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# ‘Own’ price influences in a Stackelberg leadership with demand uncertainty

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**Abstract**: We consider a Stackelberg duopoly model with demand uncertainty only for the first mover. We study the advantages of leadership and flexibility with the variation of the demand uncertainty and also with the variation of the ‘own’ price effect. We compute, in terms of the demand uncertainty and of the ‘own’ price effect parameter, the probability of the second firm to have higher profit than the leading firm. We prove that, even in presence of low uncertainty, the expected value of the profit of the second firm increases to higher values than the ones of the leading firm with the increase of the ‘own’ price effect.

**Key words**: Perfect Bayesian equilibrium, Stackelberg duopoly, uncertainty.
JEL Classification: C72, D43, L11.

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## 1 Introduction

The model introduced by Stackelberg (1934) is one of the most widely used models in industrial organization for analyzing firms' behavior in a competitive environment. It studies the strategic situation where firms sequentially choose their output levels in a market. The question we ask is: Do first movers really have strategic advantage in practice? The belief of first-mover advantage was widely held among entrepreneurs and venture capitalists, but is now questioned by numerous practitioners. We indicate some examples of successful pioneering firms (as described in Liu (2005)): Dell was the first to introduce the direct-sale business model into the PC market, and it achieved great success, growing from Mr. Dell's small-dorm business into a giant in the PC market. However, we can find many counterexamples. During the dot-com booming era, Pets.com, Webvan, Garden.com and eToys were all first movers in their respective market segments, but they all ended up burning through their investment capital before attracting enough customers to sustain a business (see Stalter (2002)). Why do these pioneering firms get very different results? The probability of success of pioneering in a market clearly depends on many factors, including technology, marketing strategy, market demand and product differentiation. Several research papers focus their attention in giving answers to such question. In this paper, we extend Liu's results by focus not only on the effects of the market demand uncertainty, but also on the 'own' price effect, to explain the advantages and disadvantages of being the leading firm. Usually, the followers in markets get more market information than first movers before sinking their investments. In some industries that we consider to have fairly stable and predictable market demand, the pioneering firm tends to be the biggest player. However, if a market has a high degree of uncertainty, the followers can wait and see the customers' response to the new product introduced by the first movers, as well as move along the "differentiation curve" of innovation.

As in the model studied by Liu (2005), we consider that only the first mover (leading firm) faces demand uncertainty. The demand uncertainty is given by a random variable uniformly distributed, with mean $\mu$ and standard deviation $\sigma$ characterizing the *demand uncertainty parameter* $\theta = \left(\mu + \sqrt{3}\sigma\right)/\left(\mu - \sqrt{3}\sigma\right)$. By the time the second mover chooses its output level, that uncertainty is resolved. Therefore, the leading firm possesses first-mover advantage, but the second mover enjoys an informational advantage because it can adjust the production level after observing the realized demand (flexibility). We study the advantages of flexibility over leadership as the 'own' price effect parameter $\beta \geq 1$ changes. We find explicit functions $I_\beta$ and $J_\beta$, in terms of the 'own' price effect parameter $\beta$, characterizing the demand uncertainty parameter $\theta$ for which the leading firm looses its advantage for some realizations of the demand random variable. We show that the leading firm looses its advantage for high values of the demand intercept, if the demand uncertainty parameter $\theta$ is greater than $I_\beta$, and for low values of the demand intercept, if the demand uncertainty parameter $\theta$ is greater than $J_\beta$ (see Figure 7). Hence, for high values of the demand uncertainty parameter ($\theta > J_\beta$) only in an intermediate zone of the realized demand does the first mover preserve its advantage. We observe that for 'own' price effect equal to cross price effect ($\beta = 1$), the functions $I_\beta$ and $J_\beta$ coincide, i.e $I_1 = J_1$, and for 'own' price effect greater than cross price effect ($\beta > 1$), we have that $I_\beta > J_\beta$.

We make an ex-ante analysis by computing the expected value, with respect to the demand realization $\alpha$, of the profits of both firms in terms of the demand uncertainty parameter $\theta$ and of the 'own' price effect parameter $\beta$ (see Figure 2). In particular, we prove that, even in the presence of low uncertainty, the expected value of the profit of the second firm increases to higher values than the ones of the leading firm with the increase of the `own' price effect (see Figures 3 and 4). Moreover, we show that there is a value $\theta_0$ such that if the uncertainty parameter $\theta$ is greater than $\theta_0$, then the expected profit of the follower firm is always greater than the expected profit of the leading firm. We also make an ex-post analysis by computing and comparing the firms' profits after the demand uncertainty has been resolved. We also compute the probability $P(\pi_2^* > \pi_1^*)$ of the second firm to have higher profit than the leading firm in terms of the demand uncertainty parameter and of the 'own' price effect parameter (see Figure 9). We show that the probability of the follower firm to have higher profit than the leading firm increases with the degree of the demand uncertainty, and with the 'own' price effect. Furthermore, when the 'own' price effect is equal to the cross price effect ($\beta = 1$), the probability $P(\pi_1^* > \pi_2^*)$ of the leading firm to have higher profit than the follower firm is greater than the probability of the opposite situation; however, when the 'own' price effect is sufficiently greater than the cross price effect, for sufficiently high level of uncertainty, the probability $P(\pi_2^* > \pi_1^*)$ of the follower firm to have higher profit than the leading firm is greater than the probability of the opposite situation.

