Optimal investment allocation under embodiment in a two sector model

Abstract: In this paper the impact of investment specific technological progress on investment allocation is studied by using a two sector model that takes into account investment specific technological progress. The aim of the paper is to analyze the optimal allocation of capital goods and to revisit the recent controversy on the structural change played by the Information and Communication Technology-growth enhancing role.

Keywords: Investment allocation, two sector models, investment specific technical progress.

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1 Introduction

The suspicious that technological progress and capital accumulation cannot be dissociated was raised by Solow (1957/1962) who admitted that a large portion of it is specific to capital goods. Following this initial insight some authors such as Phelps (1962), Nelson (1964) and Greenwood and Yorukoglu (1974) have also confirmed that some gains in productivity can accrue from an increase in the efficiency of capital goods. Despite the fact that these authors have reported an increase in the rate of embodied technological progress this hypothesis seems to have gained force only in the nineties after more empirical research has shown that about 60% of U.S. economic growth is due to investment specific technical change while the Hicks-neutral technical progress accounts for the remaining 40%. (GREENWOOD et al. 1997).

The investment specific technical change – ISTC hereafter – may be defined as technological change embodied in the form of new equipment, representing phenomena such as advances in computer technology, robotization of assembly lines, faster and more efficient means of telecommunications, and so on. This type of technological innovation is different from the usual changes in total factor productivity in which capital of different generations is thought of as being the same type of good, or having the same cost as previous vintages of capital (i.e. as measured in units of the consumption good). In case it is found that ISTC accounts for a considerable fraction of total growth in total factor productivity, it is suggested an important role for investment in spurring productivity growth above and beyond its traditional role of capital deepening.

Greenwood at al. (1997/2000), motivated by the negative movement between the relative price of new equipment and equipment investment, have found that ISTC, by triggering equipment investment, may be the most important source of long-term growth. Cummins & Violante (2002) measured technical change at the asset, industry and aggregate level in the U.S. from 1947 to 2000 and found that technological improvement in equipment and software accounts for an important fraction of output growth and plays a key role in the productivity resurgence of the 1990s. More precisely, improvement in the quality of equipment and software explains about 20% of growth in the US in the postwar period and about 30% of growth in the 1990s. Besides, they found that 60% of labor productivity growth in the postwar period comes from technological advances in equipment and software.

Other authors such as Iglesias (2002) and Licandro et al. (2002) have not only confirmed this result but also found that the rate of ISCT has fostered and the rate of neutral technical progress has decreased after 1974. Boucekkine et al. (2002, p. 76) have emphasized that "there has been an acceleration in the rate of embodied technical progress since 1974, corresponding to the rise of the information technologies." Kosempel and Carlaw (2003) have verified a similar result for the Canadian economy, finding that the economic growth after 1974 is almost completely explained by investment specific technical change.

Collechia and Schreyer (2002) have estimated the contribution of technical progress investment specific in total investment in many OCDE countries and have concluded that some of the countries in the cross section were able to reap larger benefits than others. From this finding it is possible to conclude that not only the investment but also its allocation play an important role in harvesting the benefits of information and communication technologies embodied in capital goods.

The flavour of this result is not a novelty. A number of authors such as Bose (1971), Weitzman (1971), Araujo and Teixeira (2002) and Araujo (2004) have shown that the decisions of investment allocation play an important role in economic growth since the rate of investment allocation determines the growth rate of output in a closed economy. But a common characteristic of these analyses is that they do not take into account technological progress embodied in capital goods. If larger portions of it are embodied in capital goods then the decisions on investment allocation are also decisions on the allocation of technological progress. In this paper this point is analyzed by introducing investment specific technological progress in a two sector model departing from the Feldman's contribution.

The structural change between the capital and consumption goods sectors that arise from this extended model may be used to reconcile two different views on the extent of the New Economy in the U.S. economy in the nineties: it shows that even though ICT is a general purpose technology as pointed out by Jorgenson and Stiroh (2002) its concentration on the durable sector as claimed by Gordon (2002) and Acemoglu (2002) may arise as the optimal outcome of a representative agent model who maximizes intertemporal consumption in a two sector model. This paper is structured as follows. In Section 2, the basic model is presented and the optimal rate of allocation of equipment is established for a centrally planed economy. The results show how the analysis can be used to explain the stylized facts of the U.S. economic growth in the nineties and section 3 concludes.

