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Assessing the Significance of Factors Effects in Output Oriented DEA Measures of Efficiency: An Application to Brazilian Banks*

Resumo: Este artigo utiliza medidas de Análise de Envoltória de Dados (DEA) de eficiência técnica, orientadas para produto, para acessar a significância de efeitos técnicos em bancos brasileiros. Medidas de eficiência são calculadas para produto múltiplo (títulos, empréstimos e depósitos à vista) e simples (combinado). Os insumos são trabalho, capital e recursos captados de terceiros. Os efeitos técnicos de interesse na análise são a natureza do banco (múltiplo ou comercial), o tipo (crédito, negócios, tesouraria ou varejo), o tamanho (grande, médio, pequeno e micro), o tipo de controle (privado ou público), a origem (doméstico ou estrangeiro) e operações de crédito de liquidação duvidosa. O último é uma medida de risco. Máxima verossimilhança no contexto da distribuição normal truncada, da distribuição exponencial e modelos Tobit, assim como uma análise de covariância não-paramétrica são as ferramentas estatísticas consideradas para inferência nos efeitos técnicos. O modelo final escolhido é do tipo Tobit e definido pela distribuição gama. A resposta com conteúdo mais informativo considerando tanto a significância dos fatores como o melhor ajuste aos dados é dada pela medida de eficiência DEA orientada a produto, com retornos variáveis de escala, proveniente de insumos e produtos múltiplos. Origem do banco e tipo do banco são os únicos efeitos significantes.

Palavras-chave: Análise de envoltória de dados; fronteiras de produção; inferência em dois estágios.

Abstract: This paper uses output oriented Data Envelopment Analysis (DEA) measures of technical efficiency to assess the significance of technical effects for Brazilian banks. Efficiency measurements are computed for a multiple output (securities, loans, and demand deposits) and for a single (combined) output. The inputs are labor, capital and loanable funds. The technical effects of interest in the analysis are bank nature (multiple and commercial), bank type (credit, business, bursary and retail), bank size (large, medium, small and micro), bank control (private and public), bank origin (domestic and foreign), and non-performing loans. The latter is a measure of bank risk. Maximum likelihood in the context of the truncated normal distribution, the exponential distribution, and general Tobit models, as well as a nonparametric analysis of covariance, are the statistical tools considered to make inferences on technical effects. The final model chosen is of the Tobit type and is defined by a gamma distribution. The response providing both the most informative content regarding the significance of factors and the best fit to the data is an efficiency measure derived from a multiple input - multiple output product oriented DEA computed under the assumption of variable returns to scale. Bank origin and bank type are the only significant effects.

Key words: Data Envelopment Analysis, production frontiers; two-stage inference.

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1. Introduction

The main objective of this paper is to compute measures of technical efficiency based on Data Envelopment Analysis (DEA) for the Brazilian banks and to relate the variation observed in these measurements to covariates of interest. This association is investigated in the context of several alternative models fit to DEA measurements of efficiency and DEA residuals. The DEA residuals are derived from a single output oriented DEA measure. They were introduced as a formal tool of analysis in DEA by Banker (1993).

Output is measured both as a 3-dimensional vector formed by the variables investment securities, total loans and demand deposits and as a combined index of these variables. The three input sources are labor, capital and loanable funds. The causal factors considered here as affecting efficiency measurements an DEA residuals are bank nature, bank type, bank size, bank control, bank origin and risky loans (nonperforming loans).

The statistical methods we use in the paper explore Banker (1993) and Souza (2001.2005) results.

Several bank studies, among them Eseinbeis et all (1999), Sathye (2001), Campos (2002) and Tortosa-Ausina (2002) have considered the use of DEA to measure the relative efficiency of a bank. Typically a DEA context is defined, such as a revenue or cost optimization, input or output orientation, under constant or variable returns to scale, and subsequently analyzed. If additionally an empirical investigation on the association between technical effects and DEA measures is demanded, as in Eseinbeis et all (1999), regression is the basic technique used in the analysis. The models suggested in the literature go from the standard analysis of covariance models as suggested in Coelli, Rao, and Battese (1998), to the Tobit model as in McCarthy and Yaisawarng (1993). Our contribution to this literature is twofold. Firstly we open the possibility of combining output in banking studies which makes the Banker (1993) kind of approach viable in a context inherited from a production model. Relative to such models it is possible, besides the assessment of significance of factor effects, to attach measures of error to DEA efficiency measurements. Secondly, even if a deterministic univariate production model is not justifiable one could still make use of a general class of censored models to fit the DEA measurements, whether they are computed in the form of residuals from a production model or simply as a measure of efficiency. In this context relates the models we use are similar in appearance to those used in the analysis of a stochastic frontier in a DEA analysis. This is achieved generalizing the Tobit. The distributions other than the normal considered in these extensions are the gamma and the truncated normal. This order of ideas appear in Souza (2005) and generalizes Banker and Natarajan (2004).

Our presentation proceeds as follows. In Section 2 we discuss the input and output specifications for our application in banking as well as the technical effects of interest. In Section 3 we introduce the DEA responses considered in the article. Section 4 covers the statistical models that are used in the application to analyze the DEA efficiency measures. Section 5 is on data analysis and deals with descriptive statistics and formal statistical inference. Finally in Section 6 a summary is presented showing the main results and conclusions of the article.