## 2 The model and the perfect Bayesian equilibrium

We start by describing the Stackelberg duopoly model. The demand, for simplicity, is linear, namely

$$\begin{cases} p_1 = \alpha - \beta q_1 - q_2 \\ p_2 = \alpha - q_1 - \beta q_2 \end{cases}, \quad (1)$$

with $\alpha > 0$ and $\beta \geq 1$, where $p_i$ is the price and $q_i$ the amount produced of good by the firm $F_i$, for $i \in \{1, 2\}$. We note that, since $\beta \geq 1$, 'cross' effects are dominated by 'own' effects. Firms have the same constant marginal cost $c$. We consider from now on prices net of marginal costs. This is without loss of generality since if marginal cost is positive, we may replace $\alpha$ by $\alpha - c$. We consider that the demand intercept is a random variable uniformly distributed in the interval $[\alpha_0, \alpha_1]$, with $\alpha_1 > \alpha_0 > 0$. We note that, in this case, the demand uncertainty parameter $\theta$ is equal to the ratio $\alpha_1 / \alpha_0$. The distribution of $\alpha$ is of common knowledge. Profit $\pi_i$ of firm $F_i$ is given by

$$\pi_i = p_i q_i = (\alpha - \beta q_i - q_j)\, q_i. \quad (2)$$

As already described in the Introduction, the timing of the game is as follows:

i) Firm $F_1$ chooses a quantity level $q_1 \geq 0$ without knowing the value of the demand realization;

ii) Firm $F_2$ first observes the demand realization and observes $q_1$, and then chooses a quantity level $q_2 \geq 0$.

In the next theorem we show that this game has a uni\que perfect Bayesian equilibrium $(q_1^*, q_2^*)$. Let

$$K_\beta = \frac{8\beta^2 - 2\beta - 3}{2\beta - 1},$$

$A = 16\beta^2 (\beta^2 - 1) + 6\beta + 1$, $B = 16\beta^2 (2\beta^2 - 1)$ and $C = 2\beta (8\beta^3 - 3)$, and denote by $\Delta$ and $\Sigma$, respectively, the expressions

$$\Delta \equiv \Delta(\beta, \alpha_0, \alpha_1) = 4\beta^2\alpha_0 - 2(2\beta^2 - 1)\,\alpha_1 + \sqrt{A\alpha_1^2 - B\alpha_0\alpha_1 + C\alpha_0^2}$$

and

$$\Sigma \equiv \Sigma(\beta, \theta) = 4\beta^2 - 2(2\beta^2 - 1)\theta + \sqrt{A\theta^2 - B\theta + C}. \quad (3)$$

We observe that $A\alpha_1^2 - B\alpha_0\alpha_1 + C\alpha_0^2 \geq 0$, for all $\theta = \alpha_1 / \alpha_0 > 1$, and so $\Delta$ and $\Sigma$ are well-defined. We also note that $\Delta = \alpha_0\, \Sigma$.

**Theorem 1** *Consider a Stackelberg duopoly model facing the demand system , where the parameter $\alpha$ is uniformly distributed in the interval $[\alpha_0, \alpha_1]$. Then, there is a unique perfect Bayesian equilibrium $(q_1^*, q_2^*)$ given as follows:*

(i) If $\theta \leq K_\beta$, then

$$q_1^* = \frac{2\beta - 1}{4(2\beta^2 - 1)}(\alpha_1 + \alpha_0)$$

and

$$q_2^* = \frac{\alpha}{2\beta} - \frac{2\beta - 1}{8\beta(2\beta^2 - 1)}(\alpha_1 + \alpha_0);$$

(ii) If $\theta \geq K_\beta$, then

$$q_1^* = \frac{\Delta}{3} \textit{ and}$$

$$q_2^* = \begin{cases} 0, & \textit{if} \quad \alpha < \Delta / 3 \\ \dfrac{\alpha}{2\beta} - \dfrac{\Delta}{6\beta}, & \textit{if} \quad \alpha \geq \Delta / 3 \end{cases}.$$

Theorem 1 is proved in the Appendix. In the next corollary we compare the quantities produced by each firm.

**Corollary 1** Let $I_\beta = (4\beta^2 - 1) / (4\beta^2 - 3)$.

*a) If $\theta < I_\beta$, then, for all $\alpha \in [\alpha_0, \alpha_1]$, the leading firm produces more than firm $F_2$, $q_1^* > q_2^*$.*

b) *If $I_\beta \leq \theta \leq K_\beta$, then there* exists

$$R_1 \equiv R_1(\beta, \alpha_0, \alpha_1) = \frac{4\beta^2 - 1}{4(2\beta^2 - 1)}(\alpha_1 + \alpha_0) \in [\alpha_0, \alpha_1]$$

such that

(i) if $\alpha \leq R_1$, the leading firm produces more than firm $F_2$, i.e $q_2^* \leq q_1^*$;

(ii) if $\alpha \geq R_1$, firm $F_2$ produces more than the leading firm, i.e $q_2^* \geq q_1^*$.

c) If $\theta \geq K_\beta$, then there exists

$$R_2 \equiv R_2(\beta,\alpha_0,\alpha_1) = \frac{(2\beta+1)\Delta}{3} \in [\alpha_0,\alpha_1]$$

such that

*(i) if $\alpha \leq R_2$, the leading firm produces more than firm $F_2$, i.e $q_2^* \leq q_1^*$;*

*(ii) if $\alpha \geq R_2$, firm $F_2$ produces more than the leading firm, i.e $q_2^* \geq q_1^*$.*

Corollary 1 is proved in the Appendix. The different situations described in this corollary are illustrated in Figure 1, for some values of the parameters.