2 The Model

Let us assume a closed economy with two sectors: the capital goods and the consumption goods sectors. The capital goods are used by both sectors but, once investment is made, they cannot be transferred from one sector to the other (irreversibility assumption). Following Greenwood et al. (1997) let us consider that the first sector is a traditional one that produces a non-durable commodity that can be consumed or invested and uses both capital goods and equipments according to a Cobb-Douglas production function:

$$Y = [(1-a)K_e]^{\alpha_e} K_s^{\alpha_s} L^{1-\alpha_e-\alpha_s} \tag{1}$$

Where $0 < a < 1$ is the fraction of the stock of equipments, K_e, which is installed in the *IT* sector and the remainder is installed in the traditional sector to produce *Y*, which is the amount of non-durable good produced in the economy; K_s refers to the stock of capital goods and *L* stands for the labor force. The symbols α_s and α_e stand for the shares of equipments and capital goods respectively. Following Boucekkine et al. (2004) let us consider that the technology in the *IT* sector is AK:

$$Y_e = aK_e \tag{2}$$

Where Y_e stands for the production of equipments. For the sake of convenience only it is assumed that the productivity of capital is equal to one, that is $A=1$. Let us consider that the investment specific technological progress is implemented through the equation of investment in equipments in the lines suggested by Greenwood et al. (1987) and followed by a number of authors [see Boucekkine et al. (2004)]. In this case, the law of motion of the stock of equipments is given by:

$$\dot{K}_e = q(aK_e) - \delta K_e \tag{3}$$

Where δ stands for depreciation and $q > 1$ captures the investment specific nature of technological progress. The stock of traditional capital goods varies according to the following expression:

$$\dot{K}_s = Y - C - \delta K_s \tag{4}$$

In order to determine the optimal rate of allocation of stock of equipments let us assume that the central planner solves the following problem in per capita terms:

$$\underset{c,a}{Max}\int_0^\infty \ln(c)e^{-(\theta-n)t}dt$$

s.a. $$\dot{k}_e = qak_e - (n+\delta)k_e \tag{5}$$

$$\dot{k}_s = [(1-a)k_e]^{\alpha_e} k_s^{\alpha_s} - c - (n+\delta)k_s \tag{6}$$

where $\theta > 0$ is the social rate of pure time discount and n is the growth rate of population. Introducing two co-state variables λ and μ related to the investment in sectors 1 and 2 respectively, allows us to write the corresponding Hamiltonian as follows:

$$H = \ln(c) + \lambda\{qak_e - (n+\delta)k_e\} + \mu\{[(1-a)k_e]^{\alpha_e} k_s^{\alpha_s} - c - (n+\delta)k_s\} \tag{7}$$

The first order conditions are:

$$c:\ \frac{\partial H}{\partial c} = 0 \Rightarrow \ \mu = c^{-1} \tag{8}$$

a: $$\frac{\partial H}{\partial a} = 0 \Rightarrow \ \lambda q k_e = \mu\alpha_e(1-a)^{\alpha_e-1} k_e^{\alpha_e} k_s^{\alpha_s} \tag{9}$$

The Hessian matrix associated to the Hamiltonian is the following:

$$\mathrm{H} = \begin{bmatrix} -c^{-2} & 0 \\ 0 & c^{-1}\alpha_e(\alpha_e-1)(1-a)^{\alpha_e-2} k_e^{\alpha_e} k_s^{\alpha_s} \end{bmatrix}$$

The minor principals are given by: $|\mathrm{H}_2| = c^{-3}\alpha_e(1-\alpha_e)(1-a)^{\alpha_e-2} k_e^{\alpha_e} k_s^{\alpha_s} > 0$, and $|\mathrm{H}_1| = -c^{-2} < 0$. It implies that the Hamiltonian is concave since the Hessian matrix is negative definite. Hence we can guarantee that the optimal solution is in fact a maximum. The Euler equations are:

$$\dot{\lambda} - (\theta-n)\lambda = -\frac{\partial H}{\partial k_e}$$

$$\dot{\lambda} - (\theta-n)\lambda = -\{\lambda[qa-(n+\delta)] + \mu[(1-a)^{\alpha_e}\alpha_e k_e^{\alpha_e-1} k_s^{\alpha_s}]\} \tag{10}$$

$$\dot{\mu} - (\theta-n)\mu = -\frac{\partial H}{\partial k_s} \Rightarrow$$

$$\dot{\mu} - (\theta-n)\mu = -\{\mu[(1-a)^{\alpha_e} k_e^{\alpha_e}\alpha_s k_s^{\alpha_s-1} - (n+\delta)]\} \tag{11}$$

The transversality conditions are given by:

$$\lim_{t\to\infty}\lambda(t)k_e(t)e^{-\rho t} = 0 \tag{12}$$

$$\lim_{t\to\infty}\mu(t)k_s(t)e^{-\rho t} = 0 \tag{13}$$

From expression (11) we obtain:

$$\frac{\dot{\mu}}{\mu} = \delta + \theta - (1-a)^{\alpha_e} k_e^{\alpha_e} \alpha_s k_s^{\alpha_s - 1} (1-\varphi)^{1-\alpha_e-\alpha_s} \tag{14}$$