2. Specification of Inputs and Outputs

The definition of outputs and inputs in banking is controversial. See Colwell and Davis (1992), Berger and Humphrey (2000) and Campos (2002) for an in depth discussion on the matter. As described in Campos (2002) basically two approaches are possible - production and intermediation. The production approach considers banks as producers of deposits and loans using as inputs capital and labor. In such a context typically output is measured by the number of deposit accounts and the number of transactions performed. Under the intermediation approach banks function as financial intermediaries converting and transferring financial assets between surplus units and deficit units. Each output is measured in value not in number of transactions or accounts. There is not a unique recommendation on what should be considered as the proper set of inputs and outputs particularly under the intermediation approach. Here we follow the intermediation approach and take as output the vector $y=(v_1,v_2,v_3)$ defined by the variables $v_1=$ securities, $v_2=$ loans, $v_3=$ demand deposits. This output vector is also combined into a single measure, denoted by y_c, representing the sum of the values of v_i. This approach follows along the lines of Leightner and Lovell (1998), Sathie (2001) and Campos (2002). Although this definition of outputs is not followed always in the banking literature, is the most common, as seen in Campos (2002). Notice for example that the usage of demand deposits in the Brazilian banking literature also varies. Nakane (1999) studying cost efficiency considers it as a covariate in the cost function although its specification in the translog cost function is similar to an output. Silva and Neto (2002), also in the context of cost functions, consider demand deposits only as a factor influencing the technical efficiency component in the model.

All production variables, as shown below, are measured as indices relative to a benchmark and are normalized by a measure of size. This approach has the advantage of making the banks more comparable through the reduction of variability and of the influence of size in the DEA analysis.

We should emphasize here that DEA is quite sensitive to the dimension and composition of the output vector. Tortosa-Ausina (2002) provide examples showing that ordering in DEA efficiency may change substantially with the dimension of y. A single output is the extreme case. The combined measure has the advantage of avoiding spurious DEA measurements resulting from unique bank specializations. In this sense it leads to more robust and less conservative measures of technical efficiencies. The drawback to its use is that it may show some double counting due to the nature of the output components. But the double counting is also present in the multiple output vector. Nonetheless, most banking studies use a multiple output approach and thus we will follow this literature.

The inputs we consider inputs are labor (l), the stock of physical capital (k) which includes the book value of premises, equipments, rented premises and equipment and other fixed assets, and loanable funds (f) which include, transaction deposits, and purchased funds.

Typically the product oriented DEA efficiency analysis variables are specified using input and output measured in physical quantities. This is not strictly necessary and does not prevent its use in the intermediation approach even in a production function context. One may work with indexes or proxies reflecting the intensity of usage of each variable (input or output) in the production process. This is the case with the present application. Total output, loanable funds and capital are values. Also we found labor costs to be a more reliable measure of the intensity of labor usage than the number of employees which was much variable within the year. In this context we defined indexes to reflect the behavior of the production variables. These indexes were then further normalized by an index of size defined by the number of employees in the end of the period under analysis.

The data base used is COSIF, the plan of accounts comprising balance-sheet and income statement items that all Brazilian financial institutions have to report to the Central Bank on a monthly basis. This is the same data base used in all studies on the subject dealing with Brazilian banking. See for example Nakane (1999) and Campos (2002). The classification of banks was provided by the Supervision Department of the Central Bank of Brazil. They use cluster analysis to group banks according to their characteristics. The total number of banks used in the analysis (sample size) is 94.

The statistical analysis carried out in this paper is restricted to the year of 2001 and involves both Ancova (analysis of covariance) models and maximum likelihood estimations under several specifications described in detail in Section 4. As pointed out above output and input variables are treated as indexes relative to a benchmark. In this paper the benchmark for each variable, whether an input, an output or a continuous covariate, was chosen to be the median value of 2001. Banks with a value of zero for one of the inputs or the outputs were eliminated from the analysis.

Outputs, inputs, and the continuous covariate were further normalized through the division of their respective indexes by an index of personnel intended to be a size adjusting factor. The construction of this index follows the same method used for the other variables, that is, the index is the ratio of the number of employees in December of 2001 by its median value in the same month.

Even after size adjustment some banks still show values out of range either for inputs or outputs. There are some outliers in the data base. This is a problem for DEA applications which is known to be very much sensitive to outliers. To eliminate nonconforming output and input vectors we use a sort of Mahalanobis distance of common use in regression analysis to identify outlying observations. This amounts to identify as outlying observations for which the ith element of the diagonal of the hat matrix $W(W'W^{-1}W')$ is at least two times its trace. Here $W=(1,Y)$ or $W=(1,X)$ where 1 is a column of ones and Y and X are the matrices of output products and input usage respectively.

The covariates of interest for our analysis - factors likely to affect inefficiency, are nonperforming loans (q), bank nature (n), bank type (t), bank size (s), bank control(c) and bank origin (o). Nonperforming loans is a continuous variate and it is also measured as a ratio of indices like an input or output. All other covariates are categorical. The variable n assumes one of two values (commercial, multiple), the variable t assumes one of four values (credit, business, bursary, retail), the variable s assumes on of four values (large, medium, small, micro) the variable c assumes one of two values (private, public) and the variable o assumes one of two values (domestic, foreign). There is a bank (Caixa Econômica Federal - CEF) in the data base that requires a distinct classification due to its nature - variable n. We introduced one more level for this bank. This amounts to add one more level to the factor bank nature n. Dummy variables were created for each categorical variable. They are denoted $n_1, n_2, n_3, t_1, \dots, t_4, s_1, \dots, s_4, c_1, c_2$, and o_1, o_2 respectively.

3. Data Envelopment Analysis Production Models

Consider a production process with n production units (banks). Each unit uses variable quantities of m inputs to produce varying quantities of s different outputs y.

Denote by $Y=(y_1,\dots,y_n)$ the $s\times n$ production matrix of the n banks and by $X=(x_1,\dots,x_n)$ the $m\times n$ input matrix. Notice that the element $y_r \geq 0$ is the $s\times 1$ output vector of bank r and $x_r \geq 0$ is the $m\times 1$ vector of inputs used by bank r to produce y_r (the condition $l \geq 0$ means that at least one component of l is strictly positive). The matrices $Y=(y_{ij})$ and $X=(x_{ij})$ must satisfy: $\sum_i p_{ij} > 0$ and $\sum_j p_{ij} > 0$ where p is x or y.

