

INVENTION AND BUSINESS CYCLES

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Abstract – Invention of new goods leads to interesting business cycle behavior. The economy's dynamic response to invention is consistent with several business cycle facts that the standard real business cycle model has difficulty explaining.

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INTRODUCTION

The usual approach to technical progress is to treat it as an improvement in the efficiency of producing the final goods and services that already exist. Thus, technical progress appears either as an increase in the scale coefficient of the production function for final goods (i.e., “total factor productivity”) or as an increase in the variety or quality of intermediate goods used to produce the final goods. The real business cycle and endogenous growth literatures provide respective examples of these two treatments. In this paper, I examine another possibility, namely, treating technical progress as the invention of final goods that previously did not exist. Much invention seems to be of this nature; television, fast food, and factory robots are but a few examples that spring to mind.

Invention of a new good alters the economy’s steady state and so calls forth a transitional effort to move the economy to its new steady state. That transition is a cyclical response, and it exhibits several business cycle characteristics that hitherto have been difficult to explain. For example, output’s response typically is hump-shaped, adjustment in consumer and producer durables leads the cycle, and employment is much more highly correlated with output than productivity is. Also, even though invention is a change in the production function, the effect it has on the economy depends on the characteristics of the demand for the new good. Consequently, the shock will appear to the econometrician as a demand shock. The theory thus can explain Blanchard and Quah’s (1989) finding that demand shocks lead to a transitory change in output that is hump-shaped. The subsequent analysis is done in a framework that excludes economic growth, but a simple example in an appendix available from the author shows that the same conclusions can arise in the presence of endogenous growth as well.

2. SOME FACTS IN NEED OF EXPLANATION

Two basic facts of business cycles, almost definitions of cycles, are persistence and comovement. First, aggregate deviations from “trend” persist for several periods. There is, of course, the problem of exactly how one measures trend. As Nelson and Plosser (1982) first showed, the method chosen has major implications for the magnitude, timing, and even existence of measured business cycles. The issues involved are well-known and beyond the scope of the present discussion. Suffice it to say here that most analyses of the time-series data suggest considerable persistence in deviations from trend, however estimated. Second, various sectors of the economy move together in business cycles. If they did not, there would be no (or at least not many) observable business cycles because most sectors of the economy are too small a part of the whole to be responsible on their own for the observed magnitude of aggregate fluctuations. So any theory of business cycles must explain persistence and comovement.

The most recent major body of thought to attempt an explanation has been real business cycle (RBC) theory. Despite its considerable appeal, RBC theory to date has been not offered a fully convincing story. Most RBC models obtain persistence only by putting it into the exogenous shocks driving the economy. The model itself thus does not deliver the persistence; the shocks do. Second, most

RBC models have been single-sector models and so cannot even address the issue of comovement. Those models that include many countries or many locations typically display what Baxter (1996) calls the “comovement problem”: the models generate comovement of labor and investment across locations that is negative unless the shocks are extremely highly correlated across locations, meaning that once again the shocks rather than the model deliver the desired behavior.

In addition, there are other business cycle facts that standard RBC models have difficulty explaining. Stadler (1994) reviews many of them; those relevant to the theory presented below are the following:¹

- (1) Employment (in terms of hours worked) is almost as variable as output and strongly procyclical, whereas real wages are no more than weakly procyclical.²
- (2) The correlation between productivity and employment is negative in most countries, and that between productivity and output is moderate in size.
- (3) Output’s response to a transitory innovation is hump-shaped with a strong trend-reverting component; output growth therefore displays positive autocorrelation at short horizons and weak (negative) correlation at longer horizons.
- (4) One quarter to one half of the variation in Solow residuals is attributable to variations in aggregate demand.³

Baxter (1996) mentions two more business cycle facts concerning durable goods:

- (5) Purchases of consumer durables and investment (i.e., producer durables) are more volatile than total output.
- (6) Purchases of consumer durables and investment lead the cycle.

RBC models do predict that investment in producer durables is more volatile than output, but they typically do not predict that investment will lead the cycle. Virtually all RBC models, like most macro models in general, ignore consumer goods altogether and so offer no explanation for their behavior. Baxter’s model is a notable exception, but it has only partial success in explaining the path of consumer durables and no more success than other RBC models in explaining facts (1) through (4).

Fluctuations in standard RBC models are driven by productivity shocks. These by definition are supply-side phenomena, so they have nothing to say about that part of the Solow residual arising from the demand side of the economy or the effects it will have on economic behavior. Micro studies find low wage elasticities of labor supply, so most productivity shocks should be absorbed in wages. As a result, standard RBC models greatly underpredict variability of hours worked. RBC models predict a positive correlation between productivity and hours and a very high correlation between productivity and output. As already mentioned, most RBC models can account for aggregate autocorrelation patterns only by imposing

¹ Stadler also presents some facts about open economies and nominal quantities, but the theory below is confined to real closed economies and so cannot address those facts. Extensions of the theory developed herein to include trade and money would be interesting.

² There is one real business cycle model that can explain this fact. Hansen (1985) supposes indivisible labor, leading to all variability in hours coming from changes in the number of workers. His simulations give large fluctuations in hours worked and small fluctuations in productivity.

³ Transitory shocks therefore are not necessarily nominal in origin; see Issler and Vahid (2001).

them on the time propagation structure of the productivity shocks. Finally, most RBC models discuss only the path of total output and do not distinguish among components such as consumer nondurables, consumer durables, and investment.⁴

The theory presented below is a new type of RBC model. In contrast to the standard model, the theory developed here is driven by the interaction of supply-side invention with the demand side of the economy. The underlying shocks are inventions in the kinds of goods that can be produced. Such shocks are changes in the production process, so the theory is a member of the RBC family. Unlike the standard RBC model, however, invention shocks do not raise current output directly. Changes in the economy arise only to the extent that the newly invented goods are demanded. High demand leads to a large response; low demand leads to a small response. The theory thus concerns precisely that part of the Solow residual about which standard RBC model is silent. The economy's dynamic response to an invention has most of the characteristics that Stadler and Baxter discuss and that standard RBC models cannot explain. Employment is highly variable, employment always can and often must be negatively correlated with productivity, employment and output both always can and often must respond in a hump-shaped manner. Several variables act as leading indicators. Length of the workweek, new orders for consumer goods, plant and equipment orders, building permits, and durable order backlogs all either are now or have been components of the Index of Leading Indicators; their predictive value is explained by the theory developed below. The theory thus addresses elements of the economy's behavior that the standard RBC model leaves unexplained and so complements that model nicely.

3. INVENTION OF DURABLE GOODS: SIMPLE CASES

We begin the analysis with two simple cases: (1) an economy, initially producing only a nondurable good using only labor as a productive input, invents a consumer durable, and (2) an economy in the same initial state invents productive capital instead of a consumer durable. These simple models give the basic intuition of the more advanced models to follow.

1. Invention of a Consumer Durable.

The representative household obtains utility from consumption of nondurables c , the service flow d from durables, and leisure h :

$$(1) \quad u(c, d, h)$$

$$u_i > D, u_{ii} < D \quad \text{for } i = c, d, h$$

⁴ Baxter's (1996) model is an exception. It has two sectors, one producing nondurables and the other producing both productive capital and consumer durables. Shocks are the standard changes in each sector's total factor productivity. In the model, the two sectors show positive comovement, and durables and investment lead the cycle. However, like most other RBC models that rely on shocks to total factor productivity, Baxter's model cannot generate observed serial correlation patterns internally.

We assume for simplicity that d is proportional to the stock D of durables with a constant of proportionality of one, allowing us to replace d in the utility function with D :

$$(2) \quad u(c, d, h) = u(c, D, h)$$

Initially, there are no durables to consume, so we allow utility to be positive when D is zero.

$$(3) \quad u(c, D, h) > 0$$

We also assume the following conditions on marginal utilities:⁵

$$(4) \quad \lim_{D \rightarrow 0} u_D = \Delta < \infty = \lim_{c \rightarrow 0} u_c = \lim_{h \rightarrow 0} u_h$$

Finally, we suppose that utility is separable in nondurables, durables, and leisure

$$(5) \quad u_{ij} = 0 \text{ for } i \neq j$$

The main reason for this assumption is analytical simplicity. One can derive results for with nonseparable utility, but it is necessary to go case by case, taking one cross-derivative at a time and first considering what happens if that derivative is positive and then what happens if it is negative. However, it is not clear what the signs of the cross-derivatives should be. Although it is easy to think of examples where goods are complements or substitutes, it is not obvious whether the total of all nondurable consumption is a complement or a substitute for the total of all durables consumption or for leisure. For example, labor-saving goods, such as washing machines, presumably are more valuable when leisure is low, meaning that u_{Dh} is negative for such goods; sporting goods, in contrast, presumably are more valuable when one has leisure time in which to enjoy them, meaning that u_{Dh} is positive for those goods. It is unclear what sign u_{Dh} should have for aggregate durables consumption. In fact, non-separability between leisure and consumption may be useful in explaining one aspect of the data, as mentioned later, but virtually all the analysis that follows maintains total separability of utility.