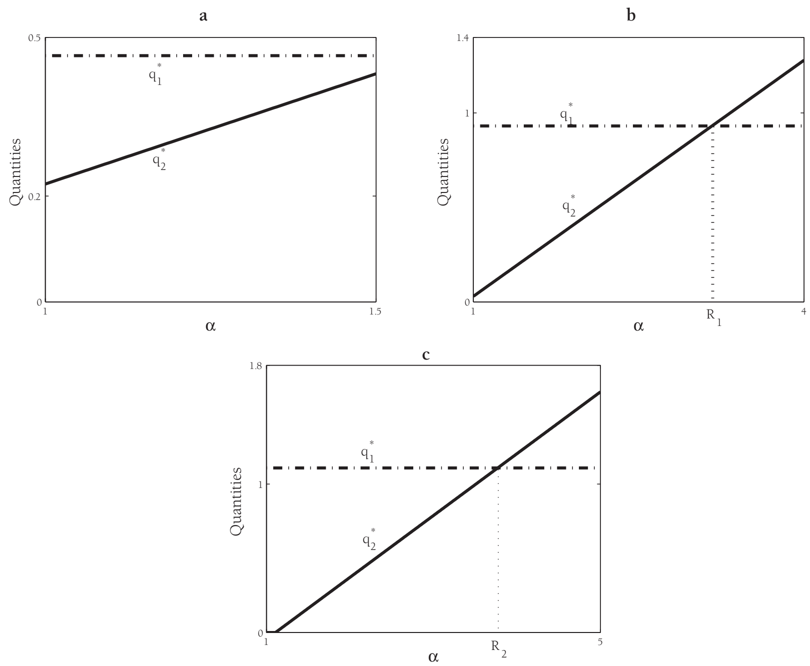



**Figure 1:** the quantities produced by each firm when the intervals of the uniform distribution of the parameter $\alpha$ are such that the ratio $\theta$ between its endpoints is as in each situation of Corollary 1, for $\beta$ = 1.2. **(a)** $[\alpha_0, \alpha_1] = [1, 1.5]$; **(b)** $[\alpha_0, \alpha_1] = [1, 4]$; **(c)** $[\alpha_0, \alpha_1] = [1,5]$.

In the next theorem, we present an ex-ante analysis by giving the profits that the firms can expect, before the knowledge of the demand realization $\alpha$. For simplicity of notation, we denote by $\pi_i^*(\alpha)$ the profit $\pi_i^*\left(q_1^*, q_2^*\left(q_1^*,\alpha\right)\right)$, for $i \in \{1, 2\}$.

**Theorem 2** *While the demand realization $\alpha$ is unknown for both firms, their expected profits, $E\left(\pi_1^*(\alpha)\right)$ and $E\left(\pi_2^*(\alpha)\right)$, are given by*

$$E\left(\pi_1^*(\alpha)\right) = \begin{cases} \dfrac{(\alpha_1+\alpha_0)^2(2\beta-1)^2}{32\beta\left(2\beta^2-1\right)}, & \text{if } (\theta,\beta) \in P \\[2ex] \dfrac{\Delta\left(18\beta\left(\alpha_1^2-\alpha_0^2\right)-12\beta^2\Delta(\alpha_1-\alpha_0)-9\alpha_1^2+6\Delta\alpha_1-\Delta^2\right)}{108\beta(\alpha_1-\alpha_0)}, & \text{if } (\theta,\beta) \in Q \end{cases}$$

and

$$E\left(\pi_2^*(\alpha)\right) = \begin{cases} \dfrac{\left(64\beta^4-48\beta^3-28\beta^2+12\beta+7\right)\left(\alpha_0^2+\alpha_1^2\right)+2\alpha_0\alpha_1\left(32\beta^4-48\beta^3+4\beta^2+12\beta-1\right)}{192\beta\left(2\beta^2-1\right)^2}, & \text{if } (\theta,\beta) \in P \\[2ex] \dfrac{(3\alpha_1-\Delta)^3}{324\beta(\alpha_1-\alpha_0)}, & \text{if } (\theta,\beta) \in Q \end{cases}$$

where

$$P = \{(\theta,\beta) : (1 < \theta < 3 \wedge \beta \geq 1) \vee (\theta > 3 \wedge \beta \geq \beta_0) \vee (\theta = 3 \wedge \beta = 1)\}$$

and

$$Q = \{(\theta, \beta) : \theta \geq 3 \wedge 1 \leq \beta \leq \beta_0\},$$

*with*

$$\beta_0 = \frac{1+\theta+\sqrt{\theta^2-6\theta+25}}{8}.$$

Theorem 2 is proved in the Appendix. In Figure 2 we plot firms' expected profits, $E\left(\pi_1^*(\alpha)\right)$ and $E\left(\pi_2^*(\alpha)\right)$, as functions of the demand uncertainty parameter $\theta$ and of the 'own' price effect parameter $\beta$. Taking $\alpha_0 = 1$, we see that there is a value[1] $\theta_0$ (approximately equal to 7.30) such that if the uncertainty parameter $\theta$ is greater than $\theta_0$, then the expected profit of the follower firm is always greater than the expected profit of the leading firm. Figure 3 illustrates cross sections of Figure 2 at $\beta = 1.2$ (Figure 3**a**) and $\beta = 1.5$ (Figure 3**b**). Figure 4 illustrates cross sections of Figure 2 at the demand uncertainty parameter's values $\theta = 2$ (Figure 4**a**) and $\theta = 8$ (Figure 4**b**).