In our application $m=3$ and $s=1$ or $s=3$ and it will be required $x_r,\ y_r > 0$ (which means that all components of the input and output vectors are strictly positive).

Definition 2.1: *The measure of technical efficiency of production of bank o under the assumption of variable returns to scale and output orientation is given by the solution of the linear programming problem* $\mathrm{Max}_{\phi,\lambda}\phi$ *subject to the restrictions*

1. $\lambda=(\lambda_1,\dots,\lambda_n)\geq 0$ *and* $\sum_i^n \lambda_i = 1$.
2. $Y\lambda \geq \phi y_o$.
3. $X\lambda \leq x_o$.

In the next section we consider statistical models adequate to the analysis of the optimum values ϕ_o^* of Definition 2.1 when covariates are thought to affect them. These models can be viewed as extensions of the univariate case, i.e, when $s=1$. In this instance it is possible to model the input-output data observations as a production model for which the DEA measurements under certain conditions behave as nonparametric maximum likelihood estimators. These results were originally presented in Banker (1993) and are extended in Souza (2001).

Suppose that $s=1$ and that the production pairs (x_j, y_j), $j=1,\dots,n$ for the n banks in the sample satisfy the deterministic statistical model

$y_j = g(x) - \varepsilon_j$, where *g(x)* is an unknown continuous production function, defined on a compact and convex set *K*. We assume *g(x)* to be monotonic and concave. The function *g(x)* also satisfy $g(x_j) \geq y_j$ for all j. The quantities ε_j are inefficiencies which are independently distributed nonnegative random variables. The input variables x_j are drawn indpendently of the ε_j.

One can use the observations (x_j, y_j) and Data Envelopment Analysis to estimate *g(x)* only in the set

$$K^* = \left\{ x \in K; x \geq \sum_{j=1}^{n} \lambda_j x_j, \lambda_j \geq 0, \sum_{j=1}^{n} \lambda_j = 1 \right\}.$$

For $x \in K^*$ the DEA production function is defined by

$$g_n^*(x) = \sup \left\{ \sum_j \lambda_j y_j ; \sum_j \lambda_j x_j \leq x \right\},$$

where the is restricted to nonnegative vectors λ satisfying $\sum_{j=1}^{n} \lambda_j = 1$.

For each bank *o*, $g_n(x_o) = \phi_o^* y_o$. This function is a production function on K^*, in other words, is monotonic, concave, $g_n(x_j) \geq y_j$, and satisfies the property of maximum extrapolation, that is, for any other production function $g_u(x), x \in K,\ g_u(x) \geq g_n^*(x), x \in K^*$.

4. Statistical Models Adequate to Study Product Oriented DEA Inefficiencies

We begin our discussion here assuming $s = 1$. It is shown in Banker (1993) that $g_n(x)$ is weakly consistent for *g*(*x*) and that the estimated residuals $\varepsilon_j^* = (1 - \phi_j^*) y_j$ have approximately, in large samples, the same behavior as the ε_j. Souza (2001) shows that the same results hold under conditions that do not rule out heteroscedasticity. These results validate the use of the DEA residuals or inefficiencies, or even the DEA measurements themselves, as dependent variables in regression problems since under the assumptions of the deterministic model they will be independent.

Banker (1993) discusses two distributions for the ε_j (assumed to be iid random variables) consistent with the asymptotic results cited above: the exponential and the half normal. Souza (2001) extends the discussion to the exponential and truncated normal relaxing the iid assumption. These more general models allow the use of typically stochastic frontier methods in the DEA analysis.

One may argue that the use of distributions like the exponential or the truncated normal are not totally adequate since in any particular application of DEA some residual observations will be exactly zero. This leads one naturally to the consideration of censored models to describe the stochastic behavior of the DEA residuals.

Let $z_0, \ldots, z_b$ be variables (covariates) we believe to affect inefficiency. Based on Souza (2001) results the following two statistical models can be used to fit the inefficiencies ε_j^* under the assumptions of the deterministic model.

Firstly one may postulate the exponential density

$$\lambda_j \exp(-\lambda_j \varepsilon)$$

where $\lambda_j = \exp(-\mu_j)$ with

$$\mu_j = z_{0j}\delta_0 + \ldots + z_{bj}\delta_b$$

sup

The z_{ij} are realizations of the z_i and the δ_i are parameters to be estimated.

Secondly one may consider the model $\varepsilon_j^* = \mu_j + w_j$ where w_j is the truncation at $-\mu_j$ of the normal $N(0, \sigma^2)$. This model is inherited from the analysis of stochastic frontiers of Coelli, Rao, and Battese (1998) and is equivalent to truncations at zero of the normals $N(\mu_j, \sigma^2)$.

For the exponential distribution the mean of the *jth* inefficiency error is exp (μ_j) and the variance exp (2 μ_j). For the truncated normal the mean is $\mu_j + \sigma\xi_j$ and the variance

$$\sigma^2 \left(1 - \xi_j \left(\frac{\mu_j}{\sigma} + \xi_j \right) \right)$$

where

$$\xi_j = \frac{\phi(\mu_j / \sigma)}{\Phi(\mu_j / \sigma)}$$

$\phi(.)$ and $\Phi(.)$ being the density function and the distribution function of the standard normal, respectively.

In both models the mean and the variance are monotonic functions of μ_j and thus both specifications allow monotonic heteroscedasticity.