We suppose for now that output y is produced using only labor as an input; productive capital does not exist. This restriction keeps the dimensionality of the problem tractable. The durable good D is a state variable; introducing productive capital would then add a second state variable and make the problem very difficult to analyze. We discuss a model with productive capital and labor in section III.2

⁵ The restriction that u_D is strictly finite even when D is zero is for analytical convenience, as discussed below. Nothing of importance is affected by this restriction. Moreover, it also seems reasonable to suppose that the marginal utility of, say, food (c) and sleep (h) could become infinite as their quantities go to zero whereas the marginal utility of household durables (D) - including even clothing in at least in some climates - would not be infinitely valuable on the margin even when totally absent. People die without food or sleep but do not necessarily die without consumer durables.

and limited versions of models with both consumer and producer durables after that. The production function is the usual concave relation:

$$(6) \quad y = f(1-h) \quad f' > 0, f'' < 0$$

where h is the fraction of time spent on leisure.

The household seeks to maximize lifetime utility subject to its budget constraint:

$$(7) \quad \max_{c,D,k} \int_0^{\infty} u(c, D, h) e^{-\rho t} dt$$

subject to

$$dD/dt = f(1-h) - c - \delta D$$

plus initial and terminal conditions, where D is the rate of time preference and δ is the depreciation rate for durables.

1.1. Pre-Invention Optimality.

Before invention of the nondurable, there is no dynamic aspect to the household problem, which reduces to a simple period-by-period maximization of utility:

$$(8) \quad \max_{c,k} u(c, D, h)$$

subject to the budget constraint

$$(9) \quad c = f(1-h)$$

One readily obtains the Euler equation

$$(10) \quad \frac{u_c(c, D, h)}{u_k(c, D, h)} = \frac{u_c(c)}{u_k(h)} \quad \text{because } u_{cD} = u_{ch} = u_{Dh} = D$$

$$= f'(1-h)$$

which is a one-to-one relation between c and h . We can use the budget constraint (9) to write

$$(11) \quad u_c \frac{[f(1-h)]}{u_k(h)} = f'(1-h)$$

The value of h chosen to satisfy (11) [equivalently, (10)]; c is then determined uniquely from the production function. Optimal c and h both are constant at the values c^B and h^B , where the superscript B denotes values before invention.

1.2. Post-Invention Optimality.

Once the durable good has been invented, the household solves problem (7) with D no longer constrained to zero. This problem is truly dynamic. Its Hamiltonian is

$$(12) \quad H = u(c, D, h) + \lambda[f(1-h) - c - \delta D]$$

where λ is the costate variable. The necessary conditions include the dynamic equations

$$(13) \quad dD/dt = \partial H / \partial \lambda = f(1-h) - c - \delta D$$

$$(14) \quad d\lambda/dt = -\partial H / \partial D + \rho\lambda = -u_D + (\delta + \rho)\lambda$$

the first-order conditions

$$(15) \quad \partial H / \partial c = u_c - \lambda = 0$$

$$(16) \quad \partial H / \partial h = u_h - \lambda f' = 0$$

and the usual endpoint (initial and transversality) conditions

$$(17) \quad D(t = 0) = D_0 \quad \text{given}$$

$$(18) \quad \lim_{t \rightarrow \infty} \lambda_t D_t e^{-\rho t} = 0$$

The endpoint conditions do not figure in the discussion that follows and so will not be mentioned again in the sequel. The two first-order conditions (15) and (16) together give the Euler equation

$$(19) \quad u_h / u_c = f'$$

which again is a one-to-one relation between c and h . Note that it also is exactly the same condition as (10), a fact we use in deriving the economy's adjustment path. Equation (19) cannot be written in the form of (11) because the simple budget constraint (9) no longer applies, so solving for optimal c and h is more

complicated. Total differentiation of (15) and (16) yields implicit functions for c and h in terms of δ :

$$(20) \quad c = c(\lambda), \quad c' < D$$

$$(21) \quad h = h(\lambda), \quad h' < D$$

The dynamic equations (13) and (14) then can be written

$$dD/dt = f[1 - h(\lambda)] - c(\lambda) - \delta D$$

$$d\lambda/dt = -u_D(D) + (\delta + \rho)\lambda$$

Setting these two equations equal to zero yields the expressions for the equilibrium loci for D and λ ; total differentiation then yields the following expressions for the slopes of those loci:

$$(22) \quad \frac{d\lambda}{dD} \Big|_{dD/dt=0} = \frac{-\delta}{c' + f'h'}$$

$$(23) \quad \frac{d\lambda}{dD} \Big|_{d\lambda/dt=0} = \frac{u_{DD}}{\lambda + \rho}$$

$$< 0$$

The $dD/dt=0$ locus slopes upward, and the $d\lambda/dt=0$ locus slopes downward. We thus obtain the phase diagram and its associated dynamics shown in Figure 1. The economy has a steady state at the saddle point (DA^* , λA^*) and approaches it along one of the dynamic adjustment paths. In this notation, the superscript A denotes values after invention, and the asterisk denotes a steady-state value. In the case at hand, the approach will be along the left-hand path, starting at the point $(0, \lambda A)$, because at the instant after invention of the durable good, the value of D is zero.⁶

We now can work out the economy's response to the invention of the durable good.

⁶ This is the point where the restrictions in (4) enter the analysis. Because u_D is finite even when $D = 0$, the $d\lambda/dt=0$ locus intersects the vertical axis at a finite value of λ . The dynamic adjustment path lies below that locus, so it also intersects the vertical axis at a finite λ . Had we allowed the usual Inada condition that u_D approaches infinity as D approaches zero, we then would have to deal with the problem of how to allocate output between durable and non-durable consumption - c and D , respectively. An infinite value of u_D when D is zero would imply that all output would be diverted to D , at least for an instant. But then, at that same instant, no output would be allocated to c , which then would make its marginal value infinite, suggesting that some output should be allocated to c , thus violating the implication that all output should go to D . In terms of the phase diagram, both the $d\lambda/dt=0$ locus and the dynamic adjustment path would go asymptotically toward the vertical axis, implying no finite starting value for λ and thus a starting value of zero for non-durables c . Assuming that u_D is strictly finite avoids these problems.

(1) Invention leads to immediate decreases in both leisure h and consumption c of the nondurable. This result is quickly shown by contradiction. Denote the values of c and h after invention by $c_t A$ and $h_t A$. The values cB and hB before invention are constant and so have no time subscript. Let the moment of invention be denoted $t = 0$. Suppose that $c_0 A = cB$. Then condition (19), being one-to-one between c and h , guarantees that $h_0 A = hB$, too. Before invention, however, the amount of output $f(hB)$ equals cB , so that no output is left over for building the durable good D . Thus at time zero, no durable would be accumulated. After time zero, the phase diagram shows that δ falls along the dynamic adjustment path, which in turn implies that both c and h rise, by (20) and (21). This solution, however, is impossible because c starts equal to output and then perpetually rises while output perpetually falls, violating the transversality condition. Also, no durable good D ever gets made. Thus we cannot have $c_0 A = cB$. *A fortiori*, we cannot have $c_0 A > cB$, either. The only remaining possibility is $c_0 A < cB$, which implies by (19) that $h_0 A < hB$. Thus c and h both fall at the instant of invention. The reduction in h raises output y , with the difference between c and y devoted to building durables D .

(2) Subsequently, the economy follows the dynamic adjustment path down from the point $(0, 80A)$ toward the steady state. Along this path, D rises and δ falls. The latter property means that c and h both rise, by (20) and (21).

