

DeSerpa's Time Allocation Model: A Comment

Resumo: O modelo de alocação de tempo de Allan DeSerpa tem sido pouco valorizado na literatura microeconômica, apesar de sua influência na análise de custo-benefício de projetos de transporte, especialmente no cálculo do custo de oportunidade do tempo. Dado que DeSerpa apresentou seu modelo como uma alternativa ao modelo de alocação de tempo de Gary Becker, o presente artigo inicialmente compara os dois modelos. Conclui-se que DeSerpa foi um dos primeiros autores a analisar formalmente as consequências da produção conjunta no contexto do modelo de Becker. Além disso, com base na análise de DeSerpa, a usual otimização em dois estágios na presença de produção doméstica conjunta é algo que precisa ser qualificado. A utilidade marginal negativa do trabalho em seus usos domésticos garante a otimização em dois estágios.

Palavras - chaves: alocação do tempo; produção doméstica; produção conjunta.

Abstract: The time allocation model of Allan DeSerpa has received little recognition from the mainstream microeconomics literature on this subject, despite its influence on cost-benefit analysis of projects of transportation, especially in the computation of the opportunity cost of time. Given that DeSerpa introduced his model as an alternative to Gary Becker's model, the present paper initially compares the two models. It follows that DeSerpa was one of the first authors to formally analyze the consequences of joint production in the context of Becker's model. Also, the accepted usage of two-stage optimization in the presence of joint domestic production is also something that may be qualified from DeSerpa's analysis. It is a sufficient condition for this procedure that all domestic uses of time involve negative marginal utility.

Keywords: time allocation, household production, joint production.

JEL classification: J22, D13.

João Rogério Sanson^{1*}

^{1*} Department of Economics, Federal University of Santa Catarina, Brazil. E-mail: jrsanson@mbox1.ufsc.br. The author thanks Duilio A. Bérni and an anonymous referee from this periodical for useful suggestions, with the usual disclaimer, and the National Council for Research (CNPq) of Brazil for research grants, which included the research assistance of Paula P. Villasbôas, Michael Ax. Wilhelm, and Antônio M. Fontoura.

Introduction

As a model of the allocation between domestic and market work, Becker (1965)'s was a landmark, consolidating a now comprehensive program of research, being one of the cornerstones of the modern theory of household production. Once the initial impact of the model was absorbed, DeSerpa (1971) proposed an alternative model of time allocation.² The proposal was intended to include Becker (1965) as a particular case. DeSerpa's model, for instance, was all set for the use of non-linear programming techniques. Nevertheless, both authors proposed models of a consumer that is not limited to selling work; the consumer also produces many of the goods for own consumption purposes. These models are specimens of the household theory of production, with emphasis on work flow allocation.

Apparently, a mainstream evaluation of Allan DeSerpa's paper is yet to be made. A basic contribution from the area like Pollak & Wachter (1975) does not mention it. In the review of Gronau (1986) I only found it in the bibliography. Also, the textbooks of Deaton & Muellbauer (1980), Barbosa (1985), and Bryant (1990), which present the theory of household production in a fairly thorough way, do not refer to DeSerpa's work.³ In transportation economics literature, however, DeSerpa (1971) is well known and highly influent, as shown, for example, by Bruzelius (1979) and Gonzalez (1997). My purpose here is to check how DeSerpa's model fits into the mainstream household production theory.

In the next section, I compare the models of Becker and DeSerpa, from the viewpoint of jointness in production. This jointness is a consequence of a possible domestic work presence in the utility function, which is one of the main assumptions in DeSerpa's model. In fact, Grossman (1971) also discussed jointness in a specialized Becker model in which the hours of work are given.⁴ In general, the subsequent literature credits Pollak & Wachter (1975) for rigorously pointing out, with the help of duality methods, the consequences of production jointness in Becker's model. Pollak & Wachter (1975, p.271) start with production

functions that have specific time use for each commodity. The jointness in production occurs because time used in the production of a commodity also enters the preference function, becoming thus a new commodity, exactly as postulated by DeSerpa (1971). The use of work in domestic production at the same time produces a commodity given by this time use itself.⁵

In the third section, I evaluate this assumption of jointness in the context of the definition of cost functions for domestic activities, since Pollak & Wachter (1975) discuss the household model in terms of cost functions. I argue that minimization of costs hinges on the correct assumption as to the preference direction for domestic work.

