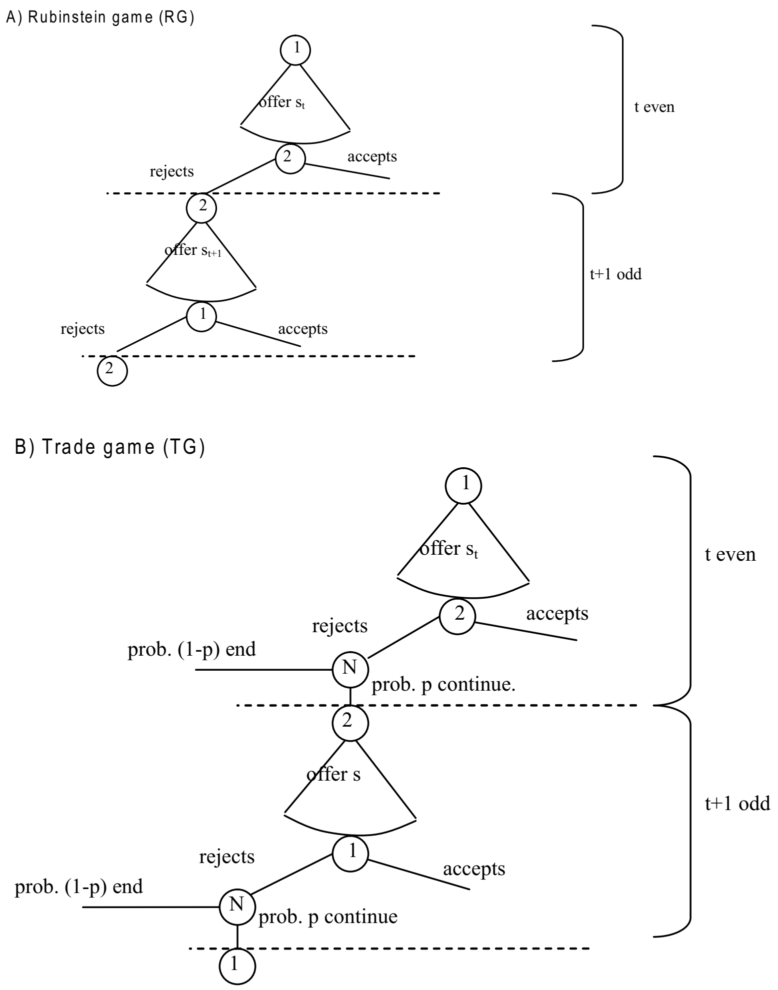
In a TG there is a benefit b per period to split between players 1 and 2, with discount factors per time unit δ_1 y δ_2. If the agreement is delayed for one period this benefit b is lost. At the beginning of period t one player i makes an offer s to j. If the offer s is accepted the players begin to receive sb and (1-s)b respectively per period in each of the following periods as long as the trade opportunity lives. At the end of each period there is a probability p of nature choosing the continuation of the trade opportunity and (1-p) of choosing its end. This random result is the same whether an agreement was already reached previously or not. Decisions by nature in different periods are independent random variables and are not affected by the players' strategies.

When the bargain ends with "accept" with partition s in period t:

- In the RG players 1 and 2 receive respectively sure benefits sB and (1-s)B dated at the beginning of period t.
- In the TG players 1 and 2 receive respectively random benefits with expected values, given the game is in t, discounted at the beginning of t, denoted by $B^C_1(s,b,\delta_1,p)$ and $B^C_2(s,b,\delta_2,p)$.

The probability of the TG born at period 0 surviving until period t is equal to p^t. The benefits for each player in the TG discounted to the beginning of t=0, given an agreement at t=0, are random variables, as the duration of the trade opportunity after the agreement is random.

Diagram 1



The expected values of the benefits for each player in a TG, discounted to the beginning of period 0, when agreement is reached at t=0 with partition s, benefit per period b, and surviving probability p, when discount factors are δ_1 y δ_2 , are:

$$B^C_1(s,b,\delta_1,p) = sb\sum_{t=0}^{\infty}\delta_1{}^t p^t = \frac{sb}{1-\delta_1 p} = sB^{\infty}(b, p\delta_1) \quad (1)$$

$$B^C_2(s,b,\delta_2,p) = (1-s)b\sum_{t=0}^{\infty}\delta_2{}^t p^t = \frac{(1-s)b}{1-\delta_2 p} = (1-s)B^{\infty}(b, p\delta_2)$$

where $B^{\infty}(b, \rho) = \dfrac{b}{1-\rho}$

$B^{\infty}(b, \rho)$ is the value of an infinite flow of benefits b per period, beginning at t=0, discounted to instant t=0 with factor ρ per period.

Let us assume $\delta_1 = \delta_2 = \delta$, resulting in:

$$B^C_1(s,b,\delta,p) = \frac{sb}{1-\delta p} = sB^{\infty}(b, p\delta) \qquad (2)$$

$$B^C_2(s,b,\delta,p) = \frac{(1-s)b}{1-\delta p} = (1-s)B^{\infty}(b, p\delta)$$

Those values $B^C_1(.)$ y $B^C_2(.)$ are also the expected benefits of an agreement with partition s, obtained at any period t, discounted to the beginning of this period, given the event that the game has survived until period t.

The following table compares benefits in RG and TG for an agreement reached in period t, with partition s.

Game	Player	Periods	0, 1, …., t-1	t(agreement)	t+1, t+2,….
RG	1	Sure benefit	0	sB	--
	2	Sure benefit	0	(1-s)B	--
TG	1	Current benefit per time period as long as the game survives	0	sb	sb
		Expected benefit discounted to t		$sB^{\infty}(b, p\delta)$	
		Current benefit per time period as long as the game survives	0	(1-s)b	(1-s)b
	2	Expected benefit discounted to t		$(1-s)B^{\infty}(b,p\delta)$	

Let us denote with S_R and S_C respectively the sets of subgames of the Rubinstein game RG and the trade (commerce) game TG.

There is a biunivocal correspondence B_H between the subgames of a TG and the subgames of a particular RG, where corresponding subgames have in their histories the same actions for each of the players, and in the TG nature has played always "continue".