A censored model discussed in Souza (2005) that could also be used impose the assumption that the ε_j^* satisfy the statistical model

$$\varepsilon_j^* = \begin{cases} w_j & \text{if } w_j > 0 \\ 0 & \text{if } w_j \le 0 \end{cases}$$

where $w_j = \mu_j + u_j$ the u_j being iid normal errors with mean zero and variance σ^2. This is the Tobit model of McCarty and Yaisawarng (1993). An extension allowing heteroscedasticity can be introduced assuming that the variance σ^2 is dependent on j and on some set of observables l_j, in other words, $\sigma_j^2 = \exp\{(1, l'_j)\zeta\}$, where the parameter vector ζ is unknown. In our application this dependency will be on bank size.

The Tobit model is adequate when it is possible for the dependent variable to assume values beyond the truncation point, zero in the present case. McCarty and Yaisawarng (1993) argue that this is the case in the DEA analysis. Their wording on this matter is as follows. It is likely that some hypothetical banks might perform better than the best banks in the sample. If these unobservable banks could be compared with a reference frontier constructed from the observable banks, they would show efficiency scores less than unity (over efficiency). This would lead to a potential nonpositive residual.

Clearly the Tobit could also be defined for the efficiency measurements ϕ_j^* in which case the truncation point would be one. We would have

$$\phi_j^* = \begin{cases} w_j & \text{if } w_j > 1 \\ 0 & \text{if } w_j \le 1 \end{cases}$$

Maybe a more reasonable assumption in the context of the Tobit model is to allow only for positive over efficiencies. In this case the distributions that readily come to mind to postulate for w_j are the truncation at zero of the normal $N(\mu_j, \sigma^2)$ and the gamma with shape parameter constant p and scale λ_j.

The standard technique to analyze all these models is maximum likelihood. The likelihood functions to be maximized with respect to the unknown parameters are defined as follows.

For the exponential distribution is

$$L(\delta) = \prod_{j=1}^{n} \lambda_j \exp\{-\lambda_j \varepsilon_j^*\}$$

For the truncated normal is

$$L(\delta, \sigma) = \prod_{j=1}^{n} \frac{\phi\left(\frac{\varepsilon_j^* - \mu_j}{\sigma}\right)}{\sigma \Phi\left(\frac{\mu_j}{\sigma}\right)}$$

where $\phi(.)$ is the density of the standard normal and $\Phi(.)$ its distribution function.

For the heteroscedastic Tobit model with censoring point at $a = 0$ or $a = 1$ is

$$L(\delta, \zeta) = \prod_{j:y_j^*=a} \Phi\left(\frac{a - \mu_j}{\sigma_j}\right) \prod_{j:y_j^*>a} \frac{1}{\sigma_j} \phi\left(\frac{y_j^* - \mu_j}{\sigma_j}\right)$$

where $y_j^* = \varepsilon_j^*$ or $y_j^* = \phi_j^*$.

For the Tobit with censoring defined by a truncated normal is

$$L(\delta, \sigma) = \prod_{j:y_j^*=a} \frac{\Phi\left(\frac{a-\mu_j}{\sigma}\right) - \Phi\left(-\frac{\mu_j}{\sigma}\right)}{\Phi\left(\frac{\mu_j}{\sigma}\right)} \prod_{j:y_j^*>a} \frac{\phi\left(\frac{y_j^*-\mu_j}{\sigma}\right)}{\sigma \Phi\left(\frac{\mu_j}{\sigma}\right)}$$

For the Tobit with censoring defined by a gamma distribution, let $\Gamma()$ denote the gamma function and let $G_p()$ denote the distribution function of the gamma distribution with shape parameter p and unit scale. The likelihood is

$$L(\delta, p) = \prod_{j:\phi_j^*=1} G_p(\lambda_j) \prod_{j:\phi_j^*>1} \frac{\lambda_j^p (\phi_j^*)^{p-1} \exp\{-\lambda_j \phi_j^*\}}{\Gamma(p)}$$

Some of the results in Banker (1993) and Souza (2001) can be extended to multiple output models not necessarily associated to a production model. Consistency of the ϕ_j^* is one of them. See Kneip, Park, and Simar (1998) and Banker and Natarajan (2001, 2004). This suggests that with the exception of the censoring at zero case and the models for DEA residuals all approaches are viable for multiple outputs since in large samples the DEA measurements ϕ_j^* will behave as in random sampling.

Another class of models that can be used in any instance is defined by the class of analysis of covariance models as suggested in Coelli, Rao, and Battese (1998). Here we apply a nonparametric version of the analysis of covariance taking as responses the rankings r_j of the observations on the variables under investigation (Conover, 1998). In other words we also use the model

$$r_j = z_{0j}\delta_0 + ... + z_{bj}\delta_b + u_j$$

where the u_j are independent $N(0,\sigma^2)$ non observable errors. This model shows approximate nonparametric properties.

5. Data Analysis

We begin the discussion in this section with Tables 1-4 which show basic statistics for DEA measures. DEA analysis was carried out using the software Deap 2.1 (Coelli, Rao and Battese, 1998). Entries in Tables 1-3 relate to the behavior and the association of the DEA measures of efficiency considered here and Table 4 is a runs test of randomization (Wonnacott and Wonnacott, 1990). We do not see evidence from this table against the assumption of independent observations. Table 1 refers to $(\phi_j^*)^{-1}$ when the output is y_c, i.e, combined. Table 2 refers to the same variable when the output is trivariate. Table 3 presents a matrix of Spearman rank correlations between the three responses of interest - DEA residuals and DEA measurements computed assuming combined and multiple output. The rank correlations seem to point to differences in the analysis with each variable. Although efficiency measurements computed considering the multiple output are much larger than the corresponding measurements for the combined output the ordering induced by the two measures show a reasonable agreement. For bank size and bank nature the averages of both measurements point to the same direction. Commercial banks dominate multiple banks and small and micro banks outperform medium and large banks. For bank control and bank origin however the story is different. The combined output indicate that private and foreign banks perform better. The multiple output puts private and public banks on equal footing and point to a better performance of domestic over foreign banks. For bank type both output types point to bursary banks as the best performance. They seem to differ significantly in the worst performance however. Credit institutions for the combined output and retail institutions for the multiple output. It should be said however that most of these differences are not statiscally significant. Most pair of confidence intervals will have a non empty intersection as can be seen in Tables 1 and 2. This fact is also captured in the nonparametric analyses of covariance shown in Tables 5, 6, and 7. The only significant effects detected are bank origin, marginally, for ε^* and bank type for ϕ^* under combined and multiple output, the last result being marginal.