In the fourth section I briefly deal with the case of variable supply of market work. In sections two and three I consider only the case of fixed supply of market work which is used all the way by DeSerpa (1971). This differs from Becker's model, since he explicitly models the allocative decision between market work and domestic activity. DeSerpa (1971, p. 829 n.1) argues that the basic qualitative properties of the model are not affected.

Joint production

If the time of work is given, Becker's model may be specified as:

$$\max. \quad u(Z_1, \dots, Z_n) \quad (1)$$

$$\text{subject to} \quad X_i = Z_i \quad i = 1, \dots, n \quad (2)$$

$$T_i \geq a_i Z_i \quad i = 1, \dots, n \quad (3)$$

$$\sum T_i = T_0 \quad (4)$$

$$\sum P_i X_i = Y \quad (5)$$

Expressions (2) and (3) are the production functions of domestic activities, Z_i , that have as inputs the market goods, X_i , and own time of the consumer, T_i . Notice that each unit of consumption activity uses up one unit of market good. Thus, there is identification between the domestic product and the market good. In its original specification, Becker's model utilizes a more general definition of the consumption activity than simply a market good combined with the time necessary to consume it, involving both variable coefficients and a vector of goods and time uses. The present specification is consistent with the

² DeSerpa (1973 and 1975) reinforces and extends the same arguments.

³ A burgeoning research area from household production theory is that of collective household models. For a survey with emphasis on econometric implementation see Vermeulen (2002), which, as expected, does not consider DeSerpa's model.

⁴ In fn.2 of this unpublished and often cited paper the author leaves out the input of own work from the production functions and then, on fn.27, mentions the possibility of "time effects" on relative prices of commodities, provided own work were an input to the production function. These effects were central to DeSerpa (1971) and Pollak & Wachter (1975) analyses. It is of course difficult to still classify Grossman's model as Beckerian when own labor is not considered as an input. Similarly, DeSerpa & Huntington (1978) also consider a similar model in which the market labor supply by the household is given. Nevertheless, it is easy to accept that Grossman (1971) implicitly considers a household production model, despite casting aside time allocation. The paper is undisputed as one of the first systematic discussions of production jointness within a model that has some of the basic characteristics of a Beckerian household production model. Of course disregarding DeSerpa (1971) as such, some important authors give him this precedence, as is the case of Michael & Becker (1973), Gronau (1986, p.275n.2), and Becker (1991, p.24n.5).

⁵ DeSerpa & Huntington (1978), in their discussion of the applicability of the Rybczynski theorem to Becker's model, show that one of the difficulties for this application is the possible presence of work in the preference function.

one by DeSerpa (1971). So as to make both models comparable, expression (2) appears as equality and (3) as inequality. In Becker's model with Leontief production functions, the natural form for these restrictions is inequality. However, the usual equality presentation hinges upon the non-satiety axiom, something implicitly considered by Becker (1965, p.498n.2).

Expression (4) is the time restriction. Following DeSerpa (1971), I suppose the time allocation for market work to be exogenously given. Although this assumption throws away some of the main insights of Gary Becker, such as the choice between consumption activities and market work, it is unessential.⁶ Expression (5) is the monetary restriction.

I left the preference function, (1), as the last one to be considered because it was in it that Allan DeSerpa made a significant change. Becker's preference function is now reformulated to include among its arguments the inputs of domestic work into each consumption activity, which appear as n additional consumption activities. DeSerpa's model is then specified⁷ as

$$\max. u(Z_1, \dots, Z_n, Z_{n+1}, \dots, Z_{n+n}) \quad (6)$$

subject to expressions (2) through (5) above, plus the restrictions

$$T_i = Z_{n+i} \quad i = 1, \dots, n \quad (7)$$

Therefore, the domestic productive process generates more than one consumption activity at the same time. When producing Z_i the household jointly produces Z_{n+i} .