Let Σ_1 and Σ_2 be the sets of all possible strategies in the Rubinstein game RG and Σ^C_1 and Σ^C_2 the sets of all possible strategies in trade game TG.

Given any strategy $\sigma_i \in \Sigma_i$ in RG, we define $\sigma^C_i(\sigma_i)$ the corresponding strategy in TG, as the one that chooses in each subgame of TG the same actions as σ_i in the corresponding subgame in RG determined by $\mathbf{B_H}$.

Symmetrically we can define $\sigma_i(\sigma^C_i)$ for each $\sigma^C_i \in \Sigma^C_i$.

The pair of functions σ^C_i: $\Sigma_i \rightarrow \Sigma^C_i$ y σ_i:$\Sigma_i \rightarrow \Sigma^C_i$, i=1,2, set for each player a biunivocal correspondence $\mathbf{B}_\Sigma$ between the strategies of each player in both games RG and TG.

Proposition I:

Given:

- A trade game TG(δ,b,p) with discount factor δ equal for both players, benefit per period of time b, and probability of survival of the trade opportunity in each period p,
- and a Rubinstein game RG(B^{∞}(b, pδ), pδ) splitting an amount B^{∞}(b,pδ) where both players have discount factor pδ,
- the expected benefits discounted to the beginning of period t, in TG(δ,b,p), given that the game is at the beginning of subgame H^C dated at t, when strategies σ^C_1, σ^C_2 are played, are respectively equal to the certain benefits discounted to the beginning of t in RG(B^{∞}(b, pδ), pδ) when the game is a the beginning of subgame H, the corresponding of H^C, and strategies σ_1, σ_2 correspondings in B_Σ of σ^C_1, σ^C_2 are played. This is valid in particular for the entire games TG y RG, which are also subgames dated at t=0.

Proof:

Let RG be the Rubinstein game to split an amount B, and $T^R(H, \sigma_1, \sigma_2)$: $S_{\mathbf{R}} \times \Sigma_1 \times \Sigma_2 \rightarrow \{N \cup \infty\}$ the function determining the period t of the agreement (or infinite, meaning no agreement is ever reached), when strategies σ_1 and σ_2 (of the entire game RG) are played starting at the beginning of subgame $H \in S_{\mathbf{R}}$. If subgame H is dated at t, $T^R(H, \sigma_1, \sigma_2) \geq t$.

Let $S^R(H, \sigma_1, \sigma_2)$: $S_{\mathbf{R}} \times \Sigma_1 \times \Sigma_2 \rightarrow \{[0, 1] \cup SA\}$ be the function determining the partition reached in RG, when strategies σ_1 and σ_2 are played starting at the beginning of $H \in S_{\mathbf{R}}$, where SA means no agreement.

For any a subgame H beginning at instant t the benefits for the players in the RG with total benefit B and discount factors ρ_1 y ρ_2, discounted to instant t, when players start playing strategies σ_1 y σ_2 at the beginning of subgame H, are:

$$\Pi_1^R(H,\sigma_1,\sigma_2) = \rho_1^{T^R(H,\sigma_1,\sigma_2)-t} S^R(H,\sigma_1,\sigma_2)B \qquad (3)$$
$$\Pi_2^R(H,\sigma_1,\sigma_2) = \rho_2^{T^R(H,\sigma_1,\sigma_2)-t}\left[1 - S^R(H,\sigma_1,\sigma_2)\right]B$$

Let $T^C(H^C, \sigma^C_1, \sigma^C_2) : S_{\mathbf{C}} \times \Sigma_1 \times \Sigma_2 \rightarrow \{N \cup \infty\}$ be the function determining the period t of the agreement in TG, when strategies σ^C_1, σ^C_2 (of the entire game) are played starting at the beginning of subgame H^C, and nature plays always "continue". If subgame H^C_t is dated at t, then $T^C(H_t, \sigma_1, \sigma_2) \geq t$.

Let $S^C(H^C, \sigma^C_1, \sigma^C_2) : S_{\mathbf{C}} \times \Sigma_1 \times \Sigma_2 \rightarrow \{[0, 1] \cup SA\}$ be the function determining the partition reached in TG, when strategies σ^C_1, σ^C_2 are played starting at the beginning of subgame H^C, and nature plays always "continue".

As strategies and subgames are related by $\mathbf{B}_\Sigma$ and $\mathbf{B}_H$, if H^C and H are corresponding subgames in TG and RG, and nature plays always "continue", then:

$T^C(H^C, \sigma^C_{\mathbf{1}}(\sigma_1), \sigma^C_{\mathbf{2}}(\sigma_2)) = T^R(H, \sigma_1, \sigma_2)$ (4)

$S^C(H^C, \sigma^C_{\mathbf{1}}(\sigma_1), \sigma^C_{\mathbf{2}}(\sigma_2)) = S^R(H, \sigma_1, \sigma_2)$

for any $\sigma_1 \in \Sigma_1$ and $\sigma_2 \in \Sigma_2$,

and symmetrically:

$T^C(H^C, \sigma^C_1, \sigma^C_2) = T^R(H, \sigma_{\mathbf{1}}(\sigma^C_1), \sigma_{\mathbf{2}}(\sigma^C_2))$

$S^C(H^C, \sigma^C_1, \sigma^C_2) = S^R(H, \sigma_{\mathbf{1}}(\sigma^C_1), \sigma_{\mathbf{2}}(\sigma^C_2))$

for any $\sigma^C_1 \in \Sigma^C_1$ and $\sigma^C_2 \in \Sigma^C_2$.

Let H and H^C be corresponding subgames in RG y TG respectively, dated at t.