(3) In the new steady state, both c and h are lower than before invention of the durable, and y is higher. To see these results, note that in the steady state $dD/dt=0$, so that investment in D is the replacement amount δDA^* . Again, there are three possibilities for cA^* - greater than, equal to, or less than cB . If $cA^* = cB$, then again by (19) $hA^* = hB$. We thus also have $yA^* = yB$. Consequently, $yA^* - cA^* = yB - cB = 0$, leaving no output for the replacement investment required to maintain DA^* . Thus cA^* cannot equal cB . *A fortiori*, cA^* cannot exceed cB . So $cA^* < cB$, which then implies by (19) that $hA^* < hB$. This last condition also implies that steady state output $yA^* = f(1-hA^*)$ exceeds its pre-invention level $yB = f(1-hB)$. The lower values of c and h along the entire optimal path imply a loss of utility compared to the pre-invention steady state. Obviously, that loss must be at least compensated by the lifetime consumption of the new durable good; otherwise, the durable good would not be demanded even after its invention.

We can summarize the economy's optimal response to the invention of the durable good as follows:

- (A) consumption of the non-durable good c immediately jumps down and then gradually rises, stopping at a level below its pre-invention level;
- (B) leisure h behaves the same way, immediately jumping down and then gradually rising to stop at a level below its pre-invention level, implying that labor immediately jumps up and then gradually falls to a level above its pre-invention level;
- (C) the durable good stock D gradually rises from zero to its steady state value DA^* ;
- (D) output $y = f(1-h)$ jumps up, then gradually falls, stopping at a value above its pre-invention value, reflecting the behavior of labor supply;
- (E) productivity (either average or marginal product) is negatively related to output because the production function is unchanged, marginal product

of labor is diminishing, and labor supply has risen (to produce the durable good);

(F) even though non-durable consumption expenditure falls in response to the shock, total consumption expenditure, equal to non-durable consumption plus purchases of durables, is always higher than before the shock because total consumption equals output.

This response has most of the characteristics mentioned earlier that Stadler and Baxter discuss. Employment has approximately the same variability as output and is strongly procyclical; it and output display a Blanchard-Quah (1989) hump-shaped response with strong mean-reversion (see summary points B and D).⁷ Consequently employment and output growth show positive autocorrelation over short horizons and weak autocorrelation over longer horizons (that is, if output growth is high [low] now it probably was high [low] last period, whereas it probably was not high [low] several periods ago). Productivity is negatively correlated with employment (summary point E). Durables purchases are more variable than total output because the pre-invention amount of durables purchases is small (zero in this model), which makes the variation in durables purchases large in percentage terms. Also, invention shocks work through the economy's demand side. The increase in output is directly related to the size of the new demand for it. For example, if there were no demand, there would be no increase in output at all. This central planner's version of the model does not have prices, but in the competitive equilibrium version of the solution, firms' increase in demand for labor reflects the increase in the price of output, which in turn is driven by the households' demand for the new good. Total output therefore is positively correlated with the price level. The shock to the economy therefore would be perceived by the econometrician as originating on the demand side, consistent with many observations and estimation results showing demand-side shocks to be a significant driving force for aggregate fluctuations.

There are three limitations of the model. First, it has only one sector, so it cannot address comovement in any meaningful way. We will consider some two-sector models later. Second, it does not predict that investment leads the cycle. Total output and investment in durables move together, rising and falling simultaneously. Third, demand for nondurables is countercyclical in the model but procyclical in the data. We address these deficiencies in versions of the model discussed below.

An interesting aspect of the model economy's behavior is that inventions that appear similar from the production side, in the sense that similar goods are invented and/or require similar resources to produce one unit of the new good, can produce very different output responses. The exact character of the dynamic adjustment path depends on subtle aspects of the consumer's utility function. For example, if the marginal utility for the new good is low immediately after invention but declines very slowly as the new good is accumulated (i.e., utility's second derivative is of small magnitude), then the response of the economy will be small initially but will last a long time (80 and 8A* will not be much above the pre-

⁷ The employment response is more abrupt than in reality, having a discontinuous jump up in labor supply rather than the gradual increase seen in the data. A gradual increase can be obtained straightforwardly though tediously by including adjustment costs, which otherwise leave the solution's characteristics unchanged, exactly as in the case of investment adjustment costs analyzed by Abel and Blanchard (1983).

invention level 8B, DA^* will be large, and the transition path from $DOA = 0$ to DA^* will take a long time to traverse). In contrast, if marginal utility for the new good is high but declines very rapidly, the response of the economy will be large initially but will die away quickly (80A and 8A* will be well above the pre-invention level 8B, DA^* will be small, and the transition path from $DOA = 0$ to DA^* will take little time to traverse). Obviously, other combinations are possible with corresponding differences in the associated dynamic adjustment paths. The co-movement and serial correlation properties of the economy's response to various inventions of durable goods thus should be qualitatively similar, but the magnitudes and durations can be quite different. The cyclic responses thus are "all alike" in their overall patterns but different in magnitude and duration, just like real-world aggregate fluctuations.

Finally, note that the shock here is permanent in some ways; the durable good, once invented, does not subsequently disappear. This shock therefore differs from the purely transitory shock discussed by Blanchard and Quah (1989) in their analysis of the hump-shaped behavior of output. The results obtained here are therefore not directly comparable to Blanchard and Quah's empirical findings. The comparison may be valid nonetheless. Blanchard and Quah's analysis excludes by construction the type of shock considered here. Any observed behavior due to such shocks then must be interpreted as arising from whatever shocks are included in the analysis; in particular, one would expect the transitory effects of omitted invention shocks to be attributed to the included transitory shocks. A full resolution of this issue requires extending Blanchard and Quah's analysis to include invention shocks. Some difficulties that may arise in attempting such an extension are discussed later.

2. Invention of a Producer Durable (Physical Capital).

We now examine the complementary model in which the invention is a producer rather than consumer durable. Consumer durables are now assumed not to exist. This kind of model has many of the same characteristics as the previous model, in which the invention was a consumer durable, but there also are some important differences.

Household utility now depends only on consumption c of non-durables and on leisure h :

$$u(c, h)$$

Production is concave in labor and capital:

$$\begin{aligned} y &= f(K, I-h) \\ (24) \quad f_1 &> D, f_{11} < D \\ f_2 &> D, f_{22} < D \end{aligned}$$

To avoid uninteresting complications, we also assume that (i) production is separable

$$f_{12} = D$$

(ii) the marginal product of capital is finite even when $K = 0$

$$\lim_{k \rightarrow 0} f_2 = k < \infty$$

and (iii) there are no costs to adjusting K . Non-separability between capital and labor would enrich the dynamics but not alter the main points to be made. Similarly, adjustment costs would not alter the general shape of the adjustment path and so would leave the main results unaffected. As before, the assumption of a finite marginal product of capital when there is no capital is only a mathematical convenience.

The household seeks to maximize lifetime utility subject to the law of motion for K :

$$\max_{c, k} \int_D^{\infty} u(c, h) c^{-\rho t} dt$$

(25) subject to

$$dK / dt = f(K, 1 - h) - c - \delta K$$

2.1. Pre-Invention Optimality.

Before the invention of productive capital, there is no dynamic aspect to the household problem. We have the same simple period-by-period maximization of utility as in section III.1.1 with the same solution. As before, one obtains the Euler equation

$$(26) \quad \frac{u_2(c)}{u_k(h)} = f_2(1 - h)$$

which is the same as (10) and which again can be written as (11). From it, one determines h , which in turn determines c from the production function (recall that capital is fixed at zero).

2.2. Post-Invention Optimality.

After invention of productive capital, the household solves problem (25) with K no longer constrained to zero. The Hamiltonian for this problem is

$$(27) \quad H = u(c, h) + \lambda [f(K, 1 - h) - c - \delta K]$$

The dynamic equations and first-order conditions are

$$(28) \quad dK / dt = \partial H / \partial \lambda = f(K, 1 - h) - c - \delta K$$

$$(29) \quad d\lambda / dt = -\partial H / \partial K + \rho\lambda = -\lambda [f_1 - (\delta + \rho)]$$

$$(30) \quad \partial H / \partial c = u_c - \lambda = 0$$

$$(31) \quad \partial H / \partial h = u_k - \lambda f_2 = 0$$

This is just the standard Cass model of general equilibrium. Its solution is well-known, so we can state the results we need with only a quick explanation.⁸ Because the cross-partial derivatives of both the utility and production functions are zero, the first-order conditions (30) and (31) are independent of each other and also of the capital stock K . From them we easily obtain by total differentiation the relations

$$\begin{aligned} c &= c(\lambda), & c' < 0 \\ h &= h(\lambda), & h' < 0 \end{aligned}$$

Substituting these into the dynamic equations (28) and (29), setting those equal to zero, and totally differentiating gives the slopes of the equilibrium loci for K and λ . The equilibrium locus for K has a slope of

$$(32) \quad \frac{d\lambda}{dK} \Big|_{dK/dt=0} = \frac{f_1 - \delta}{c' + f_2 h'} \\ > 0 \text{ as } f_1 < \delta$$

which implies that the $dK/dt=0$ locus is convex with a minimum at KA^{**} , where KA^{**} is the value of K for which the numerator of (32) equals zero (i.e., the Golden Rule level of capital). The equilibrium locus for λ is vertical at KA^* (the Modified Golden Rule level of K). The phase diagram is shown in Figure 2. Note that conditions (30) and (31) give the Euler condition

$$(33) \quad \frac{dd_c(c)}{u_k(h)} = f_2(1 - h)$$

which, as with (19), cannot be written in the form of (11).

