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OPTIMAL DESIGN OF PERFORMANCE-BASED REWARDS IN TAX ADMINISTRATION*

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Abstract: *This paper illustrates some of the positive incentive effects of rewarding tax performance outcome indicators. By rewarding total collections, the government induces the autonomous tax administration to exert effort in improving the quality of tax enforcement. Improvements in quality effort, in turn, may increase the efficiency of both auditing of underreporters and inspecting of nonfilers. Although audits of underreporters and inspections of nonfilers are substitutes for each other, at least one of these enforcement quantities is necessarily positively related to enforcement quality. We also find that, under some reasonable circumstances, tax evasion falls with either an improvement in the computerization of the tax administration or an increase the highest income tax rate.*

Keywords: *tax administration, tax evasion, multitask organizations, performance-based rewards.*

JEL Classification: *H26, J33, K42, M52.*

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Introduction

Several developing countries have recently attempted to reform their tax collection agencies (see, e.g., Bear, Castro and Vehorn (1999), Das-Gupta and Mookherjee (1998), Kahn, Silva and Ziliak (2001), Mookherjee (1997), Silvani and Baer (1997), and Vazquez-Caro, Reid and Bird (1992)). Most reforms introduced or improved performance-related bonuses. In Brazil, for example, a tax collection reform introduced in 1988 provided monetary compensation to tax collectors based on their individual and group performance in finding and collecting taxes from

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tax evaders. Performance rewards were directly related to the amount of fines collected. Kahn, Silva and Ziliak (2001) find that the Brazilian bonus scheme led to a significant improvement in tax enforcement performance. Mexico also reformed its tax collection agency in 1988. According to Mookherjee (1997), the new Mexican system involves a bonus consisting of about 60% of additional collections within each office.

The implementation of a bonus system, however, is not per se sufficient for the success of the reform[2]. Das-Gupta and Mookherjee (1998), for example, show that tax enforcement in India has not been very effective in spite the existence of performance-based pay. It appears that the overall incentive effects of bonuses have been weak because both the likelihood of receiving a bonus and the size of the bonus are influenced by many factors beyond the control of a tax official.

The Indian case provides a good example of the many difficulties that there exist in improving tax administration performance with bonuses. Tax administrations are multitask institutions. Many of the tasks performed by tax administrations are not observable or not verifiable by the government, responsible for the payment of bonuses. Consider, for example, tax enforcement quality and field inspections of nonfilers. It is hard for the government to observe the degree of accuracy or diligence employed by a tax collector in the examination of returns. It is also very difficult for the government to monitor the performance of tax collectors when they conduct field inspections. This informational asymmetry implies that the government faces a moral hazard problem vis-a-vis the tax administration and that only verifiable outcomes can be rewarded. As Holmstrom and Milgrom (1991) have pointed out in more general contexts, the tax administration may have incentives to concentrate its efforts on verifiable margins of performance in lieu of unverifiable margins, such as quality and field inspections.

We show that the potential perverse effects associated with the implementation of tax performance bonuses – e.g., low tax enforcement quality and low field inspection effort – need not to occur. If the government rewards the tax administration for its performance in aggregate outcomes, such as collection of taxes and fines, the tax administration may respond by improving tax enforcement quality and quantity measures. The underlying rationale for this behavior is quite straightforward. Performance rewards provide the tax administration with incentives to improve its effectiveness in the collection of overdue taxes and fines. The effectiveness of the tax administration is positively associated with both quantity and quality measures of tax enforcement. Indeed, improvements in quality measures may be necessary for improvements in the effectiveness of auditing of underreporters and inspecting of nonfilers. Consider, for example, an economy characterized by a low level of development and widespread tax evasion. It may initially be easy to catch tax evaders without raising tax enforcement quality because there is a large pool of tax evaders. Eventually, however, as the easy targets are gone, effective collection of overdue tax and fines will necessarily depend on the level of effort exerted by the tax administration in quality activities. By rewarding the tax administration for its performance in the collection of taxes and fines, the government may induce the tax administration to work harder in all enforcement activities, and subsequently raise the additional revenues needed to reach the government's targeted level of overall tax revenue.