The basic insight of DeSerpa (1971) was the analysis of joint production, while the remainder of the model may then be seen as a special case of Becker's model.⁸ Naturally, in the household production literature, analysis of joint production was no novelty, besides Grossman (1971). The most important case is model of Lancaster (1966) who supposes, in fact, a productive process with 'complete jointness.' In contrast, Becker (1965) supposes that each domestic product has specific inputs.⁹ Therefore, the novelty of DeSerpa (1971) was to discuss joint production within Becker's model, something usually associated to Pollak & Wachter (1975).

The joint production considered by DeSerpa and Pollak & Wachter is a domestic

good and the work time used by the home producer in generating that good. According to Allan DeSerpa, when working at home, the consumer may obtain satisfaction in that activity, and may even allocate more time than the minimum that is technically necessary. Thus work itself is also a source of benefit in addition to the benefits of consuming the product of that work. To Becker (1965), though, all the benefits of a domestic activity, including the act of producing it, are included in the activity. If the domestic production of a meal is more pleasurable than buying the same meal in a restaurant, then they are not perfect substitutes and should be considered as different consumption activities.

Minimization of domestic production costs

Provided the explicit treatment of welfare connected to domestic work is accepted, there is a difficulty with the model of DeSerpa, when he writes that "... the individual is free to allocate more than the required amount of time to any activity" (p.830). In such a case more time devoted to domestic work is preferred to less, although the allocation of the minimum is "... a matter of individual preference." Pollak & Wachter (1975, p.271) give an example in which "... household time spent in cooking and cleaning is a direct source of utility or disutility ..." However, the direction of preference for work has different consequences in each case.

In microeconomics literature, work is usually presumed as a bad, that is, less work is preferred to more, even when this is implicitly done as in the leisure-income model. Naturally, a contrary preference direction means that work is a good. One consequence of it, proved by DeSerpa (1971), is that the household does not necessarily minimize the costs associated to domestic time for consuming a good, as a Beckerian household would do, provided work is a good. This occurs when restriction (3) above is not binding at the equilibrium.¹⁰ With the production function of the Leontief type, this means that costs are not minimized. Here, the cost of time

⁶ With such procedure, total prices disappear from the equilibrium conditions. These prices consider the sum of money plus time costs in consuming a given good. They are also known as Mincer-Becker prices. The main thesis of Pollak & Wachter (1975) was exactly the inexistence of Mincer-Becker prices under more general assumptions that include either increasing returns to scale or joint production. Another consequence of taking time allocation for market work as exogenously given is that the market price for labor is not considered in the discussion of the opportunity cost of domestic use of time for production purposes.

⁷ This specification of DeSerpa's model, designed so as to be comparable to Pollak & Wachter (1975, p. 271), may be converted back to the original version of DeSerpa by simply eliminating the variables Z_i .

⁸ Like Pollak & Wachter (1975), DeSerpa (1975) also shows the lack of generality of Becker's model and is now seen, as a complement to Pollak & Wachter (1975), at least in transport economics (Gonzalez, 1997).

⁹ For comparisons between the household production models of Gary Becker and Kelvin Lancaster, see Atkinson & Stern (1981) and Gronau (1986, p.274-275).

¹⁰ DeSerpa (1971, p.831) presents the Kuhn-Tucker conditions for the inequality restrictions (3) above. One of these conditions is $\kappa_i (T_i - a_i X_i) = 0$, where κ_i is the corresponding Lagrangean multiplier. It follows that the inequality in the time input requirement results in $\kappa_i = 0$. Then from the first order condition $U_{n+i} = \mu - \kappa_i$, at the same page, it follows that the marginal utility of domestic work time (U_{n+i}) must be positive, since the multiplier of the time restriction (μ) is positive. In addition, from the Kuhn-Tucker conditions a weaker result may occur. The optimum T_i under positive U_{n+i} may coincide with the cost minimizing T_i . Thus $U_{n+i} = \mu$, $\kappa_i = 0$, and $\kappa_i > 0$. Finally, from this same first order condition, $U_{n+i} < 0$ is only consistent with $\kappa_i > \mu$, in fact also $\kappa_i > \mu$. The Kuhn-Tucker condition thus implies cost minimization.

use is given in terms of the alternative use in domestic production. If consumers, on domestically producing a good, are not cost minimizers because they enjoy that kind of work, or if they simply are efficiently producing more of the domestic good in substitution to a market good, as Gary Becker would prefer to say, might only be answered through empirical studies.