⁸ The model is usually called the “Cass growth model” because it first was used to characterize the dynamic adjustment path of a growing economy with endogenous saving. Growth is exogenous to the model, however, so it offers no explanation for the presence of growth or determination of the growth rate. The model’s solution is nevertheless a complete characterization of the economy’s general equilibrium behavior (under the usual assumptions equating the representative agent’s problem to the competitive equilibrium problem). The model therefore is well-suited to the issues at hand.

The properties of the optimal solution are now easy to obtain. The arguments are essentially the same as those used in the case of invention of a consumer durable, so the details are omitted here. The important results are the following:

(1) Invention leads to immediate decreases in both leisure h and consumption c of the nondurable. The reduction in h raises output y . The resulting difference between c and y is devoted to building productive capital K .

(2) Subsequently, the economy follows the dynamic adjustment path down from the point $(0, 80A)$ toward the steady state, implying increases in both c and h . The behavior of output is unclear. The decrease in h reduces y , but the increase in the capital stock K raises it, leaving ambiguous the direction of change in y along the dynamic adjustment path.

(3) In contrast to the previous model of inventing D , in the new steady state both c and h are higher than before invention of the durable. As in the previous model, y is higher. To see this, just note that the initial response of both c and h is to fall, lowering utility. Unlike a consumer durable, physical capital does not produce utility directly, so its accumulation in itself adds nothing to the household's lifetime utility. In order to compensate for the initial loss in utility, then, c and h eventually must exceed their pre-invention levels. Because the approach to the steady state is monotonic, once c and h exceed their pre-invention levels, they stay above it. Therefore, the steady state values cA^* and hA^* must exceed their pre-invention levels cB and hB . In the new steady state, c exceeds its pre-invention level, and in addition there is replacement investment to maintain the capital stock at its steady state level KA^* . Thus we must have $yA^* > yB$.

The economy's response is mostly the same as in the case of invention of a consumer durable; there are two interesting differences. First, output's path may be hump-shaped but is not necessarily so. Output does jump up in response to the invention, but after that its direction is uncertain because of the opposing effects of declining labor supply and a rising stock of productive capital. The time path of output may have quite a different shape from that resulting from invention of a consumer durable, but it also may look much the same. There thus may not be a single characteristic path of output's response to "invention" in general.

Second, this ambiguity in the shape of output's time path allows the possibility that investment leads the cycle. One possible path is for output to jump initially, continue growing for a while as physical capital is accumulated, and then drop down to a final steady state level as labor supply falls. Such a path is not a necessary outcome, but apparently is possible if the marginal productivities of labor and capital and the marginal utilities of leisure and goods have the right relations to each other along the economy's adjustment path. Output then has a hump shape with the peak occurring some time after the moment of invention. In contrast, physical capital accumulation decelerates along the entire adjustment path and thus is highest immediately after the moment of invention. Investment thus peaks before output and so leads the cycle.

4. TWO KINDS OF DURABLE GOODS IN SIMULTANEOUS EXISTENCE

We want to examine an economy that invents one kind of durable good when the other already exists. This more realistic case is far more difficult to analyze than the models of the previous section. The two kinds of durables and their corresponding costate variables leave us with a four-dimensional system. A phase diagram usually cannot be drawn, and analysis of the dynamic path usually is impossible. The problem at hand suffers from these difficulties, and an analytical solution to the most general formulation of the model is unobtainable. A few remarks on the nature of these difficulties follow at the end of this section. We nevertheless can get some insight into the two-variable problem by using a device first suggested by Mankiw (1987), which is to suppose that each kind of durables good, K and D , can be instantly transformed into the other. We then define total wealth W as the sum of K and D . This device reduces the dimensionality of the problem from four to two because now W is the only state (non-jumping) variable, so it and its corresponding costate variable constitute the state space.

Total wealth of the representative household is

$$(34) \quad W = K + D$$

and its evolution is governed by the equation

$$(35) \quad dW / dt = f(k, 1 - h) - c - \delta W$$

where the production function is the same as (24) and we assume that productive capital and consumer durables have the same depreciation rate δ . The household utility function is

$$u = u(c, D, h)$$

which is the same as (2). We thus have the production function of the model in section III.2 and the utility function of the model in section III.1.

1. Invention of D When K Already Exists.

As before, we have to compare pre-invention and post-invention optimality.

1.1. Pre-Invention Optimality.

With D constrained to zero, the household problem is

$$\begin{aligned} \max_{c, k} \int_0^{\infty} u(c, D, h) e^{-\rho t} dt \\ \text{subject to} \\ dW / dt = dK = f(K, 1 - h) - c - \delta K \end{aligned}$$

This is exactly the same problem as in section III.2.2, so we need not dwell on the analytics. The phase diagram is again given by Figure 2, and there is a steady state at $(WB^*, 8B^*) = (KB^*, 8B^*)$. The slope of the $dW/dt=0$ locus is given by the right side of (32).

1.2. Post-Invention Optimality.

After the invention of D , the household problem becomes more complex:

$$\max_{c,D,k} \int_D^{\infty} u(c, D, h) c^{-\rho t} dt$$

subject to

$$dW / dt = f(W - D, 1 - h) - c - \delta W$$

Recall that W is the only state variable. K and D are now control variables, but of course only one is independent, by (34). We express everything in terms of the control variable D by replacing K with $W - D$. The Hamiltonian for this problem is

$$(36) \quad H = u(c, D, h) + \lambda [f(W - D, 1 - h) - c - \delta W]$$

The dynamic equations and first-order conditions are

$$(37) \quad dW / dt = \partial H / \partial \lambda = f(W - D, 1 - h) - c - \delta W$$

$$(38) \quad d\lambda / dt = -\partial H / \partial W + \rho \lambda = -\lambda [f_1 - (\delta + \rho)]$$

$$(39) \quad \partial H / \partial c = u_c - \lambda = 0$$

$$(40) \quad \partial H / \partial D = u_D - \lambda f_1 = 0$$

$$(41) \quad \partial H / \partial h = u_h - \lambda f_2 = 0$$

At the moment of invention, the household instantly transforms some of the existing capital K into consumer durables, so that K falls and D rises with total wealth W unchanged. That much is straightforward. Determining the steady state and the approach to it requires more work.

As before, the first-order conditions are independent of each other, implying the following relations between the three control variables c , h , and D on the one hand and the state and costate variables W and δ on the other hand:

$$c = c(\lambda), \quad c' < D$$

$$h = h(\lambda), \quad h' < D$$

$$D = D(W, \lambda) \quad D_1 > 0 \quad D_2 < 0$$

We assume that all goods are normal, implying that $0 < 1 - DW < 1$. These relations yield the phase diagram, shown in Figure 3, which shows both the pre-invention and post-invention equilibrium loci and also the dynamic adjustment path to the new steady state. The derivation of Figure 3 is much the same as in the preceding section, so details are relegated to an appendix available from the author. We discuss here only the implications for the economy's response to the invention of the consumer durable.