By substituting expression (9) into expression (11) it yields after some algebraic manipulation:

$$\frac{\dot{\lambda}}{\lambda} = \delta + \theta - q \tag{15}$$

In steady state by equalizing (14) to (15) we conclude that:

$$(1-a)^{\alpha_e} k_e^{\alpha_e} \alpha_s k_s^{\alpha_s - 1} = q \tag{16}$$

By differentiation of expression (8) we conclude that $\frac{\dot{c}}{c} = -\frac{\dot{\mu}}{\mu}$. Hence the growth rate of consumption in steady state is given by:

$$\frac{\dot{c}}{c} = q - \delta - \theta \tag{17}$$

The per capita output of the traditional sector is given by the following expression:

$$y = (1-a)^{\alpha_e} k_e^{\alpha_e} k_s^{\alpha_s} (1-\varphi)^{1-\alpha_e-\alpha_s} \tag{18}$$

By taking logs in both sides of (18) and by taking the derivative in relation to time we conclude that:

$$\frac{\dot{y}}{y} = \alpha_e \frac{\dot{k}_e}{k_e} + \alpha_s \frac{\dot{k}_s}{k_s} \tag{19}$$

Considering that in steady state $\frac{\dot{k}_e}{k_e} = \frac{\dot{k}_s}{k_s}$ it is possible to obtain the relationship between the growth rate of output of this sector and the growth rate of the stock of equipments: $\frac{\dot{y}}{y} = (\alpha_e + \alpha_s)\frac{\dot{k}_e}{k_e}$. As $\alpha_e + \alpha_s < 1$ the growth rate of per capita output of the traditional sector is smaller than the growth rate of the ICT sector. Since in steady state $\frac{\dot{c}}{c} = \frac{\dot{y}}{y}$ and considering that from (5):

$$\frac{\dot{k}_e}{k_e} = qa - n - \delta \tag{20}$$

In steady state we have:

$$\frac{\dot{c}}{c} = (\alpha_e + \alpha_s)\frac{\dot{k}_e}{k_e} \tag{21}$$

By inserting expressions (17) and (20) in the above expression we obtain after some algebraic manipulation the optimal value of capital allocation a* in steady state:

$$a^* = \frac{(n+\delta)(\alpha_e + \alpha_s) + q - \theta - \delta}{(\alpha_e + \alpha_s) q} \tag{22}$$

Taking the derivative of a* with respect q allows us to conclude that:

$$\frac{\partial a^*}{\partial q} = \frac{\delta + \theta + (n + \delta)(\alpha_e + \alpha_s)}{(\alpha_e + \alpha_s)q^2} > 0 \tag{23}$$

This expression shows us that the higher the pace of investment specific technological progress the higher the rate of allocation of equipment that must be made in the ICT sector in order to meet the optimality conditions. On the other hand when the pace of ICT slowdowns it is optimal to spread this technology throughout the economy in order to compensate the smaller productivity of equipments by transferring a larger physical amount to the traditional sector. This result may help us to understand the optimality of a stylized fact of the U.S. economy in 1990's.

According to Gordon (2000, p. 72), "The New Economy has created a dynamic explosion of productivity growth in the durable manufacturing sector (…). However the New Economy has meant little to the 88 percent of the economy outside durable manufacturing." This result can be rationalized by expression (22) since it shows that it is optimal to concentrate the stock of equipments in the ICT sector if the pace of investment specific technological progress increases.

Hence if the economy faces a boost in the rate of investment specific technological progress, which means that every vintage of equipment embodies higher levels of technological progress, it is optimal to concentrate the allocation of equipments in the ICT sector which seems the case of U.S. economy in the 1990's. Following this interpretation although ICT is a general purpose technology as supported by Jorgenson and Stiroh (2000) it may be optimal to concentrate it in the durable sector.

3 Conclusion

In this paper departing from a two sector model of investment allocation the optimal rate of investment allocation that maximizes intertemporal consumption is established. The result for the case that the embodied technological progress is constant for every vintage shows that the higher the investment specific technological progress the higher the optimal rate of investment allocation. In this case the gains in productivity that accrue from the increasing pace of embodied technological progress will reach the traditional sector without the necessity of keeping or increasing the initial allocation of equipments in this sector. This may be an explanation for the Gordon's view that the ICT were restricted to the ICT sectors. Despite the fact that ICT is a general purpose technology it is optimal to concentrate it in the durable sector since its productivity gains may be conveyed to the traditional sector through a higher efficiency of the electronic equipments.

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