It is important to mention here that in none of the models the variable nonperforming loans (q) seems to affect efficiency (inefficiency) significantly. Berger and Young (1997) find mixed evidence regarding the role of nonperforming loans in banking efficiency studies. They find evidence supporting both the bad luck and the bad management hypothesis. The bad luck hypothesis suggests that bank failures are caused primarily by uncontrolled events and thus a better proxy for riskiness of banks could be the concentration of loans and the loans-to-assets ratio. On the other hand, the bad management hypothesis implies that major risks for banking institutions are caused internally, which suggests that supervisors and regulators should analyze bank efficiency along with credit losses and credit risk.

Table 1. Descriptive statistics for categorical variables. Response is $1/\phi_j^*$ for a model with combined output. L and U are lower and upper 95% confidence limits

Variable	Level	N	Mean	L	U
Bank Nature	Commercial	12	0.462	0.267	0.657
	Multiple	81	0.378	0.317	0.440
Bank Type	Credit	33	0.408	0.320	0.496
	Business	24	0.526	0.405	0.646
	Bursary	3	0.746	0	1
	Retail	34	0.508	0.487	0.056
Bank Size	Large	18	0.317	0.181	0.452
	Medium	30	0.386	0.270	0.502
	Small	25	0.419	0.316	0.522
	Micro	21	0.409	0.280	0.538
Bank Control	Private	79	0.405	0.341	0.469
	Public	15	0.288	0.160	0.415
Bank Origin	Foreign	28	0.404	0.291	0.516
	Domestic	66	0.379	0.311	0.447

Table 2. Descriptive statistics for categorical variables. Response is $1/\phi_j^*$ for a model with multiple output. L and U are lower and upper 95% confidence limits

Variable	Level	N	Mean	L	U
Bank Nature	Commercial	12	0.633	0.466	0.800
	Multiple	81	0.585	0.525	0.646
Bank Type	Credit	33	0.642	0.559	0.726
	Business	24	0.646	0.531	0.761
	Bursary	3	0.750	0	1
	Retail	34	0.478	0.384	0.572
Bank Size	Large	18	0.522	0.388	0.655
	Medium	30	0.528	0.422	0.633
	Small	25	0.634	0.527	0.741
	Micro	21	0.674	0.553	0.795
Bank Control	Private	79	0.594	0.533	0.654
	Public	15	0.555	0.390	0.720
Bank Origin	Foreign	28	0.534	0.419	0.650
	Domestic	66	0.610	0.546	0.674

Table 3. Rank correlation between DEA residuals ε^*, combined output DEA ϕ^{*1} and multiple output DEA ϕ^{*2}.

Variable	ε^*	ϕ^{*1}	ϕ^{*2}.
ε^*	1	0.412	0.527
ϕ^{*1}	-	1	0.798
ϕ^{*2}.	-	-	1

Table 4. Runs test for DEA residuals ε^*, combined output DEA ϕ^{*1} and multiple output DEA ϕ^{*2}.

Variable	Runs	z	p-value
ε^*	43	-1.037	0.230
ϕ^{*1}	43	-1.037	0.230
ϕ^{*2}.	44	-0.830	0.407

Table 5. Nonparametric analysis of covariance for DEA residuals.

Source	df	Sum of Squares	Mean Square	F	p-value
Model	11	16,676.598	1,516.054	2.37	0.014
Bank Nature	2	1,742.179	871.090	1.36	0.262
Bank Type	3	1,612.793	537.598	0.84	0.476
Bank Size	3	2,910.929	970.310	1.52	0.217
Bank Control	1	1,565.253	1,565.253	2.45	0.122
Bank Origin	1	2,175.467	2,175.467	3.40	0.069
q	1	95.029	95.029	0.15	0.701
Error	82	52,488.902	640.109	-	-
Total	93	69,165.500	-	-	-

Table 6. Nonparametric analysis of covariance for DEA measurements computed for a combined output.

Source	df	Sum of Squares	Mean Square	F	p-value
Model	11	26,438.222	2,403.475	4.61	<0.001
Bank Nature	2	2,282.392	1,141.196	2.19	0.118
Bank Type	3	17,049.212	5,683.071	10.91	<0.001
Bank Size	3	2,167.122	722.374	1.39	0.253
Bank Control	1	198.099	198.099	0.38	0.539
Bank Origin	1	67.824	67.824	0.13	0.719
q	1	565.008	565.008	1.08	0.301
Error	82	42,723.778	521.022	-	-
Total	93	69,162.000	-	-	-

Table 7. Nonparametric analysis of covariance for DEA measurements computed for a multiple output.