Our economy consists of two sectors, black and white. Agents in the black sector may

[2] See, e.g., Baer, Castro and Vehorn (1999), Crotty (1998), Das-Gupta (1999) and Gill (1998) for a comprehensive list of measures and strategies that, if effectively implemented, may improve tax administration performance.

evade taxes by not filing at all or by underreporting income. Agents in the white sector, on the other hand, are assumed to always file tax returns. However, they may evade taxes by underreporting income. Our crucial finding is that, under some reasonable circumstances, the tax administration exerts effort in auditing of underreporters and inspecting of nonfilers at levels that are positively related to the amount of effort it exerts in quality. In other words, auditing effort and inspecting effort may both be strategic complements to quality effort.

Given the fact that black markets are widespread in developing economies (see, e.g., Goswami, Sanyal and Gang (1991) and McLaren (1996)), we believe that our model may be quite useful in understanding how developing economies may effectively lower tax evasion. We find that improvements in computerization and increases in the highest tax rate may both motivate the tax administration to work harder in all of its enforcement activities, and an improvement in tax enforcement effectiveness may lower tax evasion because it raises the expected cost associated with tax evasion.

The model

Consider an economy with two sectors, white (W) and black (B). The populations of agents in black and white sectors are P^B and P^W, respectively. There are three types of income earners, high (H), medium (M), and low (L). The populations of income earners are P_H, P_M and P_L. We assume that $P^B = P_L + P_H^B$ and $P^W = P_M + P_H^W$. In words, agents in the black sector earn either low or high income. Agents in the white sector earn either medium or high income. Hence, $P_H = P_H^B + P_H^W$.

In order to finance its expenditures, the government levies a tax on income. Before tax, the income levels are $Y_H > Y_M > Y_L$. We assume that the income tax rates satisfy $1 > t_H > t_M > t_L$. We also assume that: (i) $Y_L = 0$; and (ii) $t_L = 0$. Low income earners ("low types") earn the subsistence level of income. Assumption (i) normalizes this income level to zero. Assumption (ii) tells us that low types do not pay taxes because they earn the subsistence level of income.

The government knows the population sizes and the levels of income earned by "whites" and "blacks," but it cannot immediately distinguish individuals according to their income types. To obtain information about individual incomes, the government relies on individual income reports and on the enforcement effort of its Tax Administration (TA). To simplify the analysis, we assume that whites must pre-register with the TA and that the TA can verify without cost whether or not whites filed tax returns. Our objective is to reserve the option of filing or not filing to earners of high black income ("high blacks").

Earners of white income have two options open to them: (a) report Y_M; or (b) report Y_H. High blacks, on the other hand, have three options open to them: (a) report Y_M; (b) report Y_H; or (c) do not file a tax return. We refer to high blacks that do not file as "high nonfilers" (N_H). Since low blacks do not file tax returns, the total population of nonfilers, N, is $N \equiv P_L + N_H$.

The TA may monitor income earners in two possible ways: (i) by auditing returns or (ii) by conducting field inspections of nonfilers. We refer to audits of returns and field inspections of nonfilers simply as "audits" (A) and "inspections" (I), respectively. The TA may audit taxpayers who report medium income or audit taxpayers who report high income. Let A_M and A_H denote the quantities of medium-income and high-income audits performed. For simplicity, we assume that $A_M \equiv e_M$ and $A_H \equiv e_H$ where e_M and e_H represent the effort levels exerted by the TA in auditing activities. Further, we assume that the quantity of inspections performed, I, equals the effort level, e_I, spent on inspections, $I \equiv e_I$.