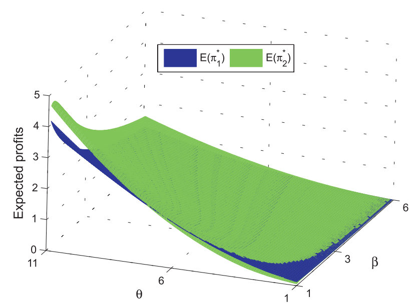



**Figure 2:** firms' expected profits varying with the demand uncertainty parameter $\theta$ and with the 'own' price effect parameter $\beta$, by taking $\alpha_0 = 1$.

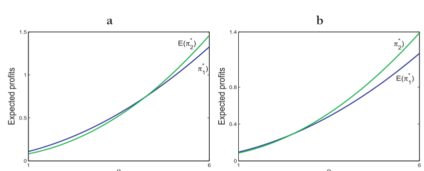



**Figure 3:** cross sections of Figure 2 at (**a**) $\beta = 1.2$ and (**b**) $\beta = 1.5$.

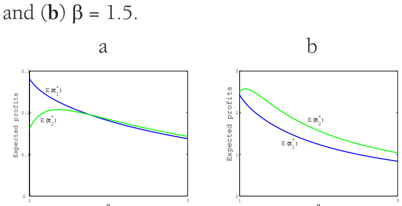



**Figure 4:** cross sections of Figure 2 at the uncertainty parameter's values (**a**) $\theta = 2$ and (**b**) $\theta = 8$.

Now, we are going to analyze the profits that the firms obtain after the observation of the demand realization. In the next theorem, we describe three regions for the demand uncertainty parameter $\theta = \alpha_1 / \alpha_0$ corresponding to three distinct profits relations between the leading and the follower firms (see Figure 5). The low-medium uncertainty boundary value is

$$I_\beta = \frac{4\beta^2-1}{4\beta^2-3},$$

and the medium-high uncertainty boundary value is

$$J_\beta = \frac{4\beta^2+4\beta-5}{(2\beta-1)^2}$$

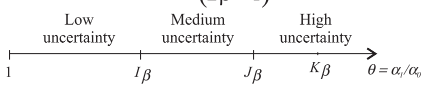



**Figure 5:** three regions for the demand uncertainty parameter $\theta$ corresponding to three distinct profits relations between the leading and the follower firms.

The functions $I_\beta$ and $J_\beta$ characterize the demand uncertainty parameter $\theta$ for which the leading firm looses its advantage for some realizations of the demand random variable. In fact, in the next theorem, we will show that the leading firm looses its advantage for high values of the demand intercept, if the demand uncertainty parameter $\theta$ is greater than $I_\beta$, and for low values of the demand intercept, if the demand uncertainty parameter $\theta$ is greater than $J_\beta$. Hence, for high values of the demand uncertainty parameter ($\theta > J_\beta$) only in an intermediate zone of the realized demand does the first mover preserve its advantage. We observe that for 'own' price effect equal to cross price effect ($\beta = 1$), the functions $I_\beta$ and $J_\beta$ coincide, i.e, $I_1 = J_1$, and for 'own' price effect greater than cross price effect ($\beta > 1$), we have that $I_\beta < J_\beta$. In Figure 6, we show the plots of $I_\beta$, $J_\beta$ and $K_\beta$ as functions of the 'own' price effect parameter $\beta$.

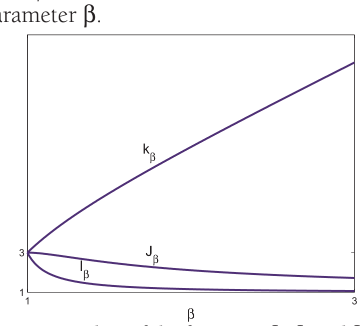



**Figure 6:** plots of the functions $I_\beta$, $J_\beta$ and $K_\beta$.

**Theorem 3** *a) If $\theta < I_\beta$, then, for all $\alpha \in [\alpha_0, \alpha_1]$, the leading firm profits more than firm $F_2$, i.e $\pi_2^* < \pi_1^*$.*

[1] The value $\theta_0$ is given by

$$\theta_0 = \frac{9}{4}+\sqrt{\frac{143}{48}-\left(\frac{3539}{3456}-\frac{3011\sqrt{17}}{13824}\right)^{1/3}-\left(\frac{3539}{3456}+\frac{3011\sqrt{17}}{13824}\right)^{1/3}}+ \\ +\sqrt{\frac{143}{24}+\left(\frac{3539}{3456}-\frac{3011\sqrt{17}}{13824}\right)^{1/3}+\left(\frac{3539}{3456}+\frac{3011\sqrt{17}}{13824}\right)^{1/3}+\sqrt{H}},$$

where

$$H = D^{1/3}+E^{1/3}+F^{1/3}+G^{1/3}+\frac{19013}{576}$$

with

$$D = \frac{10348768573}{5971968}-\frac{8804787277\sqrt{17}}{23887872},$$

$$E = \frac{10348768573}{5971968}+\frac{8804787277\sqrt{17}}{23887872},$$

$$F = \frac{118172131}{995328}-\frac{10655929\sqrt{17}}{373248}$$ and

$$G = \frac{118172131}{995328}+\frac{10655929\sqrt{17}}{373248}.$$

We observe that $\theta_0 \approx 7.30$.