Source	df	Sum of Squares	Mean Square	F	p-value
Model	11	12,083.195	1,098.472	1.59	0.118
Bank Nature	2	1,856.744	928.372	1.34	0.267
Bank Type	3	4,829.718	1,609.906	2.33	0.081
Bank Size	3	1,292.420	430.807	0.62	0.602
Bank Control	1	325.078	325.078	0.47	0.495
Bank Origin	1	1,445.029	1,445.029	2.09	0.152
q	1	104.035	104.035	0.15	0.699
Error	82	56,715.305	691.650	-	-
Total	93	68,798.500	-	-	-

In search for more powerful tests we now investigate the several parametric models discussed in the previous section. Tables 8-10 show goodness of fit statistics for the 14 alternatives implied by the consideration of different hypothesis on the output and different censoring points. The models were fitted using SAS procedures QLIM and NLMIXED. Initial values used for the Tobit alternatives involving the gamma, exponential, and the truncated normal distributions are the estimates of the classical Tobit models. No convergence or singularities were reported by SAS in the fitting process of any of the models. The information measures of Akaike and Schwarz were used to pick the best model for each response. The truncated normal (no Tobit censoring) was the best fit for DEA residuals. For DEA measurements both with combined and multiple outputs the best alternative is provided by the Tobit censored at 1 defined by the gamma distribution.

Tables 11,12, and 13 show the results of estimation for the best models. Table 14 shows the significance of each effect of interest by means of a likelihood ratio test. The models seem to be more informative in regard to technical effects than the ancovas. Significance of effects change with the model used. We see agreement only in bank nature and nonperforming loans. These two effects are not significant in any of the models.

As a further check on model adequacy we use the conditional moment test of specification described in Greene (2003). This is as follows. Let $r(y, x, \theta)$ be a vector of m moment conditions, where y is the response variable, x is the vector of exogenous variables and θ is the unknown parameter, if the model is properly specified, $E(r(y_i, x_i, \theta)) = 0$ for every i. The sample moments are

$$\bar{r}(\hat{\theta}) = \frac{1}{n}\sum_{i=1}^{n} r(y_i, x_i, \hat{\theta})$$

where $\hat{\theta}$ is the maximum likelihood estimator of θ. Let M be the $n \times m$ matrix whose ith row is $r(y_i, x_i, \hat{\theta})$ and G be the $n \times p$ matrix whose ith row is the gradient of the log-likelihood with respect to θ evaluated at $(y_i, x_i, \hat{\theta})$. Let

$$S = \hat{\Sigma} = \frac{1}{n}[M'M - M'G(G'G)^{-1}G'M]$$

If the model is properly specified $n\bar{r}(\hat{\theta})'S^{-1}\bar{r}(\hat{\theta})$ converges in distribution to a chi-square random variabel with m degrees of freedom.

We apply the conditional moment test of specification to the model defined by production residuals under the distributional assumption of truncated normal and for the combined and multiple output efficiency measures defined by the Tobit with gamma truncation. We begin the discussion with the production residuals and the truncated normal distribution $N(\mu_i, \sigma^2)$ at $-\mu_i$. Let

$$\lambda_i = \frac{\phi(\mu_i/\sigma)}{\Phi(\mu_i/\sigma)}$$

The moment conditions we use are

1. $y_i - \hat{\mu}_i - \hat{\sigma}\hat{\lambda}_i$
2. $y_i^2 - \hat{\sigma}^2 - \hat{\lambda}\hat{\sigma}\hat{\mu} - \hat{\mu}^2$
3. $y_i^3 - \hat{\sigma}^3\left(\sqrt{2\pi}\,\Phi(\hat{\mu}/\hat{\sigma})\right)^{-1} h_3(-\hat{\mu}/\hat{\sigma})$
4. $y_i^4 - \hat{\sigma}^4\left(\sqrt{2\pi}\,\Phi(\hat{\mu}/\hat{\sigma})\right)^{-1} h_4(-\hat{\mu}/\hat{\sigma})$

where,

$$h_n(y) = \int_y^{+\infty} (x-y)^n \exp(-0.5x^2)dx$$

For the two Tobit gamma models with parameters (p, λ_i), for observations of the response y greater than one, we compute

1. $y_i - \hat{p}/\hat{\lambda}_i$
2. $y_i^2 - \hat{p}(\hat{p}+1)/\hat{\lambda}_i^2$
3. $\ln(y_i) - \Psi(\hat{p}) + \ln(\lambda_i)$
4. $1/y_i - \hat{\lambda}_i/(\hat{p}-1)$

where $\Psi(.)$ is the digamma function. For the censored observations we compute

1. $[(G_{\hat{p}+1}(\hat{\lambda}_i) - G_{\hat{p}}(\hat{\lambda}_i)/G_{\hat{p}}(\hat{\lambda}_i)]\,\hat{p}/\lambda_i$
2. $[(G_{\hat{p}+2}(\hat{\lambda}_i) - G_{\hat{p}}(\hat{\lambda}_i)/G_{\hat{p}}(\hat{\lambda}_i)]\hat{p}(\hat{p}+1)/\hat{\lambda}_i^2$
3. $\rho(\hat{p}, \hat{\lambda}_i)/G_{\hat{p}}(\hat{\lambda}_i) - \Psi(\hat{p}) + \ln(\lambda_i)$
4. $[(G_{\hat{p}-1}(\hat{\lambda}_i) - G_{\hat{p}}(\hat{\lambda}_i)]/G_{\hat{p}}(\hat{\lambda}_i)]\,\hat{\lambda}_i/(\hat{p}-1)$

where

$$\rho(p, \lambda) = \frac{1}{G_p(\lambda)\Gamma(p)}\int_0^1 \lambda^p x^{p-1}\exp(-\lambda x)\ln(x)dx$$

The moment conditions for the gamma distribution may be seen in Greene (2003).

The chi-square statistics we find for the truncated normal model for the production residuals and the gamma censoring for the single and combined outputs are 0.57, 0.050 and 0.039 clearly nonsignificant.