In addition to audits and inspections, the effectiveness of tax enforcement depends on the TA's quality performance. Indeed, in the analysis below, quality performance is a necessary condition for the effectiveness of tax enforcement. The quality level, Q, is assumed to be equal to the level of effort exerted by the TA in this activity, $Q = e_Q$.

Let $\Psi(e_M, e_H, e_I, e_Q)$ denote the TA's total operational cost. To keep things simple, we assume that:

$$\Psi(e_M, e_H, e_I, e_Q) \equiv \frac{e_M^2 + e_H^2 + e_I^2 + e_Q^2}{2}.$$

Effort levels are not perfect substitutes but their individual impacts on total and marginal costs are identical.

Let R_H and R_M denote the numbers of high and medium tax filers. Then, $R_H \equiv R_H^B + R_H^W$ and $R_M \equiv R_M^B + R_M^W$. The probabilities that high and medium tax filers are audited are assumed to be $\left(\frac{A_H}{R_H}\right)$ and $\left(\frac{A_M}{R_M}\right)$, respectively. Similarly, the probability that a nonfiler is inspected is assumed to be $\left(\frac{I}{N}\right)$. These assumptions imply that the TA is unable to distinguish tax law offenders from abiders prior to auditing or inspecting activities.

Let us now consider the incentives facing income earners. Although convicted tax offenders are required to pay fines for tax evasion, we assume that there is no reward for compliance with tax laws. Hence, medium income earners have no incentive to report high income. Because these individuals always report truthfully, their after tax income is $Z_M \equiv (1 - t_M)Y_M$.

Consistent with developing country experience, we postulate that high income earners may be able to avoid punishment for tax evasion even when they are audited or inspected. For simplicity, we assume that high types can be ordered according to their "innate" ability of avoiding punishment for tax evasion. For an individual $b \in [0, P_H^B]$, let b denote his innate ability of punishment avoidance. Hence, individual $b = 0$ has the lowest ability and individual $b = P_H^B$ has the highest ability. Similarly, for an individual $w \in [0, P_H^W]$, let w denote his innate ability of punishment avoidance.

When the TA's level of effort exerted in quality activities is e_Q, an audited individual w, who has ability of punishment avoidance w, is punished with probability:

$$\mu^W(e_Q, w) \equiv \mu(e_Q, w; P_H^W) \equiv e_Q\left(\frac{P_H^W - w}{P_H^W}\right).$$

Hence, $\mu^W(0, w) = \mu^W(e_Q, P_H^W) = 0$ and $\mu^W(e_Q, 0) = e_Q$.

It seems reasonable to assume that, everything else equal, it is harder to punish a high black than a high white. We model this increased hardship with the assumption that the TA is less informed about the black sector than about the white sector. Let κ, $0 < \kappa < 1$, denote the degree of useful hard tax information about the black sector available to the TA. When the TA's quality effort level is e_Q, an audited or inspected individual b, who has ability of punishment avoidance b, is punished with probability:

$$\mu^B(e_Q, b) \equiv \mu(e_Q, b; P_H^B)\kappa \equiv (e_Q\kappa)\left(\frac{P_H^B - b}{P_H^B}\right).$$

Each high white individual faces three possible contingencies: (1) reports Y_M / does not face punishment; (2) reports Y_M / faces punishment; or (3) reports Y_H . Each high black, on the other hand, faces five possible contingencies: (1) reports Y_M / does not face punishment; (2) reports Y_M / faces punishment; (3) reports Y_H ; (4) does not file / does not face punishment; (5) does not file / faces punishment. Let σ denote a contingency and $\Sigma^W = \{1,2,3\}$ denote the set of contingencies for high whites. Define $Z_H^W(\sigma)$ as the consumption level of a high white when contingency $\sigma \in \Sigma^W$ occurs. Similarly, let $\Sigma^B = \{1,2,3,4,5\}$ denote the set of contingencies for high blacks, and define $Z_H^B(\sigma)$ as the consumption level of a high black when contingency $\sigma \in \Sigma^B$ occurs. Let f denote the fine rate for underreporting and ϕ denote the fine rate for not filing a tax return. Then, the consumption levels for high blacks and whites are as follows:

$$Z_H^W(1) = Z_H^B(1) \equiv Y_H - t_M Y_M ;$$
$$Z_H^W(2) = Z_H^B(2) \equiv (1-(1+f)t_H)Y_H + f t_M Y_M ;$$
$$Z_H^W(3) = Z_H^B(3) \equiv (1-t_H)Y_H ;$$
$$Z_H^B(4) \equiv Y_H ;$$
$$Z_H^B(5) \equiv (1-(1+\phi)t_H)Y_H .$$

We assume that $0 < f < \phi$. To simplify the analysis, let $\phi = \dfrac{1-t_H}{t_H}$. Hence, $Z_H^B(5) \equiv 0$.

To minimize ambiguities in our comparative statics analysis below, we assume that income earners are risk neutral. The expected utility for a high type who reports truthfully is simply $(1-t_H)Y_H$. The expected utility for an individual w who reports medium income is:

$$Y_H - t_M Y_M - \left(\frac{A_M}{R_M}\right) e_Q \left(\frac{P_H^W - w}{P_H^W}\right)\left[Y_H - t_M Y_M - \left((1-(1+f)t_H)Y_H + f t_M Y_M\right)\right].$$

Then, individual w reports truthfully if and only if

$$U((1-t_H)Y_H) \geq U(Y_H - t_M Y_M) - \left(\frac{A_M}{R_M}\right) q \left(\frac{P_H^W - w}{P_H^W}\right)\left[U(Y_H - t_M Y_M) - U([1-(1+f)t_H]Y_H + f t_M Y_M)\right]. \qquad (1)$$

It is important to observe that condition (1) is violated at $w = P_H^W$. It follows that we are always guaranteed to have $R_H^W < P_H^W$.

The expected utility for an individual b who does not file is

$$\left[1-(q\kappa)\left(\frac{P_H^B - b}{P_H^B}\right)\right]U(Y_H).$$

The expected utility for an individual b who reports medium income is

$$U(Y_H - t_M Y_M) - \left(\frac{A_M}{R_M}\right)(q\kappa)\left(\frac{P_H^B - b}{P_H^B}\right)\left[U(Y_H - t_M Y_M) - U([1-(1+f)t_H]Y_H + f t_M Y_M)\right].$$

Then, individual b files a tax return if and only if

$$\left[1-(q\kappa)\left(\frac{I}{N}\right)\left(\frac{P_H^B - b}{P_H^B}\right)\right]U(Y_H) \leq \max\Big\{U((1-t_H)Y_H), U(Y_H - t_M Y_M)$$
$$-(q\kappa)\left(\frac{A_M}{R_M}\right)\left(\frac{P_H^B - b}{P_H^B}\right)\left[U(Y_H - t_M Y_M) - U([1-(1+f)t_H]Y_H + f t_M Y_M)\right]\Big\}. \qquad (2)$$

Since condition (2) is violated at $b = P_H^B$, we always have $N_H > 0$.

Let C denote the total number of individuals who comply with tax laws. Individuals may comply voluntarily or involuntarily. If C_V denotes the total number of taxpayers who comply voluntarily and C_O denotes the total number of tax law offenders that the court convicts for tax evasion, we have $C \equiv C_V + C_O$. Furthermore, the group of convicted tax evaders consists of those convicted following successful audits, C_A, and those convicted following successful inspections, C_I. Hence, $C_O \equiv C_A + C_I$. For future reference note also that:

$$C_V \equiv C_V^B + C_V^W \equiv R_H^B + P_M + R_H^W.$$

$$C_A \equiv C_A^B + C_A^W.$$

We assume that the government cannot observe either the TA's inputs (efforts) or outputs (audits, inspections and quality level). The government, however, can observe some of the TA's performance outcomes, namely, collections of total tax and fine revenues. This implies that the government can use these indicators to reward the TA's performance. The autonomous TA, therefore, controls its own revenues and allocation of effort levels among its four activities. The TA's objective function is

$$\Omega \equiv S + \rho_T T + \rho_F F + \rho_\Phi \Phi - \Psi(e_M, e_H, e_I, e_Q; \theta).$$