The benefits discounted to instant t, in trade game TG, given the event of the game having reached the beginning of subgame H^C at instant t, when strategies in the entire game are σ^C_1, σ^C_2, and discount factors are equal to δ, can be written as:

$$\Pi_1^C(H^C,\sigma_1^C,\sigma_2^C) = \delta^{T^C(H^C,\sigma_1^C,\sigma_2^C)-t} p^{T^C(H^C,\sigma_1^C,\sigma_2^C)-t} B_1^C(S^C(H^C,\sigma_1^C,\sigma_2^C),b,\delta,p)$$
$$\Pi_2^C(H^C,\sigma_1^C,\sigma_2^C) = \delta^{T^C(H^C,\sigma_1^C,\sigma_2^C)-t} p^{T^C(H^C,\sigma_1^C,\sigma_2^C)-t} B_2^C(S^C(H^C,\sigma_1^C,\sigma_2^C),b,\delta,p)$$

Substituting for B^C_1 (.) y B^C_2 (.) from (1):

$$\Pi_1^C(H^C,\sigma_1^C,\sigma_2^C) = (p\delta)^{T^C(H^C,\sigma_1^C,\sigma_2^C)-t} S^C(H^C,\sigma_1^C,\sigma_2^C) B^\infty(b,p\delta) \qquad 5$$

$$\Pi_2^C(H^C,\sigma_1^C,\sigma_2^C) = (p\delta)^{T^C(H^C,\sigma_1^C,\sigma_2^C)-t} \left[1 - S^C(H^C,\sigma_1^C,\sigma_2^C)\right] B^\infty(b,p\delta)$$

where:

$\delta_i^{T(\cdot)-t}$ is the discount factor between instants t and T() for player i

$p^{T(\cdot)-t}$ is the probability of the trade opportunity surviving until reaching period T of the agreement, given the event of having reached period t.

Let us call σ_1 and σ_2 the strategies of the RG respectively corresponding to σ^C_1 and σ^C_2 in $\mathbf{B}_\Sigma$.

From (4):

$$\Pi_1^C(H^C,\sigma_1^C,\sigma_2^C) = (p\delta)^{T^R(H,\sigma_1,\sigma_2)-t} S^R(H,\sigma_1,\sigma_2) B^\infty(b,p\delta) \qquad (6)$$

$$\Pi_1^C(H^C,\sigma_1^C,\sigma_2^C) = (p\delta)^{T^R(H,\sigma_1,\sigma_2)-t} \left[1 - S^R(H,\sigma_1,\sigma_2)\right] B^\infty(b,p\delta)$$

The benefits in Rubinstein game to split B, when both discount factors are ρ come from (3) and are:

$$\Pi_1^R(H,\sigma_1,\sigma_2) = \rho^{T^R(H,\sigma_1,\sigma_2)-t} S^R(H,\sigma_1,\sigma_2) B$$

$$\Pi_2^R(H,\sigma_1,\sigma_2) = \rho^{T^R(H,\sigma_1,\sigma_2)-t} \left[1 - S^R(H,\sigma_1,\sigma_2)\right] B$$

Then there is a biunivocal correspondence between the SPNEs in TG(δ, b, p) and RG(B^∞(b, pδ), pδ) where sure benefits discounted to t=0 in the equilibria in RG are equal to the expected benefits discounted to t=0 in the corresponding equilibria in TG. Moreover as there is only one SNPE in RG there is also a unique equilibrium in the corresponding TG with the same partition s and with immediate agreement.

End of proof.

Immediate agreement in real bargaining is rarely obtained. It is therefore necessary to explain why inefficient outcomes with delayed agreement (in t>0) occur. In the next section we will consider modified Rubinstein and trade games with inefficient outcomes, and settle correspondences between them.

3 Extension to trade games with money burning by delaying the offer

The unicity result found by Rubinstein (1982) contradicts the intuitive perception that bargaining often leads to delayed agreements, or no agreement at all, and Pareto inefficient outcomes. Since then new models were developed with adequate changes in the hypothesis of the RG to allow the existence of multiple SPNE with delayed agreement.

Avery and Zemsky (1994) synthesize the results of that literature. The intuition is that when a player can worsen the result of possible agreements each time his offer is rejected (an action of "money burning"), there is an incentive to use this threat to improve his/her bargaining position. To our purposes we will consider one form of money burning consisting of a player delaying the game when his/her offer is rejected.

Let us call RGM and TGM respectively the Rubinstein and trade games modified to allow money burning by delaying negotiation. In both games there is the possibility for player i to delay the game during k periods if his offer is rejected by player j, which is equivalent to modify the discount factor (assumed equal for both players) between period t and t+1, from δ to δ^{k+1}.

We will show that there still exists a correspondence between a TGM and a RGM defined adequately. Diagrams 2 and 3 try to describe graphically both games.

Avery and Zemsky show that in a RGM exist infinite SPNE with partitions s belonging to a segment $[s_{min}, s_{max}]$ and with agreement in any period t between 0 and a higher bound depending on the parameters of the problem. We will assume that the parameters of the problem are such that there exist SPNEs with inefficient equilibria.

Diagram 2 - Rubinstein game with the option for 2 to delay the game k periods

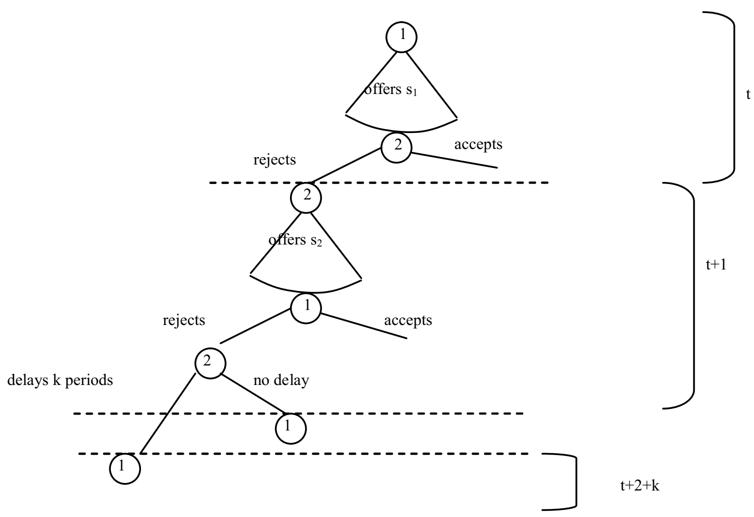
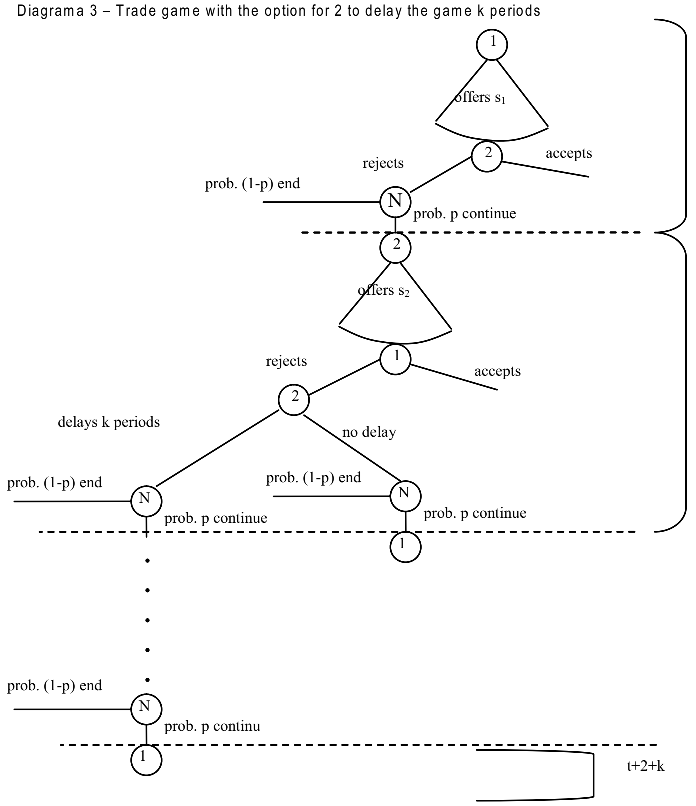