b) If $I_\beta \le \theta < J_\beta$, then there exists

$$R_1 \equiv R_1(\beta,\alpha_0,\alpha_1) = \frac{4\beta^2 - 1}{4\left(2\beta^2 - 1\right)}(\alpha_0 + \alpha_1) \in [\alpha_0, \alpha_1]$$

such that

i) if $\alpha \le R_1$, the leading firm profits more than firm $F_2$, i.e $\pi_2^* \le \pi_1^*$;

ii) if $\alpha \ge R_1$, firm $F_2$ profits more than the leading firm, i.e $\pi_2^* \le \pi_1^*$.

c) If $\theta \ge J_\beta$, then there exist

$$L \equiv L(\beta,\alpha_0,\alpha_1) = \begin{cases} \dfrac{(2\beta - 1)^2}{4\left(2\beta^2 - 1\right)}(\alpha_0 + \alpha_1), & \text{if} \quad \theta \le K_\beta \\ \dfrac{(2\beta - 1)\Delta}{3}, & \text{if} \quad \theta \ge K_\beta \end{cases}$$

and

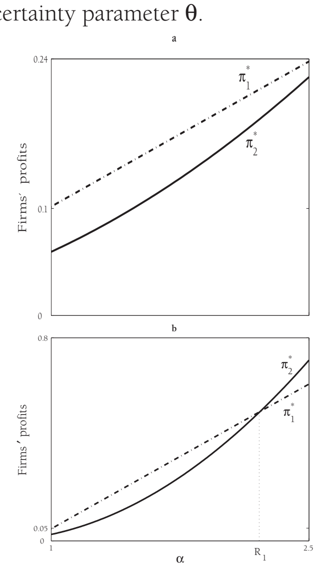
$$R \equiv R(\beta,\alpha_0,\alpha_1) = \begin{cases} \dfrac{4\beta^2 - 1}{4\left(2\beta^2 - 1\right)}(\alpha_0 + \alpha_1), & \text{if} \quad \theta \le K_\beta \\ \dfrac{(2\beta + 1)\Delta}{3}, & \text{if} \quad \theta \ge K_\beta \end{cases}$$

such that $L$, $R \in [\alpha_0, \alpha_1]$ and

i) *if $L \le \alpha \le R$, the leading firm profits more than firm $F_2$, i.e $\pi_2^* \le \pi_1^*$;*

ii) *if either $\alpha \le L$ or $\alpha \ge R$, firm $F_2$ profits more than the leading firm, i.e $\pi_2^* \ge \pi_1^*$.*

Theorem 3 is proved in the Appendix. The different situations described in this theorem are illustrated in Figure 7. In Figure 7**a**, we consider $\theta < I_\beta$; in Figure 7**b**, we consider $I_\beta \le \theta < J_\beta$; in Figure 7**c**$_1$, we consider $J_\beta \le \theta < K_\beta$; and in Figure 7**c**$_2$, we consider $\theta \ge K_\beta$. In Figure 8, we show the plots of $L$ and $R$ as functions of the demand uncertainty parameter $\theta$.



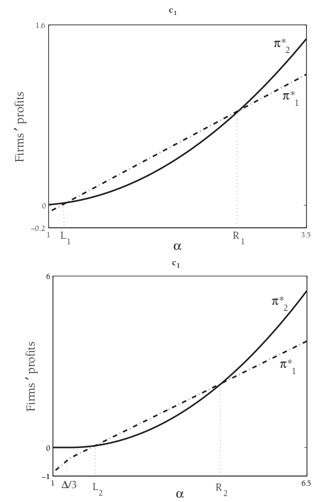




**Figure 7:** the profits of both firms when the intervals of the uniform distribution of the parameter $\alpha$ are such that the ratio $\theta$ between its endpoints is as in each situation of Theorem 3, for $\theta = 1.2$. (**a**) $[\alpha_0, \alpha_1] = [1, 1.5]$; (**b**) $[\alpha_0, \alpha_1] = [1, 2.5]$ ; (**c**$_1$) $[\alpha_0, \alpha_1] = [1, 3.5]$; (**c**$_2$) $[\alpha_0, \alpha_1] = [1, 6.5]$.

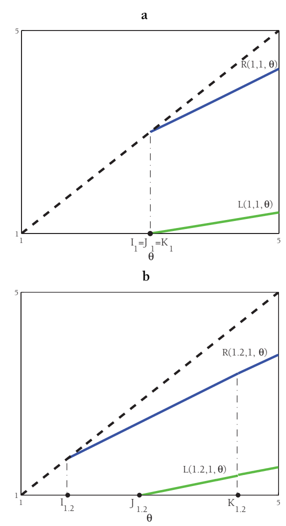



**Figure 8:** plots of the functions $L \equiv L(\beta, \alpha_0, \alpha_1)$ and $R \equiv R(\beta, \alpha_0, \alpha_1)$, by considering $\alpha_0 = 1$ and in the cases of (**a**) $\beta = 1$; and (**b**) $\beta = 1.2$.

Now, we are going to compute, in terms of the demand uncertainty parameter $\theta$ and of the 'own' price effect parameter $\beta$, the probability $P\left(\pi_2^* > \pi_1^*\right)$ of the second firm to have higher profit than the leading firm. Using the results presented in Theorem 3, we get the following corollary.