The economy starts at the steady state point SB. At the moment of invention, two things happen instantly. First, some productive capital K is transformed into the consumer durable D , leaving total wealth W unchanged. Second, the economy jumps up to point A, where the value of the costate variable is $\delta A > \delta B$, implying that both nondurable consumption c and leisure h fall. As in the simpler model of section III.1 in which physical capital was absent, nondurable consumption expenditure initially falls in response to the invention of the consumer durable, but total consumption expenditure initially rises. As time passes, the economy moves down along the dynamic adjustment path toward the new steady state SA, at which both c and h are above their pre-invention values. During this transition, W continuously rises and δ continuously falls, implying that the stock of durables D continues to rise. The fall in δ implies that both c and h continuously rise. This behavior implies in particular that the path of labor is hump shaped, first rising at the moment of invention and then falling as the economy makes the transition to the new steady state. The behavior of both productive capital K and output y have some ambiguities in their transition paths. We know that K instantly drops at the moment of invention, and we also know that its steady state value is unchanged from its pre-invention level. Consequently, eventually K must rise. However, there seems to be no way to rule out the possibility that K continues to fall along the early part of the transition path. The behavior of output reflects in part the behavior of K but also reflects the behavior of leisure. At the moment of invention, leisure instantly falls, but so does productive capital K . These changes have opposite effects on output. The initial change in output is ambiguous, and we do not know if its value yOA immediately after invention is above or below its preinvention level yB . We know that the level of output yA^* in the post-invention steady state is higher than yB , but we do not know the relation between yA^* and yOA . The path of output may have a hump shape, but it need not. Because of this leeway in the path of output, it also is possible for total investment to lead the cycle. Thus invention of a consumer durable in the presence of productive

capital can lead to a total response of the economy that is more consistent with the business cycle facts than in the simple case where productive capital is absent.

2. Invention of K When D Already Exists.

We turn now to the case of invention of productive capital when the household durable already exists. The analysis is similar to what has gone before, so we can abbreviate the discussion.

Before invention, K is constrained to zero and the household's problem is the same problem as in section III.1.2. The phase diagram is again given by Figure 1, and there is a steady state at $(WB^*, 8B^*) = (DB^*, 8B^*)$. After invention of K , the household problem becomes the same as that of section IV.1.2. The difference between the problem here and that of section IV.1.2 is the starting point. Here, initial D is positive and initial K is zero; in section IV.1.2, we had the reverse. The phase diagram is shown in Figure 4. The post-invention equilibrium loci for W and 8 respectively lie everywhere below and above the corresponding pre-invention loci. The post-invention steady state occurs at a higher value of W and a lower value of 8 than its pre-invention counterpart. Mathematical details are in an appendix available from the author.

The main implications are that the post-invention steady state values of nondurables consumption cA^* , leisure hA^* , and durables DA^* all exceed their pre-invention values cB^* , hB^* , and DB^* . Also, because 8 falls along the optimal path, we know that c , h , and D continually rise after the moment of invention. What is not clear is what happens to 8 , c , h , and D at the moment of invention. At that moment, some existing consumer durables D are instantly transformed into productive capital K , which raises output. However, 8 could jump up or down at that same instant. The dynamic adjustment path in Figure 4 has been drawn with an upward jump (downward jump in c and h), but a downward jump (upward jump in c and h) to point B also is possible. It also is not clear what path output follows. The final value yA^* exceeds yB^* because c , D , and K all are higher after invention than before. However, the relation between yA^* and the value $y0A$ of output immediately after invention is unclear. The upward jump in K raises output, but it is possible that labor simultaneously jumps down, which would lower output. Furthermore, as time passes, K continues to grow and labor definitely falls, continuing to leave the path of output ambiguous. Once again, a hump shaped path of output is possible though not necessary.

3. Non-Transformable Consumer and Producer Durables.

For most types of capital, direct transformation between uses as consumer or producer durables is not feasible. A more realistic model then would not permit such behavior. Unfortunately, as mentioned earlier, such a model is four-dimensional and analytically intractable. Sometimes, models with two state variables can be solved. Treadway (1971) proves a theorem that in models of interrelated factor demands for capital inputs, the model's structure restricts the quartic characteristic equation in such a way that the four roots all are symmetric

about a single value. This symmetry forces the characteristic equation's cubic and linear terms (i.e., those in odd powers) to drop out, leaving only the fourth-order, second-order, and constant terms. Such an equation can be trivially re-written as a quadratic in the square of the original variable.⁹ The roots then are easy to find. Rossana (1984) uses this theorem to derive a solution in an inventory model. Treadway's theorem does not apply to the models used here, however. In Treadway's model, the production function is the only function of the variables of interest, whereas here there also is a utility function. The presence of the two functions destroys the super-symmetry of the characteristic roots and leaves the quartic equation unmanageable.

One minor result is obtainable from the general model of two state variables. A bit of ambiguity is removed from the path of output in the case of invention of the durable good when productive capital already exists. When the two kinds of goods are instantaneously convertible into each other, the representative household immediately responds to the invention by converting some K into D while simultaneously increasing labor supply, leaving the change in output ambiguous. In the absence of instantaneous convertibility, there is no ambiguity. Labor still increases, but there is no immediate change in K , so output unambiguously rises immediately after invention of D . More than that apparently cannot be said without analyzing the dynamics of the system, which the high dimensionality of the system does not allow.

5. INVENTION OF NONDURABLE GOODS

So far, we have considered only invention of goods that have an element of durability – either physical capital or consumer durables. We have seen that invention of such goods can lead to complex business cycle behavior. We now turn our attention to the invention of a non-durable good. Whether such an invention has business cycle effects depends on the production function for the newly invented good.

1.A One-Sector Model.

We suppose that there are two non-durable goods, c_1 and c_2 . The utility function is essentially the same as (1) except that the durable good is replaced by the second non-durable:

$$u(c_1, c_2, h)$$

⁹ That is, an equation of the form

$$x^4 + Ax^2 + B = 0$$

can be written as the quadratic

$$z^2 + Az + B = 0$$

where $z = x^2$.

Initially, c_2 does not exist, so we allow utility to be positive when c_2 is zero.

$$u(c_1, D, h) > 0$$

We also suppose the following conditions:

$$\lim_{c_2 \rightarrow D} u_{c_2} = \Delta < \infty = \lim_{c_1 \rightarrow D} u_{c_1} = \lim_{k \rightarrow D} u_k$$

Finally, we once again suppose that utility is separable in nondurables, durables, and leisure, that production is concave in labor and capital, and that production is separable in capital and labor.

Before the invention c_2 , the household's problem is exactly the same as in section III.2.2 and so needs no further discussion here. The necessary conditions are given by (28) - (31) plus the endpoint conditions. The phase diagram is the same as that in Figure 2, and the steady state is at $(KA^*, 8A^*)$.

After invention of c_2 , the problem remains the same except that c_2 no longer is constrained to zero. The necessary conditions are nearly the same as (28) - (31). The differences are that becomes

$$dK / dt = f(K, 1-h) - c_1 - c_2 - \delta K$$

and a third first-order condition (for c_2) is added:

$$\partial H / \partial c_2 = u_{c_2} - \lambda = D$$

The dynamic equation for δ is unchanged, so the steady state value of K also is unchanged. The only change in the phase diagram is that the equilibrium locus for K shifts up. We thus have the situation depicted in Figure 5, which shows the pre-invention and post-invention equilibrium loci.

The economy's response has no interesting dynamics. It simply jumps immediately from SB to SA. Leisure decreases, total output rises, consumption of the original nondurable c_1 drops, and consumption of the new non-durable c_2 rises (from zero). These are once and for all changes; and the invention of the non-durable causes no business cycle behavior.

2. A Two-Sector Model

Invention of a non-durable good can cause business cycle behavior if the new non-durable is produced in a different sector than the first non-durable. For expository ease, we will assume very simple production functions. Output is of two types. The first type, y_1 , is produced in the first sector, which requires only labor as an input:

$$y_1 = f(1-k)$$

This kind of output can be used either as physical capital K or the first non-durable c_1 :

$$y_1 = c_1 + \frac{dK}{dt}$$

The second type of output, y_2 , is produced in the second sector, which requires only physical capital as an input:¹⁰

$$y_2 = g(K)$$

This type of output can be used only as the second non-durable c_2 :

$$y_2 = c_2$$

The utility function is the same as before.