Diagrama 3 – Trade game with the option for 2 to delay the game k periods



As in the previous pure Rubinstein and trade games, there is a biunivocal correspondence $\mathbf{B_H}$ between the subgames in RGM and TGM. Again, for each strategy $\sigma_i \in \Sigma_i$ in the modified Rubinstein game RGM, there is a corresponding strategy $\sigma^{\mathbf{C}}_{\mathbf{i}}(\sigma_i)$ in the TGM, defined as the strategy with the same decisions for each player at the beginning of corresponding subgames (offers, acceptation or rejection, and now also delaying of the game). Symmetrically we can define $\sigma_{\mathbf{i}}(\sigma^C_i)$ for every $\sigma^C_i \in \Sigma^C_i$.

Then the following proposition similar to Proposition I but extended to modified games holds:

<u>Proposition II</u>,

Given:

- A modified trade game with the possibility of delays TGM (δ,b,p) with discount factor δ equal for both players, benefit per period b and probability of surviving p,
- and a modified Rubinstein game with the possibility of delays for the same number of periods RGM(B^∞(b, pδ), pδ) where the amount to be split is B^∞(b, pδ), and both players have discount pδ**,**
- the expected benefits discounted to instant t in TGM(δ,b,p), given the event that the game has reached the beginning of subgame H^C dated at t, when players use strategies σ^C_1, σ^C_2 in the entire game, are respectively equal to the sure benefits discounted to t, in RGM(B^∞(b, pδ), pδ), when players use the respectively corresponding strategies σ_1 and σ_2, given the event that the game has reached subgame H corresponding of H^C. This holds in particular for the entire games TGM and RGM.

Proof

Let S_{RM}, Σ_{M1} and Σ_{M2} be respectively the set of subgames and the sets of all possible strategies for both players in RGM.

Let:

$T^{RM}(H, \sigma_1, \sigma_2)$: $S_{RM} \times \Sigma_{M1} \times \Sigma_{M2} \rightarrow \{N \cup \infty\}$

$S^{RM}(H, \sigma_1, \sigma_2)$ $S_{RM} \times \Sigma_{M1} \times \Sigma_{M2} \rightarrow \{[0, 1] \cup SA\}$

be the functions determining the period t of the agreement and the partition in the RGM, when starting at subgame $H \in S_{RM}$, players use strategies σ_1 and σ_2. Subgame H_t is dated at t so $T^{RM}(H_t, \sigma_1, \sigma_2) \geq t$.

In the same way we can define $T^{CM}(H, \sigma^C_1, \sigma^C_2)$ and $S^{CM}(H, \sigma^C_1, \sigma^C_2)$ for the TGM.

The analogues of (1) to (7) hold, substituting indexes RM and CM for R and C respectively.

Therefore there is a biunivocal correspondence between the SPNEs of TGM(δ,b,p) and RGM(B^∞(b, pδ), pδ), where the sure benefits of equilibria in RGM are equal to the expected benefits in the corresponding equilibria in TGM.

End of proof.

As a consequence of the existence of multiple SPNEs with inefficient delayed agreement in RGM, the corresponding inefficient equilibria in TGM also exist.

4 Sustainability of efficient rules of trade when an infinite sequence of TGMs is played

Real bilateral negotiations to split the benefits of international energy trade will probably produce inefficient outcomes. Then the following question of prime practical importance arises: would it be possible for both parts, knowing in advance they will face a practically infinite series of trade opportunities (represented by TGMs), to settle a permanent rule to split benefits without delay, and fully exploit the potential benefits of trade? Such a rule should determine right away the energy price as a function of the observed parameters of the TGM.

Let us consider a game SG consisting of an infinite sequence of TGMs, and a rule determining strategies for both players to play each TGM. The rule does not induce a SPNE in each TGM. However, it is intuitive that the rule could persist in time if: i) each player thinks his/her deviation from the rule would cause the game to fall to an infinite sequence of inefficient SPNEs, one for each TGM; ii) this sequence of SPNEs is undesirable when compared to the survival of the rule. In what follows, a condition is found for a rule to be sustainable in this way, by ensuring that the pair of strategies consisting of the two players following the rule is a SPNE of game SG.

4.1 Inefficient SPNE in SG

Let us denote by t=0, 1, …., an infinite sequence of periods of time of equal duration, and by $\{TGM_n\}$, n = 1,……, the infinite sequence of TGMs between two players.

$v_1, v_2, \ldots$ is the infinite sequence of random variables, determining the parameters that describe the respective TGMs, taking values in a sequence of sets $\{\Omega_n\}$ with respective distributions $\{\varphi_n(.)\}$, n=1…..

$v_n = (r_n, b_n, p_n)$, where:

- r_n is the variable describing the roles of the players in TGM_n, who plays first, who is the seller and who the buyer. This last fact can be relevant to determine which of the multiple equilibria is played.
- b_n is the benefit per period in TGM_n.
- p_n is the probability of trade opportunity in TGM_n surviving from one period to the next one.