**Corollary 2** *a) If $\theta < I_\beta$, then* $P\left(\pi_2^* > \pi_1^*\right) = 0$.

b) *If $I_\beta \le \theta \le J_\beta$, then*

$$P\left(\pi_2^* > \pi_1^*\right) = \frac{\left(4\beta^2 - 3\right)\theta - 4\beta^2 + 1}{4\left(2\beta^2 - 1\right)(\theta - 1)}.$$

*c) If $I_\beta \leq \theta \leq K_\beta$, then*

$$P\left(\pi_2^* > \pi_1^*\right) = \frac{\left(4\beta^2 - 2\beta - 1\right)\theta - 4\beta^2 - 2\beta + 3}{2\left(2\beta^2 - 1\right)(\theta - 1)}.$$

*d) If $\theta > K_\beta$, then*

$$P\left(\pi_2^* > \pi_1^*\right) = 1 - \frac{2\Sigma}{3(\theta - 1)},$$

where $\Sigma$ is defined by (3).

The economic interpretations of this result include: (i) given the 'own' price effect parameter, the probability $P\left(\pi_2^* > \pi_1^*\right)$ of the follower firm to have higher profit than the leading firm increases with the degree of the demand uncertainty; and (ii) given the demand uncertainty, the probability $P\left(\pi_2^* > \pi_1^*\right)$ of the follower firm to have higher profit than the leading firm increases with the 'own' price effect. Furthermore, when the 'own' price effect is equal to the cross price effect, the probability $P\left(\pi_1^* > \pi_2^*\right)$ of the leading firm to have higher profit than the follower firm is greater than the probability of the opposite situation; however, when the 'own' price effect is sufficiently greater than the cross price effect, for sufficiently high level of uncertainty, the probability $P\left(\pi_2^* > \pi_1^*\right)$ of the follower firm to have higher profit than the leading firm is greater than the probability of the opposite situation (see Figure 9).

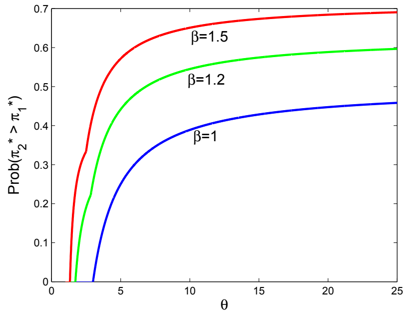



Figure 9: the probability of the second firm to have higher profit than the leading firm, as a function of the demand uncertainty parameter $\theta$, for different values of the 'own' price effect parameter $\beta$, by taking $\alpha_0 = 1$.

## 3 Conclusions

For the Stackelberg model considered, (i) we observed that in the case of demand certainty the leading firm has advantage over the follower one; and (ii) we showed that in the case of demand uncertainty the leading firm does not necessarily have advantage over the one that follows. In order to analyze the leadership and flexibility advantages, we had to consider three different situations, that depend upon the size of the ratio $\theta = \alpha_1 / \alpha_0$ between the endpoints of the interval $[\alpha_0, \alpha_1]$ in which the parameter $\alpha$ were uniformly distributed. That is, we found two functions $I_\beta$ and $J_\beta$, that depend upon the 'own' price effect parameter $\beta$, such that (i) if $\theta < I_\beta$, then the leading firm profits more than the follower; (ii) if $I_\beta \leq \theta < J_\beta$, then when the realized demand is very high, the leading firm profits less than the follower, otherwise the leading firm profits more than the follower; and (iii) if $\theta \geq J_\beta$, then when the realized demand is either very low or very high, the leading firm profits less than the follower, and when the realized demand is in an intermediate region, the leading firm profits more than the follower. We observed that for 'own' price effect equal to cross price effect ($\beta = 1$), the functions $I_\beta$ and $J_\beta$ coincide, i.e $I_1 = J_1$, and for 'own' price effect greater than cross price effect ($\beta > 1$), we have that $I_\beta < J_\beta$.

We showed that, when the 'own' price effect is equal to the cross price effect, the probability $P\left(\pi_1^* > \pi_2^*\right)$ of the leading firm to have higher profit than the follower firm is greater than the probability of the opposite situation; however, when the 'own' price effect is sufficiently greater than the cross price effect, for sufficiently high level of uncertainty, the probability $P\left(\pi_2^* > \pi_1^*\right)$ of the follower firm to have higher profit than the leading firm is greater than the probability of the opposite situation.

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## Appendix

In this Appendix, we give the proofs of the results presented throughout the paper.

*Proof of Theorem 1*. Using backwards-induction, we first compute firm $F_2$'s reaction, $q_2^*(q_1,\alpha)$, to an arbitrary quantity $q_1$ fixed by firm $F_1$, and to the realized demand parameter $\alpha$. The quantity $q_2^*(q_1,\alpha)$ is given by

$$\underset{q_2 \geq 0}{\arg\max}\, q_2\left(\alpha - q_1 - \beta q_2\right),$$

which yields

$$q_2^*(q_1,\alpha) = \max\left\{\frac{\alpha - q_1}{2\beta}, 0\right\}.$$

Therefore, firm $F_1$'s problem in the first stage of the game amounts to determine

$$q_1^* = \underset{q_1 \geq 0}{\arg\max}\, E\left(p_1 \cdot q_1\right),$$

where $p_1 = \alpha - \beta q_1 - q_2^*(q_1,\alpha)$ and E(•) is the expectation with respect to the demand intercept $\alpha$. We are going to study separately the cases: (*I*) $\alpha_0 \geq q_1$ and (*II*) $\alpha_0 \leq q_1$. Note that the density function of $\alpha$'s distribution is $1/(\alpha_1 - \alpha_0)$.