Which model should we choose? Our criterion was to pick the model that would mimic the direction of performance of the sample means for all significant effects. The only model showing the proper signs and parameters estimates with this property was the response defined by the multiple output DEA measurement ϕ^{*2}. Significant effects indicated by this model are bank type and bank origin. Domestic banks outperform foreign banks and the significance in bank type is due only to pairwise contrasts with the level retail. Finally we mention that these results marginally agree with those provided by the corresponding ancova and both models show approximately the same Pearson correlation between observed and predicted values (about 40%).

Table 8. Parametric models for DEA residuals ε^*. -2ll is twice the log-likelihood, Parms is the number of parameters and AIC and BIC are the Akaike and Schwarz information criteria, respectively.

Model	-2ll	Parms	AIC	BIC
Truncated Normal	167.7	13	193.7	226.8
Exponential	196.2	12	220.2	250.7
Tobit (at zero)	210.1	13	236.1	269.1
Heteroscedastic Tobit (at zero)	206.2	16	238.3	279.0

Table 9. Parametric models for DEA responses for combined output ϕ^{*1}. -2ll is twice the log-likelihood, Parms is the number of parameters and AIC and BIC are the Akaike and Schwarz information criteria, respectively.

Model	-2ll	Parms	AIC	BIC
Tobit (at 1)	429.3	13	455.3	488.4
Heteroscedastic Tobit (at 1)	413.6	16	445.6	486.3
Truncated Normal (Tobit at 1)	408.5	13	434.5	467.6
Gamma (Tobit at 1)	393.4	13	419.4	452.5
Exponential (Tobit at 1)	436.0	12	460.0	490.5

Table 10. Parametric models for DEA responses for multiple output ϕ^{*2}. -2ll is twice the log-likelihood, Parms is the number of parameters and AIC and BIC are the Akaike and Schwarz information criteria, respectively.

Model	-2ll	Parms	AIC	BIC
Tobit (at 1)	328.7	13	354.7	387.8
Heteroscedastic Tobit (at 1)	307.2	16	339.2	379.2
Truncated Normal (Tobit at 1)	310.8	13	336.8	369.9
Gamma (Tobit at 1)	296.3	13	317.3	350.4
Exponential (Tobit at 1)	428.3	12	452.3	482.8

Table 11. Parametric model for DEA residuals ε^*. Truncated normal distribution.

Variable	Estimate	Standard Error	t	p-value
Intercept	0.012	1.412	0.01	0.993
n_1	-1.299	1.412	-0.92	0.360
n_2	-0.491	1.327	-0.37	0.712
t_1	0.411	0.483	0.85	0.397
t_2	0.258	0.383	0.67	0.502
t_3	-0.827	0.908	-0.91	0.365
s_1	1.136	0.606	1.87	0.064
s_2	0.846	0.481	1.76	0.082
s_3	0.895	0.437	2.05	0.043
c_1	0.914	0.535	1.71	0.091
o_1	-0.567	0.278	-2.04	0.044
q	0.040	0.076	0.52	0.605
σ^2	0.788	0.198	3.97	<0.001

Table 12. Parametric model for DEA measurements from combined output ϕ^{*1}. Tobit with censoring at 1, gamma distribution with shape parameter p.

Variable	Estimate	Standard Error	t	p-value
Intercept	1.773	0.673	2.63	0.010
n_1	-0.823	0.624	-1.32	0.190
n_2	-0.617	0.606	-1.02	0.311
t_1	-1.067	0.240	-4.44	<0.001
t_2	-1.144	0.198	-5.79	<0.001
t_3	-1.752	0.397	-4.41	<0.001
s_1	-0.778	0.276	-2.82	0.006
s_2	-0.284	0.217	-1.31	0.193
s_3	-0.051	0.198	-0.26	0.797
c_1	0.238	0.200	1.19	0.237
o_1	-0.167	0.150	-1.11	0.269
q	-0.046	0.039	-1.18	0.243
p	3.079	0.463	6.65	<0.001

Table 13. Parametric model for DEA measurements from multiple output ϕ^{*2}. Tobit with censoring at 1, gamma distribution with shape parameter p.

Variable	Estimate	Standard Error	t	p-value
Intercept	1.135	0.687	1.65	0.012
n_1	-0.967	0.635	-1.5	0.132
n_2	-0.860	0.615	-1.40	0.165
t_1	-0.588	0.245	-2.40	0.018
t_2	-0.523	0.204	-2.56	0.012
t_3	-0.938	0.411	-2.28	0.025
s_1	-0.354	0.279	-1.27	0.208
s_2	-0.015	0.220	-0.07	0.944
s_3	0.127	0.199	0.64	0.525
c_1	0.119	0.210	0.57	0.572
o_1	-0.395	0.153	-2.58	0.011
q	-0.024	0.042	-0.58	0.565
p	2.976	0.486	6.12	<0.001

Table 14. Likelihood ratio test statistic -LR for the effects of interest. -2ll is twice the log-likelihood. ε^* is the DEA residual. ϕ^{*1} and ϕ^{*2} are DEA measurements for combined and multiple outputs respectively.