Let us denote $\Omega = \Omega_1 \cup \Omega_2 \cup$ ......

Let us call TGM(v) the trade game determined by a realization $v \in \Omega$ of the random variables, therefore $TGM_n = TGM(v_n)$.

Let us define the game **SJ** described by:

1) Initially n=1, t=0.

2) Nature chooses the parameters $v_n = (r_n, b_n, p_n)$ of the nth game. A game $TGM_n = TGM(v_n)$ is thus defined.

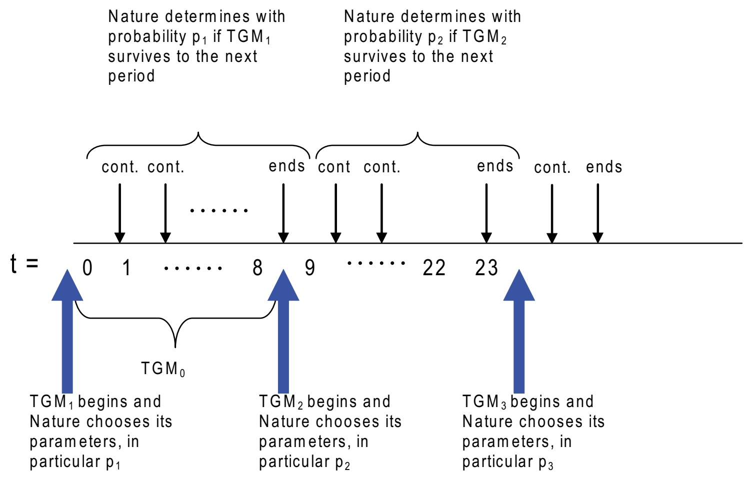
3) At period t the game TGM_n is played. In t an agreement could have already been achieved with partition s_n, leading both players 1 and 2 to receive benefits $s_n b_n$ and

$(1-s_n)b_n$ respectively. Or on the contrary, with no agreement, no benefits are collected and the sequence of offers and possibly delays is still going on.

4)At the end of period t Nature decides whether TGM_n continues to period t+1. The choice "continue" has probability p_n.

- If TGM_n finishes by nature choosing "end", the game returns to step 2) to initiate a new trade game TGM_{n+1} in period t+1.
- If TGM_n continues the game returns to step 3) in period t+1.

The following diagram describes a realization of game SJ.



The period of time when TGM_n (the nth TGM) begins, denoted by t_n, is not predetermined but is a random variable, equal to the addition of n-1 waiting times to obtain the first failure in n-1 sequences of Bernoulli independent trials with success probabilities p_μ, with $\mu=1,\ldots,n-1$.

A strategy for player i to play SJ should determine an action at the beginning of each of the subgames of SJ, in other words, for every t and every possible history of the game prior to t.

In time period t, depending on the previous choices of Nature, the current TGM can be any TGM_n with n between 1 and t+1. In the former case Nature has always played "continues" and SJ is still in TGM_1. In the latter Nature has played always "end" and SJ is in TGM_{t+1}.

Let us call $\Sigma_1(v)$ y $\Sigma_2(v)$ the strategy sets of each player in TGM(v) and let us define the functions:

$\sigma^\star_1(v): \Omega \rightarrow \Sigma_1(v)$

$\sigma^\star_2(v): \Omega \rightarrow \Sigma_2(v)$

that for every $v \in \Omega$ determine a pair of strategies $(\sigma^\star_1(v), \sigma^\star_2(v))$ constituting a particular SPNE of game TGM(v).

We have supposed there are infinite SPNEs in TGM(v), with different partitions and delays until agreement. We are supposing now that as a result of the previous experience playing SJ, both parts expect a particular SPNE $(\sigma^\star_1(v), \sigma^\star_2(v))$, to be played in TGM(v).

Depending on the value of v_n the result $(\sigma^\star_1(v_n), \sigma^\star_2(v_n))$ is more or less favorable to one player, for instance benefits can be systematically greater for sellers as a result of the weak bargaining position of a country relying on imports to avoid energy rationing.

Let us define a pair of strategies $S^\star_1$ y $S^\star_2$ to play SJ in the following way: player i always plays in TGM_n, of parameters v_n, the strategy $\sigma^\star_i(v_n)$, disregarding the outcomes of the previous TGMs.

Proposition III:

The pair of strategies $(S^\star_1, S^\star_2)$ is a SPNE in SJ.

Proof

Let us consider any subgame H of SJ dated at period t. We must prove that $(S^\star_1, S^\star_2)$ is a Nash equilibrium of H. A subgame dated at t is determined by the choices of both players and Nature up to period t-1(if the subgame begins with an offer) or t (if the subgame begins with an acceptance or rejection of an offer).

Let us denote by TGM_n the TGM containing the beginning of subgame H.

Suppose that $(S^\star_1, S^\star_2)$ is not a Nash equilibrium in subgame H. This means that one of the players, say player 1, has another strategy S'_1 to play SJ reporting him an expected discounted benefit strictly greater than his benefit from $S^\star_1$, given the event of the game beeing at the beginning of subgame H, when player 2 plays $S^\star_2$. The expected discounted benefit for 1 is the infinite addition of expected discounted benefits from the games: TGM_n (starting at H), TGM_{n+1}, $TGM_{n+2,}$ …..

Let us call $BS^\star_m$ the term adding to 1's expected discounted benefit corresponding to TGM_m, when 1 plays $S^\star_1$ and 2 plays $S^\star_2$,with m=n, n+1,…..

Similarly BS'_m is the term of 1's benefits from TGM_m, when 1 plays S'_1 and 2 plays $S^\star_2$.

As the benefit for 1 with strategy S'_1 is strictly greater than with strategy $S^\star_1$, there exists at least one $\mu \geq n$ so that in TGM_μ, BS'_μ is strictly greater than $BS^\star_\mu$.