*Case I:* $\alpha_0 \geq q_1$ (see Figure 10). In this case, $\alpha \geq \alpha_0 \geq q_1$, and so $q_2^*(q_1,\alpha) = (\alpha - q_1)/(2\beta)$. Therefore,

$$E\left(q_1\left(\alpha - \beta q_1 - q_2^*(q_1,\alpha)\right)\right) = \int_{\alpha_0}^{\alpha_1} q_1\left(\alpha - \beta q_1 - \frac{\alpha - q_1}{2\beta}\right)\frac{1}{\alpha_1 - \alpha_0}\,d\alpha = \frac{(2\beta - 1)(\alpha_1 + \alpha_0)q_1 - 2\left(2\beta^2 - 1\right)q_1^2}{4\beta}. \quad (4)$$

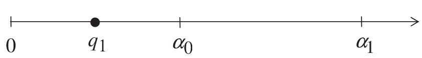


**Figure 10:** $\alpha_0 \geq q_1$ (*Case I*).

Then, firm $F_1$'s best quantity $q_1^*$ solves the equation

$$-4\left(2\beta^2 - 1\right)q_1 + (2\beta - 1)(\alpha_1 + \alpha_0) = 0.$$

Hence,

$$q_1^* = \frac{(2\beta - 1)(\alpha_1 + \alpha_0)}{4\left(2\beta^2 - 1\right)}, \quad (5)$$

and so

$$q_2^* = \frac{\alpha}{2\beta} - \frac{(2\beta - 1)(\alpha_1 + \alpha_0)}{8\beta\left(2\beta^2 - 1\right)}. \quad (6)$$

We observe that the value $\overset{*}{q_1}$ obtained in satisfies the hypothesis $\alpha_0 \geq q_1$ considered in *Case I* if, and only if, $\theta \leq K_\beta$.

***Case II:*** $\alpha_0 \leq q_1$ (see Figure 11). In this case,

(i) if $\alpha \leq q_1$, then $q_2^*(q_1,\alpha)=0$; and

(ii) if $\alpha \geq q_1$, then $q_2^*(q_1,\alpha)=(\alpha-q_1)/(2\beta)$.


**Figure 11:** $\alpha_0 \leq q_1$ (*Case II*).

Therefore,

$$E\left(q_1\left(\alpha-\beta q_1-q_2^*(q_1,\alpha)\right)\right) = \int_{\alpha_0}^{q_1} q_1(\alpha-\beta q_1)\frac{1}{\alpha_1-\alpha_0}d\alpha + \int_{q_1}^{\alpha_1} q_1\left(\alpha-\beta q_1-\frac{\alpha-q_1}{2\beta}\right)\frac{1}{\alpha_1-\alpha_0}d\alpha$$

$$=\frac{\left((2\beta-1)\alpha_1^2-2\beta\alpha_0^2\right)q_1-2\left(\left(2\beta^2-1\right)\alpha_1-2\beta^2\alpha_0\right)q_1^2-q_1^3}{4\beta(\alpha_1-\alpha_0)} \qquad (7)$$

Then, firm $F_1$'s best quantity $\overset{*}{q_1}$ solves the equation

$$-3q_1^2-4\left(\left(2\beta^2-1\right)\alpha_1-2\beta^2\alpha_0\right)q_1+(2\beta-1)\alpha_1^2-2\beta\alpha_0^2=0.$$

Hence,

$$\overset{*}{q_1}=\frac{\Delta}{3}, \qquad (8)$$

and so

$$\overset{*}{q_2}=\begin{cases}0, & \text{if } \alpha<\Delta/3 \\ \dfrac{\alpha}{2\beta}-\dfrac{\Delta}{6\beta}, & \text{if } \alpha\geq\Delta/3\end{cases}. \qquad (9)$$

We observe that the value $\overset{*}{q_1}$ obtained in satisfies the hypothesis $\alpha_0 \leq q_1$ considered in *Case II* if, and only if, $\theta \geq K_\beta$.

We note that, in *Case I* ($\theta \leq K_\beta$) we have that $\arg\max_{q_1\geq 0} E(p_1\cdot q_1)\leq\alpha_0$, and in *Case II* ($\theta \geq K_\beta$) we have that $\arg\max_{q_1\geq 0} E(p_1\cdot q_1)\geq\alpha_0$ (see Figure 12).

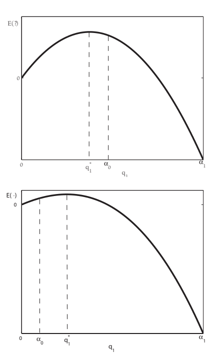

**Figure 12:** *case I* ($\theta \leq K_\beta$): $\arg\max_{q_1\geq 0} E(p_1\cdot q_1)\leq\alpha_0$; *case II* ($\theta \geq K_\beta$): $\arg\max_{q_1\geq 0} E(p_1\cdot q_1)\geq\alpha_0$.

*Proof of Corollary 1.* First, suppose that $\theta \leq K_\beta$. By Theorem 1, we get that

$$\overset{*}{q_1}-\overset{*}{q_2}=\frac{4\beta^2-1}{8\beta\left(2\beta^2-1\right)}(\alpha_1+\alpha_0)-\frac{\alpha}{2\beta}.$$

Therefore, $\overset{*}{q_1}-\overset{*}{q_2}\geq 0$ if, and only if,

$$\alpha\leq\frac{3\beta^2-1}{4\left(2\beta^2-1\right)}(\alpha_1+\alpha_0).$$

Since $\frac{3\beta^2-1}{4\left(2\beta^2-1\right)}(\alpha_1+\alpha_0)>\alpha_1$ if, and only if, $\theta < I_\beta$, and $I_\beta < K_\beta$ we get the statement **a**). Since $\frac{3\beta^2-1}{4\left(2\beta^2-1\right)}(\alpha_1+\alpha_0)\in[\alpha_0,\alpha_1]$ if, and only if, $\theta \geq I_\beta$, we get the statement **b**).