Initially, the economy does not know how to make c_2 , so all of output y_1 is devoted to c_1 . We thus have the same optimization problem as in section III.1.1. Invention of c_2 is discovery of the process for making it. Perhaps the economy initially does not know how to make capital out of output y_1 but does understand how to produce c_2 , or perhaps the economy initially knows how to make capital but knows of no use for it until it discovers the process for making c_2 . The results are the same. Upon inventing c_2 , the economy's problem becomes

$$\max_{c_1, c_2, K} \int_D^{\infty} u(c_1, c_2, h) c^{-pt} dt$$

subject to

$$dK / dt = f(1-h) - c_1 - \delta K$$

$$c_2 = g(K)$$

The Hamiltonian can be written as

$$H = u(c_1, g(K), h) + \lambda [f(1-h) - c_1 - \delta K]$$

The dynamic equations and first-order conditions are

$$dK / dt = \partial H / \partial \lambda = f(1-h) - c_1 - \delta K$$

¹⁰ The assumptions that capital is not used in first sector and labor is not used in the second sector can be relaxed without changing the results.

$$d\lambda / dt = -\partial H / \partial K + \rho\lambda = -u_2 g' + (\delta + \rho)\lambda$$

$$\partial H / \partial c_1 = u_1 - \lambda = D$$

$$\partial H / \partial h = u_3 - \lambda f' = D$$

These are the same as (13) - (16) except for the g' in the dynamic equation for 8, which alters only the magnitude (but not the sign) of the slope of the equilibrium locus for 8. Consequently, the solution to this problem is essentially the same as that obtained for the model discussed in section III.1.2 and has the same characteristics - a hump shaped response in labor and so forth. In effect, the invention of a second durable that requires the building of a new kind physical capital for its production is essentially the same as invention of a consumer durable. The physical capital acts like a stock of consumer durables, and the consumption of the new nondurable c_2 acts like the service flow obtained from a consumer durable. The one major difference between this model and that of section III.1.2 is that now the demand for non-durable goods is procyclical rather than countercyclical. Upon invention of c_2 , the demand for c_1 falls but is more than compensated by the demand for c_2 , leaving the total demand for durables higher than before invention.

The main conclusion from this analysis is that invention of a non-durable can cause interesting business cycle behavior if it changes the steady state stock of some kind of capital. Doing that introduces an element of durability into the problem. That element of durability is precisely what was missing in the example discussed in the previous section, in which the production of the new nondurable did not change the steady state stock of capital. That is why no business cycle behavior resulted from the invention of the nondurable in that case, in contrast to what happens in the case discussed in the present section.

6. EMPIRICAL RELEVANCE

It may seem that the foregoing theory cannot be important for explaining aggregate fluctuations because production of any one new good is too small to have noticeable effects on the behavior of total output. Undoubtedly, this view is correct for most goods; it seems unlikely that development of the felttip pen caused a noticeable fluctuation in aggregate output. There are, however, goods whose invention has had huge impacts on aggregate output. The latest example apparently is the personal computer. The last decade has seen a remarkable boom in US economic performance. An especially notable aspect of this boom is that it seems to have been driven in good part by investment in computer-related goods, which are both producer and consumer durables. From 1992 to 1996, for example, real

business investment in computers constituted over 13 percent of all business investment and almost percent of business investment in equipment (Haimowitz, 1998). Apparently, similar behavior occurred with the invention of cheap ways to generate and transmit electricity a century ago. An especially interesting example is the invention of inexpensive air conditioning. Before air conditioning, the southeastern part of the United States experienced net emigration; after air conditioning, the South experienced net immigration. These flows of people created an enormous demand for new houses, stores, factories, and offices and thus led to a huge building boom. The new structures had to be furnished and equipped, so there also was a boom in production of durable goods associated with new construction but not actually a part of the construction industry.

What is so interesting about the air conditioning example is that it clearly shows the linkages that can accompany invention of a new product. Invention of air conditioning did not lead simply to purchases of air conditioners; it led also to purchases of whole new residential and commercial structures and of furniture and equipment. It seems possible that similar linkages accompanying other kinds of inventions could amplify the effect of the invention on output. They also could affect the timing of cycles. A given invention may not have much impact at the time it comes out, but a complementary invention that appears subsequently may enhance the attractiveness of the first invention and lead to a surge in demand for both. Furthermore, an invention that enhances the value of an existing type of durable may not be suitable for retrofitting but instead may require purchase of whole new model. Air conditioning for automobiles and color capabilities in televisions are two examples. Thus the impact of an invention may appear as increase in output of a seemingly unrelated product.

All of this argues that the impact of particular inventions may well be more important than the inventions themselves and also may be linked to other economic activity in such a way as to make the timing of effects on aggregate output appear very complex.

I have not conducted any formal empirical test of the theory presented here, nor have I conducted a calibration exercise. The first is a research project in its own right and well beyond the scope of the present discussion. The second is problematic. A calibration exercise requires, by definition, that one impose on the model a number of parameter values and then see if the model can generate correlation patterns that mimic those of the real economy. Such a procedure seems impossible here. Using it would require that one impose parameter values for invention. Invention is not something readily observed or measured, so there seems to be no guideline for deciding what the imposed parameter values should be. For example, in the two-sector model in which a non-durable is invented, what parameter values does one put on the production function for the new good? Where are they obtained? I do not see how to proceed in such a case.

7. CONCLUSION

We have examined several models in which invention of a good that previously did not exist causes business cycle behavior. In particular, the kinds of invention studied here almost always produce a hump shaped response in work effort, and they always can and sometimes must produce a hump shaped response in output. Even though the invention is permanent and there is no serial correlation in the invention shocks themselves, business cycle fluctuations occur. The crucial requirement is that the invention changes the steady state stock of capital. Invention of goods that are durable - either consumer or producer durables - has this effect directly because the goods themselves are part of the capital stock. Invention of a nondurable good also can cause a business cycle response if the invention involves starting a new production process that requires physical capital to make the newly invented good.

The shocks considered here are not productivity shocks in the usual sense. The kinds of productivity shocks studied in the real business cycle literature directly raise the productivity of existing factors of production and would be seen by the econometrician as a supply shock. In contrast, the invention of a new good permits production that was not possible before, but it does not directly change the productivity of factors of production that already exist. The magnitude of the economy's response depends on how much demand there is for the new good. If there is no demand, there also is no response. Thus the invention of a new good would be seen by the econometrician as a demand shock. The theory developed here thus is able to explain some of the business cycle regularities noted in the empirical literature, such as Blanchard and Quah (1989), that are not consistent with productivity shocks in the standard real business cycle model.

The theory presented here complements the standard real business cycle model, which examines the effects of productivity shocks of the usual type. Each taken separately has something interesting to say about short-term fluctuations. An obvious next step is to combine the two approaches into one model and then estimate to see how well the combined model can explain observed short-term behavior of the aggregate economy.

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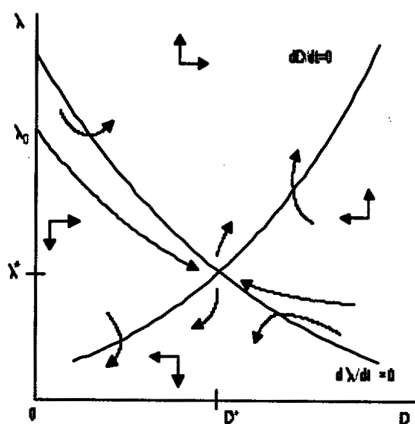


Figure 1: Invention of D

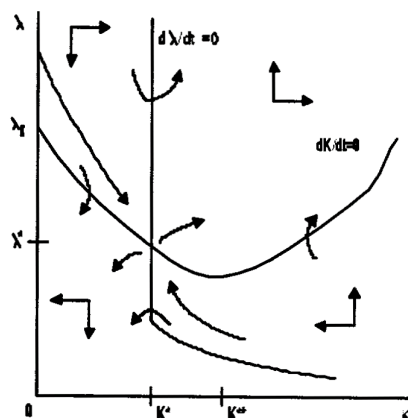


Figure 2: Invention of K

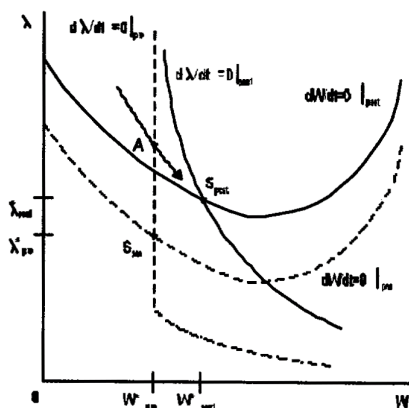


Figure 3: Invention of D when K already exists

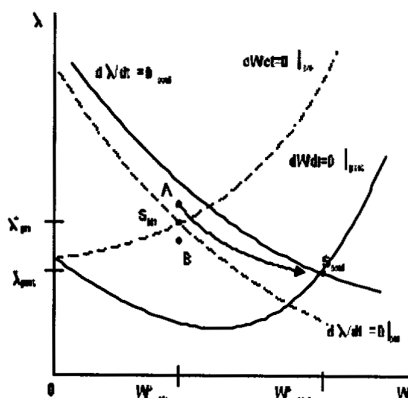


Figure 4: Invention of K when D already exists

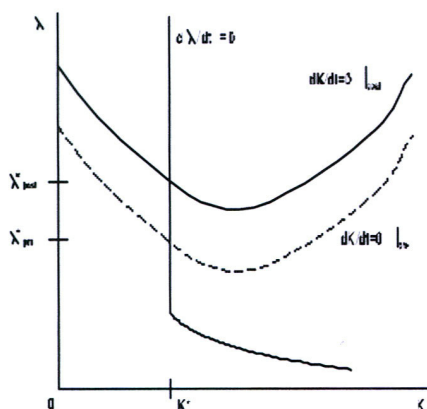


Figure 5: Invention of C_2

APPENDIX A: ANALYSIS OF PHASE DIAGRAMS

4.1.2: Phase diagram

Substituting the foregoing expressions into the dynamic equations (37) and (38), setting those equal to zero, and taking total derivatives gives the slopes of the equilibrium loci. The expression for slope of the $dW/dt=0$ locus is

$$(42) \quad \frac{d\gamma}{dW} / \lambda W / dt=0 = \frac{(1-D_1)f_1 - \delta}{c' + f_2 h' + f_1 D_2}$$