- If $\mu = n$, this means that S'_1 is a better strategy than $S^\star_1$ in the subgame G of TGM_n starting with the beginning of H. But when they play $S^\star_1$ and $S^\star_2$ both players are using $\sigma^\star_1(v_n)$ and $\sigma^\star_2(v_n)$ which form a Nash equilibrium in G. Therefore there cannot exist any strategy S'_1 of SJ reporting to player 1 in subgame G of TGM_n a greater benefit than $\sigma^\star_1(v_n)$, when confronting with $\sigma^\star_2(v_n)$.
- If $\mu > n$, this means that S'_1 is better for player 1 than $S^\star_1$ in TGM_μ to confront with $S^\star_2$, against the hypothesis of $(\sigma^\star_1(v_\mu), \sigma^\star_2(v_\mu))$ being a SPNE of TGM_μ, for every realization of v_μ.

4.2 Sustainability of a rule by means of a Nash reversion strategy

If benefits for the players in the SPNEs $(\sigma^\star_1(v_n), \sigma^\star_2(v_n))$ of the TGMs are poor enough, as agreement is badly delayed, SNPEs in SJ other than $(S^\star_1, S^\star_2)$ may exist.

The following reasoning to explore the existence of other SNPEs in SJ is of the "Nash reversion strategy" family, often used to analyze repeated games: cooperative results are sustainable in the long run as an alternative to the fall to an infinite sequence of inconvenient SPNEs, one in each game.

A rule **A** to play a TGM is a function A: $\Omega \rightarrow \Sigma_1(v) \times \Sigma_1(v)$, that for each possible value v of the parameters describing the TGM, determines the strategies $A_1(v)$, $A_2(v)$ to be played by both players in the TGM.

A rule **A** is Pareto optimal if for any $v \in \Omega$, both players agree an immediate partition s(A, v) in the first period t=0.

Let us call N(t) the ordinal of the TGM played in period t, starting with N(0)=1. N(t) is a random variable so that $N(t) \leq t+1$.

Let A be a Pareto optimal rule.

Let $G_i(A)$ be the following strategy for player i, to play SJ.

- At t=0, play $A_1(v_1)$, where v_1 is the set of parameters of TGM_1 determined by Nature.
- At t>0:
  - o Play $A_i(v_{N(t)})$ if in the entire history of the game SJ, both players have played their respective $A_1(v_n)$, $A_2(v_n)$ for every n=1, …..N(t),
  - o Play $\sigma^\star_i(v_{N(t)})$ in any other case.

If both players have played $A_i(v_\mu)$ for every $\mu < n$, the result when a new TGM_n starts is:

- If both players abide to $A_i(v_n)$, they reach immediate agreement with partition (A, v_n).
- If any of the players i deviates from his respective $A_i(v_n)$, both players turn in the next period to their respective strategies in the SPNE $(\sigma^\star_1(v_n), \sigma^\star_2(v_n))$ and keep on playing $(\sigma^\star_1(v_m), \sigma^\star_2(v_m))$ in the infinite sequence of the following TGM_m for m> n.

Our problem is to find conditions for the pair of strategies $G_1(A)$, $G_2(A)$ to be a SPNE in SJ with certainty, that is for every realization of the random variables. This is equivalent without loss of generality to conditions for $G_1(A)$ to be a better reply to $G_2(A)$ in every subgame of SJ for every possible realization of the random variables v_n, n=1, …..

Let us admit the following restrictive hypothesis:

- Probabilities of survival p_n are known instead of random. The probability of survival in TGM_1 is p^1. The probabilities of survival in TGM_n, for n>1, are equal to p^d.
- The probability distribution functions φ_n of parameters v_n are identical for every n>1: $\varphi_n \equiv \varphi^d$, for every n>1.

The sequence of survival probabilities p_n is then:

$p_1 = p^1$

$p_n = p^d$ for every n>1.

The sequence of sets where variables v_n take values is then Ω_n, n=1,…., with:

$\Omega_1 = \Omega^1$

$\Omega_n = \Omega^d$, for every n>1

Where $\Omega^1=\{v: v(r,b,\pi) \in \Omega, \pi=p^1\}$ and $\Omega^d=\{v: v(r,b,\pi) \in \Omega, \pi=p^d\}$

Proposition IV

The necessary and sufficient condition for strategies $G_1(A)$, $G_2(A)$ to be a SPNE in SJ with certainty is:

i) for any strategy S'_1 of player 1's, different from $G_1(A)$, every $v_n \in \Omega_n$ and every n, $\Delta' < 0$, where:

- $$\Delta' = \Pi_{1,n}(S'_1, G_2(A), v_n) - s(A, v_n)B^{\infty}(b_n, p_n\delta) - (\Pi_1^A - \Pi_1^*)\left[(1-p^d)\frac{\delta}{1-\delta} + (p^d - p_n)\frac{\delta}{1-p_n\delta}\right]$$
- $\Pi_{1,n}(S'_1,G_2(A),v_n)$ is the expected benefit for player 1 in TGM_n, discounted to the beginning of TGM_n, when player 1 employs in SJ the alternative strategy S'_1 against $G_2(A)$.
- Π^*_1 is the expected benefit for player 1 in every SPNEs ($\sigma^*_1(v_n)$, $\sigma^*_2(v_n)$) of TGM_n, for n>1.
- Π^A_1 is the expected benefit for player l in every TGM when rule A is played in the TGM_n, for n>1.

ii) the symmetric for player 2 substituting 1- $s(A,v_n)$ for $s(A,v_n)$.

Let us classify the subgames of SJ that start with a move by player 1 into two sets:

- The set C_a of the subgames with a history of both players using always $G_1(A)$ and $G_2(A)$, that is following rule A. They are subgames beginning at the first period of a TGM and where player 1 begins the TGM by making the first offer. (In the symmetric proof for player 2 the first offer in the TGM was made by player 1 following rule A and player 2 has to decide whether to accept it following rule A or to reject it).
- The set C_b of all the other subgames initiated by player 1, which have a history of deviation from rule A by at least one player.

Lemma 1: The pair of strategies (G_1(A), G_2(A)) is a Nash equilibrium for every subgame in set C_b, for every realization of the random variables.

Proof

Let H be a subgame of set C_b, dated at t and beginning at the n-th TGM. As part of G_2(A) player 2 plays $\sigma^\star_2(v_m)$ in every TGM_m for every m≥ n, unconditionally. Player 1's strategy with G_1(A) requires playing $\sigma^\star_2(v_m)$ for every m≥ n. The rule has been broken in a previous period and both players use their respective strategies in ($\sigma^\star_1$(v), $\sigma^\star_2$(v)) for every TGM, from period t on. Therefore strategies (G_1(A), G_2(A)) behave in the same way as strategies ($S^\star_1$, $S^\star_2$) in all the subgames in C_b. Proposición III states that ($S^\star_1$, $S^\star_2$) are a SPNE in SJ, so the pair of strategies (G_1(A), G_2(A)) is a Nash equilibrium in every subgame in C_b.