Now, suppose that $\theta \geq K_\beta$. By Theorem 1, we get that

- if $\alpha < \Delta/3$, then

$$\overset{*}{q_1}-\overset{*}{q_2}=\overset{*}{q_1}=\frac{\Delta}{3},$$

which is positive;

- if $\alpha \geq \Delta/3$, then

$$\overset{*}{q_1}-\overset{*}{q_2}=\frac{(2\beta+1)\Delta}{6\beta}-\frac{\alpha}{2\beta}.$$

Therefore, $\overset{*}{q_1}-\overset{*}{q_2}\geq 0$ if, and only if,

$$\alpha\leq\frac{(2\beta+1)\Delta}{3},$$

which implies the statement *c*).

*Proof of Theorem 2.* Firm $F_1$'s expected profit, $E\left(\pi_1^*(\alpha)\right)$, is obtained by (4) and (7). Firm $F_2$'s expected profit, $E\left(\pi_2^*(\alpha)\right)$, is determined by

$$E\left(\pi_2^*(\alpha)\right)=\int_{\alpha_0}^{\alpha_1} q_2^*\left(\alpha-q_1^*-\beta q_2^*\right)\frac{1}{\alpha_1-\alpha_0}d\alpha,$$

with $\overset{*}{q_1}$ and $\overset{*}{q_2}$ given, respectively, by (5) and (6), in the case of $\theta \leq K_\beta$, and given, respectively, by (8) and (9), in the case of $\theta \geq K_\beta$. Finally, we note that the values ($\theta$ $\beta$) in the set $P$ are the ones that satisfy $\theta \leq K_\beta$, and the values ($\theta$, $\beta$) in the set $Q$ are the ones that satisfy $\theta \geq K_\beta$.

*Proof of Theorem 3.* By Theorem 1, in the case of $\theta \leq K_\beta$, the profits $\overset{*}{\pi_1}$ and $\overset{*}{\pi_2}$ at equilibrium are given by

$$\overset{*}{\pi_1}=\frac{(2\beta-1)^2(4\alpha+\alpha_1-\alpha_0)(\alpha_1+\alpha_0)}{32\beta\left(2\beta^2-1\right)}$$

and

$$\pi_2^* = \frac{\left(4\left(2\beta^2-1\right)\alpha-(2\beta-1)(\alpha_1+\alpha_0)\right)^2}{64\beta\left(2\beta^2-1\right)^2}.$$

So, we have that $\pi_1^* - \pi_2^* > 0$ if, and only if,

$$L_1 \equiv \frac{(2\beta-1)^2}{4\left(2\beta^2-1\right)}(\alpha_0+\alpha_1) < \alpha < \frac{\left(4\beta^2-1\right)^2}{4\left(2\beta^2-1\right)}(\alpha_0+\alpha_1) \equiv R_1.$$

Again by Theorem 1, in the case of *q* $\geq K_\beta$, the profits $\pi_1^*$ and $\pi_2^*$ at equilibrium are given by

$$\pi_1^* = \begin{cases} \frac{(3\alpha-\beta\Delta)\Delta}{9}, & \text{if} \quad \alpha < \Delta/3 \\ \frac{\left(3\alpha(2\beta-1)-\left(2\beta^2-1\right)\Delta\right)\Delta}{18\beta}, & \text{if} \quad \alpha \geq \Delta/3 \end{cases}$$

and

$$\pi_2^* = \begin{cases} 0, & \text{if} \quad \alpha < \Delta/3 \\ \frac{(3\alpha-\Delta)^2}{36\beta}, & \text{if} \quad \alpha \geq \Delta/3 \end{cases}.$$

So, we have that $\pi_1^* - \pi_2^* > 0$ if, and only if,

$$L_2 \equiv \frac{(2\beta-1)\Delta}{3} < \alpha < \frac{(2\beta+1)\Delta}{3} \equiv R_2.$$

a) For $\theta < I_\beta$, we have that $L_1 < \alpha_0$ and $R_1 > \alpha_1$. Therefore, $\pi_2^* < \pi_1^*$, for all $\alpha \in [\alpha_0, \alpha_1]$.

b) For $I_\beta \leq \theta \leq J_\beta$, we have that $L_1 < \alpha_0$ and $\alpha_0 \leq R_1 \leq \alpha_1$. Therefore, if $\alpha < R_1$ then $\pi_2^* < \pi_1^*$; and if $\alpha > R_1$ then $\pi_2^* > \pi_1^*$.

c) For $J_\beta < \theta \leq K_\beta$, we have that $\alpha_0 \leq L_1$, $R_1 \leq \alpha_1$. Therefore, if $L_1 \leq \alpha. \leq R_1$ then $\pi_2^* < \pi_1^*$; and if either $\alpha_0 \leq L_1$, $R_1 \leq \alpha_1$. Therefore, if $\alpha < L_1$ or $\alpha_0 \leq L_1$, $R_1 \leq \alpha_1$. Therefore, if $\alpha > R_1$ then $\pi_2^* > \pi_1^*$.

For $\theta > K_\beta$, we have that $\alpha_0 \leq L_2$, $R_2 < \alpha_1$. Therefore, if $L_2 \leq \alpha \leq R_2$ then $\pi_2^* < \pi_1^*$; and if either $\alpha < L_2$ or $\alpha > R_2$ then $\pi_2^* > \pi_1^*$.