As before the invention of D , this expression is convex with a minimum at the value KA^{**} that makes the numerator zero, which in turn requires that

$$f_1 = (1 - D_w)^{-1\delta} > \delta$$

which means the post-invention value KA^{**} must be lower than the pre-invention value KB^{**} . The implication of this for the post-invention minimizing value WA^{**} of W is unclear. The lower KA^{**} reduces WA^{**} , but there also now is a positive value DA^{**} of consumer durables, which raises WA^{**} . The net effect on WA^{**} seems ambiguous. This aspect of the new optimal solution is unimportant for our purposes, so we simply assume that the value of W^{**} is unchanged from the value that prevails before the invention of the consumer durable.

The post-invention equilibrium locus for W lies everywhere above the pre-invention locus. To see this, simply note that, after invention, D is always positive. Consequently, for any given value of W , there is less productive capital K than before invention. If δ were the same, we would also have the same values of c and h . Keeping these latter variables unchanged but reducing K guarantees that output falls below consumption of nondurables, implying that $dW/dt < 0$. To restore equilibrium, we must raise δ , which reduces c and h .

The slope of the post-invention equilibrium locus for δ is given by

$$(43) \quad \frac{d\lambda}{dW} / d\lambda / dt=0 = \frac{1-D_1}{D_2} < D$$

so this locus now is downward-sloping. It also lies everywhere to the right of the pre-invention locus. Invention of the consumer durable has no effect on the steady state value K^* of productive capital, which is the value that makes equation (38) zero. That equation is the same for both the pre-invention [see equation (29)] and post-invention problems, so the pre-invention and post-invention values of K^* also are the same. However, the post-invention value DA^* is positive (for any value of δ), whereas the pre-invention value DB^* is constrained to zero. Thus the post-

invention value of W must exceed the pre-invention value for every δ in order to allow holding of positive D and still have the right amount of K left over to satisfy (38).

Finally, the post-invention value $8A^*$ must be higher than the pre-invention value $8B^*$. This conclusion follows from the post-invention values WA^* and KA^* being respectively higher than and the same as the pre-invention values WB^* and KB^* . The higher value of W^* requires greater replacement investment for its maintenance. For the given value of K^* , the only way to obtain the extra replacement investment is to raise δ^* , which simultaneously reduces consumption of non-durables c^* and leisure h^* .

4.2: Analysis and Phase diagram

Before invention, K is constrained to zero, and the household problem is

$$\max_{c,D,k} \int_D^{\infty} u(c,D,h) c^{-\rho t} dt$$

subject to

$$dD/dt = f(1-h) - c - \delta D$$

After invention, the household problem is

$$\max_{c,D,K} \int_D^{\infty} u(c,D,h) c^{-\rho t} dt$$

subject to

$$dW/dt = f(W-D, 1-h) - c - \delta W$$

The necessary conditions are given by equations (37) - (40) plus the relevant endpoint conditions. To determine the phase diagram, compare the expressions for the equilibrium loci and for their slopes before and after invention. These are derived above and repeated here for convenience. Before invention, we have

$$(13) \quad dW/dt = f[1-h(\lambda)] - c(\lambda) - \delta W$$

where W/D . Setting this to zero and totally differentiating gives the slope of the equilibrium locus for W :

$$(22) \quad \frac{d\lambda}{dw} \Big|_{\lambda W/dt=0} = \frac{-\delta}{c' + f'h'}$$

After invention, we have

$$(37) \quad dW / dt = f[W - D(W, \lambda), 1 - h(\lambda)] - c - \delta W$$

and the corresponding equilibrium locus has a slope of

$$(42) \quad \frac{d\lambda}{dW} /_{dW/dt=0} = \frac{(1 - D_1)f_1 - \delta}{c' + f_2 h' + f_1 D_2}$$

At $W = 0$, the two equations (13) and (37) are the same. Consequently, they equal zero at the same value of λ , which is why they have the same intersection on the vertical axis. Now let W increase. For any $W > 0$, (42) is everywhere less positive than (22) (and initially is even negative) because of the extra terms in the numerator and denominator of (42), so the post-invention $dW/dt=0$ locus rises less rapidly than the pre-invention locus and therefore must lie everywhere below it. Before invention, the λ dynamics are

$$(14) \quad d\lambda / dt = -u_D(D) + (\delta + \rho)\lambda$$

whereas after invention they are

$$(38) \quad d\lambda / dt = -\lambda[f_1 - (\delta + \rho)]$$

Using the first-order condition (40), we can rewrite this last expression as

$$d\lambda / dt = -u_D(D) + (\rho)\lambda$$

which is identical to (14). So for any given pair (D, λ) , the two expressions have the same value. Before invention, D and W are identical, but after invention they are not. In particular, after invention any given value of D is associated with a positive value of K and thus with a larger value of W than before invention. Thus the post-invention $d\lambda/dt=0$ locus lies everywhere to the right of the preinvention locus. (The slopes of both loci are always zero, by (23) and (43).)

Finally, the post-invention steady state value λ^*A of the costate variable must be below the preinvention steady state value λ^*B . This conclusion arises from the necessary conditions for the two models. The first-order conditions for c and h are identical (compare equations (15) and (16) with (39) and (41)):

$$u_c = \lambda$$

$$u_k = \lambda f_k$$

Before invention, the dynamic equation for λ is (14), which implies

$$(44) \quad u_D = (\delta + \rho)\lambda$$

in the steady state because $d8/dt=0$ in the steady state. After invention, we have the first-order condition (40) for D . However, $d8/dt=0$ in steady state, implying from equation (38) that the marginal product of capital f_1 equals $\delta + D$. Substituting $\delta + D$ for f_1 into (40) gives

$$u_D = (\delta + \rho)\lambda$$

which is identical to (44). Thus the marginal utilities for all three goods have exactly the same mathematical expressions and depend solely on 8 . After the moment of invention, the dynamic laws of the phase plane guarantee that 8 falls continuously. Hence, if 8^*A were greater than or equal to 8^*B , then $8A$ would exceed 8^*B along the entire adjustment path. From the marginal utility relations just derived, this would imply that c , h , and D would be below their pre-invention levels at all times. This in turn implies that lifetime utility is lower after invention, which would contradict the optimality of adopting the productive capital after it was invented. Consequently, we must have $8^*A < 8^*B$, which then implies that $c^*A > c^*B$, $h^*A > h^*B$, and $D^*A > D^*B$. Also, because 8 falls along the optimal path, we know that c , h , and D continually rise after the moment of invention.

5.1: Phase diagram

To see this, note that the utility function is unchanged. Therefore, at any value of 8 , the values of c_1 and h would be the same after invention of c_2 as before. At any given values of K and h , the value of output would be the same after invention as before. However, some of that output now is diverted to c_2 , and combinations of K and 8 that satisfied the equilibrium condition for K before invention would not satisfy it now. For any given K , a lower level of leisure and/or consumption would be required, implying that 8 must be higher.