End of the proof

Lemma 2: Conditions i) and ii) are necessary and sufficient for (G_1(A), G_2A)) to be with certainty a Nash equilibrium in every subgame belonging to C_a.

Proof

Let TGM_n be the trade game containing the beginning of subgame H of SJ, with H $\in C_a$. H begins at t_n, the first period of TGM_n.

Let S'$_1$ be a strategy for player 1 different from G_1(A) to play SJ, and $\sigma'_{1,\mu}$ the strategy derived from S'$_1$ in trade game TGM_μ.

For G_1(A) to be a best reply than any other strategy S'$_1$, in every subgame H$\in C_a$ beginning at t_n, the initial period of TGM_n, given that 2 plays G_2(A), the following must hold:

$$\Delta = \Pi_{1,n}(S'_1, G_2(A), v_n) + \sum_{\mu=1}^{\infty}\sum_{\tau=\mu}^{\infty}\delta^{\tau}P(\mu,\tau)E\left[\Pi_{1,n+\mu}(S'_1, G_2(A), v_{n+\mu})\right]$$
$$- s(A, v_n)B^{\infty}(b_n, p_n\delta) - \sum_{\mu=1}^{\infty}\sum_{\tau=\mu}^{\infty}\delta^{\tau}P(\mu,\tau,n)E\left[s(A, v_{n+\mu})B^{\infty}(b_{n+\mu}, p^d\delta)\right] < 0 \qquad (8)$$

for all S'$_1$, for all $v_n \in \Omega_n$, for all n.

where:

$\Pi_{1,n+\mu}$(S'$_1$, G_2(A), $v_{n+\mu}$) is the expected benefit for player 1 in $TGM_{n+\mu}$, with parameters $v_{n+\mu}$,, discounted to the beginning of this trade game, when player 1 plays S'$_1$ in SJ (and plays $\sigma'_1(v_{n+\mu})$ in $TGM_{n+\mu}$) and player 2 plays G_2(A) in SJ (and therefore $\sigma^\star_2(v_{n+\mu})$ in $TGM_{n+\mu}$). This benefit is a random variable because the parameters and the duration of the game are random variables. E[$\Pi_{1,n+\mu}$(S'$_1$, G_2(A), $v_{n+\mu}$)] is the expected value.

$s(A,v_\mu)B^\infty(b_\mu,p_\mu\delta)$ is the expected benefit for player 1, discounted to t, in TGM_μ (a trade game with survival probability p_μ) when player 1 keeps on playing G_1(A), reaching agreement at the first period of TGM_μ. $E[s(A,v_\mu)B^\infty(b_\mu,p_n\delta)]$ is the expected value of this benefit.

$P(\mu,\tau,n)$ is the conditional probability of $TGM_{n+\mu}$ (the (n+μ)-th TGM) starting τ periods after t, given the event that at period t trade game TGM_n is being played. $P(\mu,\tau, n)$ depends of the values of parameters p^1 and p^d. It is impossible for trade game TGM n+μ-th to start before period t+μ, given that at t game n-th is being played, as any TGM lives at least one period.

As ($\sigma^\star_1(v_{n+\mu})$, $\sigma^\star_2(v_{n+\mu})$) is a Nash equilibrium in TGM($v_{n+\mu}$), and given that $G_2(A)$ plays $\sigma^\star_2(v_{n+\mu})$ in every trade game following TGM_n, then:

$$\Pi_{1,n+\mu}(S'_1(v_{n+\mu}),\ G_2(A), v_{n+\mu}) \le \Pi_{1,n+\mu}(G_1(A),\ G_2(A), v_{n+\mu})$$

This means that when confronted with $G_2(A)$, any strategy S'_1 that does not play $\sigma^\star_1(v_{n+\mu})$ in the subsequent TGMs is dominated.

As the parameters v_n for n>1 have identical distributions, the expected benefits in $TGM_{n+\mu}$, for $\mu=1,2,...,$ are constant and do not depend on μ. Therefore we can define the following constants:

$$\Pi^\star_1 = E[\Pi_{1,n+\mu}(G_1(A),\ G_2(A), v_{n+\mu})] \text{ , for every } \mu \geq 1. \tag{9}$$

$$\Pi^A_1 = E[s(A,v_{n+\mu})B^\infty(b_{n+\mu},p^d\delta)], \text{ for every } \mu \geq 1.$$

Eliminating dominated strategies and substituting in condition (8) for the constants defined in (9), yields:

$$\Delta' = \Pi_{1,n}(S'_1, G_2(A), v_n) - s(A,v_n)B^\infty(b_n, p_n\delta) - (\Pi_1^A - \Pi_1^*)\sum_{\nu=1}^{\infty}\sum_{\tau=\mu}^{\infty}\delta_1^{\tau}P(\mu,\tau,n) \le 0 \tag{10}$$

for all S'_1, for all $v_n \in \Omega_n$, for all n

Substituting expression (A3) in the Annex for the double summation in (10) yields:

$$\Delta' = \Pi_{1,n}(S'_1, G_2(A), v_n) - s(A,v_n)B^\infty(b_n, p_n\delta) - (\Pi_1^A - \Pi_1^*)\left[(1-p^d)\frac{\delta}{1-\delta} + (p^d - p_n)\frac{\delta}{1-p_n\delta}\right] \le 0$$

for all S'_1, for all $v_n \in \Omega_n$, for all n. (11)

$\Gamma_{1,n} = \Pi_{1,n}(S'_1, G_2(A), v_n) - s(A,v_n)B^\infty(b_n,p_n\delta)$ is the expected "present" gain for player 1 in TGM_n when he deviates from rule A.

($\Pi_1^A - \Pi_1^\star$) is the expected value of the loss in each TGM_m (for m>n), when he deviates from rule A and the game falls to inefficient SPNEs, discounted to the beginning of TGM_m. If the SPNE is "inefficient enough" and rule A is equitable for both players one must expect ($\Pi_i^A - \Pi_i^\star$) to be positive for i=1, 2.