Response	ε^*			ϕ^{*2}			ϕ^{*1}		
Model	-2ll	LR	p-value	-2ll	LR	p-value	-2ll	LR	p-value
Full	167.710			291.338			393.406		
Bank Nature	170.662	2.953	0.228	294.135	2.798	0.247	395.882	2.476	0.290
Bank Type	170.550	2.840	0.417	299.671	8.333	0.040	424.586	31.180	<0.001
Bank Size	173.551	5.441	0.142	295.483	4.145	0.246	402.648	9.242	0026
Bank Control	171.099	3.390	0.066	291.656	0.318	0.573	394.783	1.376	0.241
Bank Origin	171.950	4.240	0.039	297.887	6.549	0.010	394.655	1.249	0.264
q	167.973	0.264	0.607	291.655	0.317	0.573	394.649	1.243	0.265

6. Summary and Conclusions

Output oriented efficiency measurements, calculated under the assumption of variable returns to scale, in the context of Data Envelopment Analysis were investigated for Brazilian banks. In this analysis bank outputs investment securities, total loans and demand deposits are analyzed combined in a single measure and as a multiple output vector to produce different DEA measurements of efficiency based on inputs labor, loanable funds, and stock of physical capital. The intermediation approach is followed and for each measure of efficiency several statistical models are considered as modeling tools to assess the significance of technical effects bank nature, bank type, bank size, bank control, bank origin, and nonperforming loans. The year of analysis is 2001.

The competing statistical models are justified in terms of the stochastic properties of the production responses in the DEA context. The range of model alternatives include the use of nonparametric analysis of covariance, the fit of the truncated normal and the exponential distribution and a general class of Tobit models allowing for heteroscedasticity. All parametric models are fit via maximum likelihood.

The response variable leading to the most informative statistical model uses as response the multiple output-input production model. DEA is oriented to output and computed under the assumption of variable returns to scale. The statistical model chosen is a like a Tobit regression induced by a gamma distribution.

The methodological contributions of the article are as follows. Firstly new alternatives to measure bank output are suggested with the objective of making banks more comparable and to reduce variability and outliers. Secondly it is suggested a collection of statistical models that one could use in a DEA application.

The empirical findings are that domestic banks outperform foreign banks and that all levels of bank type outperform retail with no other pairwise contrasts being significant. None of the models show a significant association of the response with nonperforming loans.

Relevant questions to the administration of the Central Bank of Brazil like the indication of a cut off point for inefficiency measures that would be indicative of bank failure or excessive risk taking, the effect on efficiency of privatization of a public bank or of selling a private bank to a foreign institution as well the effect of merging and acquisitions on bank efficiency were not addressed and cannot be answered in the present study. The reason for this is twofold. Firstly the measure of risk considered in the study, nonperforming loans, is not significant. Secondly to properly address the issues of risk much more complex models are necessary. A panel data structure and past information on other risk and efficiency measures (as cost and revenue efficiencies) will have to be investigated as well.

7. References

Banker, R. D. (1993), Maximum likelihood consistency and DEA: a statistical foundation, *Management Science*, 39(10):1265-1273.

Banker, R. D. and Natarajan (2001), Evaluating contextual variables affecting productivity using data envelopment analysis, presented in the *Sixth European Workshop on Efficiency and Productivity Analysis*.

Banker, R. D. and Natarajan (2004), . Statistical tests based on DEA efficiency scores. In: Cooper, WW; Seiford, L.M., Zhu, J. (eds.) *Handbook on Data Envelopment Analysis*, Kluwer International Series, New York.

Berger, A. N. and Humphrey, D. B. (2000), Efficiency of financial institutions: international survey and directions for future research, in *Performance of Financial Institutions: Efficiency, Innovation, Regulation.* Cambridge, UK.

Berger, A. N. and R.D. Young (1997), Problem loans and cost efficiency in commercial banks, *Journal of Banking and Finance* 21, 849-870.

Campos, M. B. (2002), Produtividade e eficiência do setor bancário privado brasileiro de 1994 a 1999, Dissertação de Mestrado, EASP-FGV, São Paulo.

Coelli, T., Rao, D. S. P. and Battese, G. E. (1998), *An Introduction to Efficiency and Productivity Analysis*, Kluwer, Boston.

Colwell, R. J. and Davis, E. P. (1992), Output and productivity in banking, *Scandinavian Journal of Economics*,94, Supplement, 111-129.

Conover, W. J. (1998), *Practical Nonparametric Statistics*, Wiley, NY.

Eseinbeis, R. A., Ferrier, G. D. and Kwan, S. H. (1999), The Informativeness of Stochastic Frontier and Programming Frontier Efficiency Scores: Cost Efficiency and Other Measures of Bank Holding Company Performance, *Federal Reserve Bank of Atlanta*, Working paper 99-23.

Kneip, A., Park, B. U. and Simar, L. (1998), A note on the convergence of nonparametric DEA estimators for production efficiency scores, *Econometric Theory* 14, 783-793.

Leightner, J. E and Lovell, C. A. K. (1998), The impact of finance liberalization on the performance of Thai banks, *Journal of Economics and Business* 50, 115-131.

McCarthy, T. A. and Yaisawarng, S. (1993), Technical efficiency in New Jersey school districts, in *The Measurement of Productive Efficiency*, Oxford University Press, New York.

Nakane, M. I. (1999), Productive efficiency in Brazilian banking sector, Texto para Discussão 20/99, IPE-USP, São paulo.

Sathye, M. (2001), X-efficiency in Australian banking: an empirical investigation, *Journal of Banking and Financing* 25, 613-630.

Tortosa-Ausina, E. (2002), Bank cost efficiency and output specification, *Journal of Productive Analysis*, 18, 199-222.

Silva, T. L. and Neto, M. J. (2002), Economia de escala e eficiência nos bancos brasileiros após o real, *Estudos Econômicos*, 32, 577-620.

Souza, G. S. (2001), Statistical properties of data envelopment analysis estimators of production functions, *Brazilian Journal of Econometrics*, 21(2):291-322.

Souza, G. S. (2005), Significância de efeitos técnicos na eficiência de produção da pesquisa agropecuária brasileira. Artigo aceito para publicação na *Revista Brasileira de Economia*, FGV, Rio.

Wonnacott, T. H. and Wonnacott, R. J. (1990), *Introductory Statistics for Business and Economics*, 4th ed, Wiley, New York.