The assumption as to the direction of preference for domestic work has a consequence that is unexpected. The usual technique of optimization in two stages for deriving results in the household model might not be valid anymore. This technique first finds the cost functions for each activity or a non-separable cost function for all activities when joint production is present. Then it carries out the utility maximization step. Of course, if the household does not minimize costs of domestic production, the first step is not attainable. One-step maximization, as done by DeSerpa, is then the adequate procedure.¹¹ However, the convenience of the two-step optimization for empirical work is lost.

The explicit consideration of domestic work in the preference function is another source of difficulty for DeSerpa's model. Why only one type of input in domestic production has been chosen for inclusion in the preference function? Anyone can easily imagine situations in which other inputs generate benefits in complement to what a domestic activity does. A consumer that gets pleasure from cooking may also obtain pleasure by choosing ingredients, even though the final taste and appearance of the meal may apparently be the same. One example would be two ingredients with identical characteristics except for the image each one has through advertising. The consumer may prefer one of them, obtaining pleasure from this and probably paying more for it. This may be contrasted to Becker's model, in which the activity would generate different domestic goods according to the ingredients. This of course is also valid for the amount of domestic work.

Market work and welfare

In the model presented in the body of his paper, DeSerpa (1971) presumes that the supply of work to the market is fixed. However, in a note at p.829, he suggests the inclusion of an additional consumption activity in the preference function to represent market work. This activity would use part of the individual time as input. It is unclear, however, what would be its preference direction. Any way, this would make it part of a family of alternative models, based on Johnson (1966), for whom work sold in the market is also considered in the preference function. This characterizes them as labor-leisure models, since both variables appear in the preference function. However, the presence of market labor time also in the preference function has different consequences according to the preference direction for such activity.

The model of Becker and the usual models of leisure-income do not include work in the preference function, with work time being determined residually. From total time they subtract the sum of the several uses of time in domestic activities; what is left constitutes market work time. This is a standard Walrasian procedure in the treatment of factor endowments. Given the endowment, one may find the reservation price of the factor and define whether the individual is a buyer or a seller of that factor. The exception to this is own work time, which can only be sold, since its upper limit is given by nature. Work time bought from other people, in the form of services, is a commodity that does not alter the own time upper limit for the buyer. Although it is impossible for a person to alter own time endowment in each period, work time from other people can be bought so that own time may be reallocated to uses of greater personal value.¹²

On the other hand, in models of labor-leisure, market work may alter the individual welfare differently from what giving up domestic activity does. Therefore, the issue of the preference direction for work is also relevant in labor-leisure models. Sanson (1987), on theoretical grounds, argues against the assumption of "more preferred to less" for work. The main argument is the

¹¹ Pollak & Wachter (1975, p.271) introduce a cost function in a joint production model of the above type. As mentioned above, they leave the direction of preference for domestic work undefined. However, the cost functions should only be defined in a negative marginal utility situation, which guarantees cost minimization.

¹² This impossibility of incrementing the time resource is used by DeSerpa (1971, p.833) as a basis for dismissal of the shadow price of time as given simply by the Lagrange multipliers of the time and income restrictions. He shows that the shadow price is modified by the marginal utility of time as a commodity, which results from its presence in the preference function. The relevant opportunity cost of time is in fact the best alternative domestic use of time.

possibility of a negative wage in such a situation. Instead, enjoyment of market working is easily covered by the interaction between income and substitution effects while treating all types of work as bads.

Conclusion

The preference function defined by Allan DeSerpa deals with jointly produced goods, which are explicitly not considered by Gary Becker. The considered jointly produced goods are a domestically produced good and the domestic work time that goes with it. This analysis of the joint production case in the context of Becker's model makes DeSerpa (1971) a legitimate precursor of Pollak & Wachter (1975). However, the technological, temporal and monetary restrictions of DeSerpa's model can be seen as a special case of Becker's model. Besides, DeSerpa's results are crucially dependent upon the stated preference direction for domestic work. More explicitly, the nonminimization of domestic work time requires positive marginal utility.