APPENDIX B: ANOTHER TWO-SECTOR MODEL

Invention of a non-durable good can cause business cycle behavior if the new non-durable is produced in a different sector than the first non-durable and if physical capital is a factor of production. To keep things simple, we will assume very simple production functions. Output is of two types, each produced in its own sector. The output y_1 of the first sector can be used either as capital K or as the first nondurable consumer good c_1 ; the output y_2 of the second sector can be used only as the second nondurable good c_2 . However, we now suppose that physical capital already exists and is a factor of production in both sectors, not just the second sector. The production functions are

$$\begin{aligned} y_1 &= f(K - K_2, 1 - h) \\ y_2 &= g(k_2) \end{aligned}$$

where K is the total capital stock and K_2 is the part of K used in the second sector. For simplicity, we again suppose that labor is not a factor of production in the second sector.

Initially, the economy does not know how to make y_2 ; that is, it does not know the production process g . The optimization problem is

$$\max_{c_1, k} \int_D^{\infty} u(c_1, D, h) c^{-\rho t} dt$$

subject to

$$dK / dt = f(K, 1 - h) - c_1 - \delta K$$

This is the same problem as in section III.2.2. The phase diagram thus is given by Figure 2, and there is a steady state at (K^*, δ^*) .

After invention of the process for making c_2 , the problem becomes

$$\max_{c_1, c_2, k} \int_D^{\infty} u(c_1, c_2, h) c^{-\rho t} dt$$

subject to

$$dK / dt = f(K - k_2, 1 - h) - c_1 - \delta K$$

$$c_2 = g(K_2)$$

The Hamiltonian is

$$H = u(c_1, g(K_2), h) + \lambda [f(K - K_2, 1 - h) - c_1 - \delta K]$$

The dynamic equations and first-order conditions are

$$dK / dt = \partial H / \partial \lambda = f(K - K_2, 1 - h) - c_1 - \delta K$$

$$d\lambda / dt = -\partial H / \partial K + \rho \lambda = -\lambda [f_1 + (\delta + \rho)]$$

$$\partial H / \partial c_1 = u_1 - \lambda = \delta$$

$$\partial H / \partial c = u_2 g - \lambda f_1 = D$$

$$\partial H / \partial h = u_3 - \lambda f_2 = D$$

These conditions are almost the same as (37) - (41) that pertain to the model in which a durable is invented when productive capital already exists. The only difference is that u_2 here is multiplied by g' , whereas u_2 stands alone in (40).

The presence of g' has no substantive effect on the dynamic properties of the model, and the phase diagram is given by Figure 3. The response of the economy is qualitatively the same as in section IV.1. Again, invention cause business cycle behavior. The reason is the same as in the previous section: in this example, invention of the second nondurable good changes the steady state stock of capital.

APPENDIX C: INVENTION IN A MODEL OF ENDOGENOUS GROWTH

So far, we have examined only models with no growth, but introducing exogenous growth would not change any of the results obtained. It is not immediately obvious how invention affects an economy that grows endogenously. Transition dynamics in endogenous growth models usually are so difficult that attention is confined to analyzing balanced growth paths. The focus of our interest in this paper, however, is business cycles, and those are transitional events, by definition. To obtain results, we must restrict attention to a sufficiently simple endogenous growth model in which analysis of transition dynamics is feasible.

We examine an economy with one nondurable good and one kind of productive capital. Initially, the economy does not know how to make productive capital, and the invention consists of learning to do so. Thus the model is similar in spirit to that studied in section III.2. The major difference is that the production function now allows endogenous growth:

$$(45) \quad \begin{aligned} y &= f(K, 1-h) \\ &= AK + B(1-h)^\alpha \end{aligned}$$

which generalizes the AK production function to allow extra output from labor. With this production function, y can be positive even if K is zero, so unlike the basic Solow-Swan or Cass-Ramsey model, $K=0$ does not necessarily constitute a steady state. Notice also that (45) is closely related to the bestknown example of a Jones-Manuelli production function:

$$(46) \quad y = AK + BK^\beta L^{1-\beta}$$

If we let $\beta=0$ in (46) and $\alpha=1$ in (45), the two functions are the same.

Utility is assumed to be the simplest logarithmic form:

$$u(c, h) = \ln c + \ln h$$

1. Pre-Invention Optimality.

Before invention, the economy has the same characteristics as in section III.1.1. There is no dynamic element to the maximization problem. Because we now have specific production and utility functions, we can solve for the optimal values of c and h , which are

$$c = B \left(\frac{\alpha}{1 + \alpha} \right)^\alpha$$

$$h = \frac{1}{1 + \alpha}$$

2. Post-Invention Optimality.

After invention of capital, the optimization problem becomes dynamic. The Hamiltonian is

$$H = 1nc + 1nh + \lambda [AK + B(1-h)^\alpha - c - \delta K]$$

The dynamic equations and first-order conditions are

$$(47) \quad dK / dt = \partial H / \partial \lambda = AK + B(1-h)^\alpha - c - \delta K$$

$$(48) \quad d\lambda / dt = -\partial H / \partial K + \rho\lambda = -\lambda [A - \delta - \rho]$$

$$(49) \quad \partial H / \partial c = c^{-1} - \lambda = 0$$

$$(50) \quad \partial H / \partial h = h^{-1} - \lambda \alpha B (1-h)^{\alpha-1}$$

Taking the total differential of (49), rearranging terms, and using (48) gives

$$\begin{aligned} \frac{dc}{dt} \frac{1}{c} &= - \frac{d\lambda}{dt} \frac{1}{\lambda} \\ &= A - \delta - \rho \end{aligned}$$

$$\rightarrow c = c(\lambda), \quad c' < 0$$

For positive growth to occur, we must have $A > \delta + \rho$, so we assume hereafter that this parameter restriction is satisfied. Note that in particular this restriction implies that the growth rate for λ always is negative if λ is positive. Performing similar operations on (50) leads to

$$\frac{dh}{dt} \frac{1}{h} = \frac{-\alpha B (1-h)^\alpha}{h \left[\alpha B (1-h)^\alpha + \frac{1-\alpha}{\lambda} \right]} \left(\frac{d\lambda}{dt} \frac{1}{\lambda} \right)$$

$$= Z \frac{d\lambda}{dt} \frac{1}{\lambda}$$

$$h = h(\lambda), \quad h' < 0$$

where the coefficient Z on the right side, given by

$$Z = \frac{-\alpha B(1-h)^\alpha}{h \left[\alpha B(1-h)^\alpha + \frac{1-\alpha}{\lambda} \right]}$$

is always non-positive and goes to zero as h and λ jointly go to 1 and 0, respectively. Balanced growth requires that all growth rates be constant, so h must go asymptotically to 1 for the economy to approach its balanced growth path. This result is intuitively reasonable. The production function is additively separable in K and h , so accumulation of K involves only a wealth effect that reduces labor and raises leisure.

With these results in hand, we now can derive the economy's response to invention of K . Rewrite (47) as a growth rate:

$$(51) \quad \frac{dK}{dt} \frac{1}{K} = A + \frac{B[1-h(\lambda)]^\varepsilon - c(\lambda)}{K} - \delta$$

$$= A + \frac{B[1-h(\lambda)]^\varepsilon}{K} - \frac{c(\lambda)}{K} - \delta$$

On the balanced growth path, h is constant with a value of 1 (see immediately above), so that (51) simplifies to

$$(52) \quad \frac{dK}{dt} \frac{1}{K} = A - \frac{c}{K} - \delta$$

Also, the growth rate of K is constant on the balanced growth path, so that taking the time derivative of (52) and then converting to growth rates gives

$$D = \frac{dc}{dt} \frac{1}{c} - \frac{dK}{dt} \frac{1}{K}$$

which implies that the growth rates of c and K are equal on the balanced growth path. Before invention, K is zero and not growing; consumption c equals output $B(1-h)^\alpha$. At the moment of invention, λ jumps up, which reduces both c and h and makes $B(1-h)^\alpha - c$ positive. Production of K thus becomes positive. At the instant of

invention, K still is zero, so its growth rate is infinite. As time passes, δ falls, raising both c and h . The growth rate of K falls continually as time passes because

- (1) K in the denominator of the term $[B(1-h)^{-c}]/K$ on the right side of (51) grows, making this term smaller, and
- (2) h in the numerator of the same term becomes large, again making this term smaller.

We thus have a hump shaped response of labor, which jumps up at the moment of invention and then goes asymptotically to zero. Output also jumps up at the moment of invention, but as usual with invention of productive capital, its behavior after that is difficult to determine. On the one hand, output tends to fall as h rises and labor falls; on the other hand, output tends to rise as K rises. In the long run, the increase in K dominates, and output grows without bound. But the short run path could involve fluctuations around this trend behavior.

The main conclusion is that invention of physical capital can cause business cycle behavior in an endogenous growth model.