$(\Pi_1^A - \Pi_1^*)\left[(1-p^d)\frac{\delta}{1-\delta} + (p^d - p_n)\frac{\delta}{1-p_n\delta}\right]$ is the expected "future" total loss for player 1 when he deviates from rule A, incurred in the infinite sequence of TGM_m , for all m>n.

For n=1, and as $p_n=p^1$, the condition that must hold in the first trade game for the rule A to be sustainable is:

$$\Delta' = \Pi_{1,1}(S'_1, G_2(A), v_1) - s(A, v_1)B^{\infty}(b_1, p_1\delta) - (\Pi_1^A - \Pi_1^*)\left[(1-p^d)\frac{\delta}{1-\delta} + (p^d - p^1)\frac{\delta}{1-p_1\delta}\right] < 0$$

for all S'_1, for all $v_1 \in \Omega^1$ (12)

and its symmetric for player 2, substituting 1 - $s(A,v_1)$ for $s(A,v_1)$.

Expression $\left[(1-p^d)\frac{\delta}{1-\delta} + (p^d - p^1)\frac{\delta}{1-p^1\delta}\right]$ is decreasing in p_1, and it is null when p^1 equals 1.

$(\Pi_1^A - \Pi_1^*)$ does not depend on p^1, but on p^d.

This means that when the survival probability p^1 of TGM_1 increases, the future loss due to a deviation from the rule decreases, and deviation becomes more profitable.

For n>1, as $p_n=p^d$ the condition for rule A to be sustainable is:

$$\Delta' = \Pi_{1,n}(S'_1, G_2(A)) - s(A, v_n)B^{\infty}(b_n, p^d\delta) - (\Pi_1^A - \Pi_1^*)\left[(1-p^d)\frac{\delta_1}{1-\delta_1}\right] < 0$$

for all S'_1, for all $v_n \in \Omega^d$

$\Pi_1^A - \Pi_1^*$ may depend on p, in a way resulting from rule A and the form of SNPEs in each TGM.

5 Conclusion

An intuitive interpretation of the conditions found in the preceding section is the following:

- At the beginning of any trade opportunity the probable "present" gain obtained by an opportunist deviation from a rule must be smaller than the "future" losses from the fall to the infinite sequence of inefficient equilibria.
- $(\Pi_i^A - \Pi_i^*) > 0$ for i=1,2, means that to be sustainable, the rule must be equitable and give better expected results for both players than the sequence of inefficient equilibria.
- The survival probability of the "present" trade opportunity TGM_1 must be small enough so that even a player who has been very lucky in the first TGM does not feel tempted to deviate from the rule.

The results can be applied to assess the feasibility of long run rules to determine prices for international energy spot trade:

- The more inefficient the results from case by case price negotiation (the longer the delays to achieve price agreement) the greatest incentive for countries to develop such long run rules.
- If a country's authorities estimate a high probability of maintaining favorable technical and economic conditions to negotiate trade prices, they will be reluctant to accept long run equitable rules limiting the possibility of opportunistic gains.
- It is reasonable to assume that on the contrary, supply crisis leading to very high marginal costs or risks of energy rationing put a country in a weak position to bargain over the benefits of trade.
- In the present context of the energy systems in the Mercosur countries, there seem to be significant differences in expected energy marginal costs and energy availability between countries, at least for the next few years. Therefore the short term economic incentives in the electric sector may be against the negotiation of long term stable price rules for energy trade. A fruitful negotiation of such rules could arise if the long run benefits of economic integration as a whole, beyond the energy sector, are taken into account.

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Annex

Let us call $SUM(\delta,n)=\sum_{\nu=1}^{\infty}\sum_{\tau=\mu}^{\infty}\delta^{\tau}P(\mu,\tau,n)$

Permutating the order of summation:

$$SUM(\delta,n)=\sum_{(\tau,\mu)\in N_+\times N_+,\tau\geq\mu}\delta_1^{\ \tau}P(\mu,\tau,n)=\sum_{\tau=1}^{\infty}\delta_1^{\ \tau}\sum_{\mu=1}^{\tau}P(\mu,\tau,n) \qquad \text{(A1)}$$

P(μ,τ,n) is the conditional probability of $TGM_{n+\mu}$ beginning τ periods after t, given the event that TGM_n is being played at period t.

It is convenient to separate the sumatory from (A1) in:

$$SUM(\delta_1,p_n,p,n)=\sum_{\tau=1}^{\infty}\delta_1^{\ \tau}\left[P(1,\tau,n)+\sum_{\mu=2}^{\tau}P(\mu,\tau,n)\right] \qquad \text{(A2)}$$

P(1,τ, n) is the probability of TGM_{n+1} starting at t+τ, given the event that TGM_n is being played at period t. It is the probability of τ-1 successes followed by a failure in τ Bernoulli trials with probability of success p_n. Therefore: P(1,τ, n) = $p_n^{\tau-1}(1-p_n)$

The sumatory in μ is the conditional probability of the event consisting of any of the trade games TGM_{n+2}, TGM_{n+3},... $TGM_{n+\tau}$ beginning at t+τ-1, given the event that TGM_n is being played at t. That is equivalent to the probability of being in period t+τ−1 in any of the TGMs different from TGM_n (that is $1-p_n^{\tau-1}$) times the probability of TGM_n ending at t+τ−1 (that is $1-p^d$). The value of the sumatory is then $(1-p_n^{\tau-1})(1-p^d)$.

Therefore:

$$SUM(\delta,n)=\sum_{\tau=1}^{\infty}\delta^{\tau}\left[p_n^{\tau-1}(1-p_n)+(1-p_n^{\tau-1})(1-p^d)\right]=\sum_{\tau=1}^{\infty}\delta^{\tau}\left[(1-p^d)+p_n^{\tau-1}(p^d-p_n)\right]=$$

$$=(1-p^d)\sum_{\tau=1}^{\infty}\delta^{\tau}+(p^d-p_n)\sum_{\tau=1}^{\infty}\delta^{\tau}p_n^{\tau-1}=(1-p^d)\frac{\delta_1}{1-\delta_1}+(p^d-p_n)\frac{\delta}{1-p_n\delta} \qquad \text{(A3)}$$