It is difficult to understand why DeSerpa's early work has been neglected in the mainstream literature of household production, specially the applications to time allocation, as compared to other contributions of the period. A possible explanation might be the author's emphasis on differentiating his results from Becker's, which may have distracted his readers from the main issue. DeSerpa (1971), in my opinion, should also be viewed as a theoretical bridge between Becker (1965) and Pollak & Wachter (1975). As to precedence, when compared with Grossman (1971), not only DeSerpa (1971) was then published in a first rate periodical but also based on his PhD thesis, deposited in 1970.

Bibliographical Reference

- ATKINSON, Anthony B., STERN, N.H. On labour supply and commodity demands. In: DEATON, A. (org.) *The theory and measurement of consumer behaviour*. Cambridge: Cambridge University Press, 1981. Ch.10.
- BARBOSA, Fernando de Holanda. *Microeconomia*. Rio de Janeiro: IPEA/INPES, 1985.
- BECKER, Gary S. A theory of the allocation of time. *Economic Journal*, vol.75, pp.493-517, Sept.1965.
- BECKER, Gary S. *A treatise on the family*. Enlarged ed. Cambridge, Mass.: Harvard University Press, 1991.
- BRUZELIUS, Nils. *The value of travel time*. London: Croom Helm, 1979.
- BRYANT, W.Keith. *The economic organization of the household*. Cambridge: Cambridge University Press, 1990.
- DEATON, Angus, MUELLBAUER, J. *Economics and consumer behavior*. New York: Cambridge University Press, 1980.
- DESERPA, Allan C. A theory of the economics of time. *Economic Journal*, vol.81, pp.828-846, Dec.1971.
- DESERPA, Allan C. Microeconomic theory and the valuation of travel time: Some clarification. *Regional and Urban Economics*, vol.2, No.4, pp.401-410, 1973.
- DESERPA, Allan C. On the comparative statics of time allocation theory. *Canadian Journal of Economics*, vol.8, No.1, pp.101-111, Feb.1975.
- DESERPA, Allan C.; HUNTINGTON, Hillard G. Time allocation, time value and factor intensity. *Australian Economic Papers*, vol.17, No.30, pp.124-131, Jun.1978.
- GONZALEZ, Rosa M. The value of time: A theoretical review. *Transport Reviews*, vol.17, No.3, pp.245-266, Jul.1997.
- GRONAU, Reuben. Home production: A survey. In: ASHENFELTER, O., LAYARD, R. (Org.) *Handbook of labor economics*. Amsterdam: North-Holland, 1986. 2 vol.
- GROSSMAN, Michael. *The economics of joint production in the household*. Chicago: Center for Mathematical Studies in Business and Economics, University of Chicago, [1971]. 62 p. Mimeo. (Report No. 7145.)
- JOHNSON, M.Bruce. Travel time and the price of leisure. *Western Economic Journal*, vol.4, pp.135-145, Spring 1966.
- LANCASTER, Kelvin. A new approach to consumer theory. *Journal of Political Economy*, vol.74, pp.132-157, 1966.
- MICHAEL, Robert T.; BECKER, G.S. On the new theory of consumer behavior. *Swedish Journal of Economics*, vol.75, No.4, pp.378-396, 1973.
- POLLAK, Robert A.; WACHTER, Michael L. The relevance of the household production function and its implications for the allocation of time. *Journal of Political Economy*, vol.83, No.2, pp.255-277, Apr.1975.
- SANSON, J.R. On the labor-leisure model. In: LATIN AMERICAN MEETING OF THE ECONOMETRIC SOCIETY, 7th, São Paulo, Aug.1987. *Proceedings ...* São Paulo: FIPE/USP, 1987. V.4, p.1843-1853.
- VERMEULEN, Frederic. Collective household models: Principles and main results. *Journal of Economic Surveys*, vol.16, No.4, pp.533-564, 2002.