The quantity S represents the lump-sum transfer received from the government (if positive) or paid to the government (if negative), T denotes the total tax revenue collected, F gives the total fine revenue collected from convicted underreporters, Φ represents the total fine revenue collected from convicted nonfilers, ρ_T is the reward rate per dollar of tax revenue collected, ρ_F is the reward rate per dollar of fine for underreporting collected and ρ_Φ is the reward rate per dollar of fine for nonfiling collected. The TA takes the reward rates as given; they are predetermined by the government.

The Income Tax Game

We are now ready to describe the "income tax game" played between income earners and the TA. The game is as follows:

The Income Tax Game

Stage 0: The government announces the tax structure – i.e., tax rates, fine rates and performance rewards.

Stage 1: Income earners decide whether or not to file, and those who file decide whether or not to file truthfully.

Stage 2: The TA decides how much effort to exert in each of its auditing and inspecting activities.

In order to focus on the incentives governing the decisions of income earners and the TA, we do not model the government as a strategic player. We assume that the tax structure is determined exogenously. This does not seem to be a strong assumption in what regards tax and fine rates, since in democratic societies they are determined jointly by the executive and legislative branches of government. Performance rewards, on the other hand, may be determined solely by the executive branch. Our assumption here is based on the Brazilian experience with performance bonuses. While the bonus scheme was designed by the Brazilian TA itself, its implementation depended on approval of both the executive and the legislative. A model that correctly depicts the Brazilian case, for example, would need to consider the lobbying efforts exerted by the TA as well as the strategic motivations of both the executive and legislative branches. This is an interesting avenue for future research.

The equilibrium concept used here is subgame perfection. As it is usually done, we start by examining the last stage of the game. First, note that

$$T \equiv T_M + T_H = t_M R_M + t_H R_H + t_H C_O = t_M R_M + t_H R_H + t_H (C_A + C_I),$$
$$F \equiv f t_H C_A, \quad \Phi \equiv \phi t_H C_I = (1 - t_H) C_I.$$

Then, the TA's objective function becomes:

$$\Omega \equiv S + \rho_T (t_M R_M + t_H R_H) + t_H [(\rho_T + \rho_F f) C_A + (\rho_T + \rho_\Phi (1 - t_H)) C_I] - \Psi(e_M, e_H, e_I, e_Q; \theta).$$

Let $\rho_A \equiv \rho_T + \rho_F f$ and $\rho_I \equiv \rho_T + \rho_\Phi (1 - t_H)$. Hence:

$$\Omega \equiv S + \rho_T (t_M R_M + t_H R_H) + t_H [\rho_A C_A + \rho_I C_I] - \Psi(e_M, e_H, e_I, e_Q; \theta).$$

It is important to notice that at the last stage of the game some quantities have already been determined in the previous stages and are thus taken as given by the TA. These quantities are the tax and fine rates, the performance rewards and the numbers of individuals who do not file, who file medium-income reports, R_M, and who file high-income reports, R_H. The TA is mainly interested in finding tax evaders – underreporters and nonfilers – who will not be able to avoid punishment for tax evasion.

It is straightforward to calculate the expected numbers of tax offenders who are convicted for tax evasion. These quantities are given by

$$C_A^B(e_M, e_H, e_I, e_Q) \equiv \left(\frac{\kappa e_Q}{E}\right)\left(\frac{\alpha_M e_M}{R_M}\right)\left(\frac{R_M^B}{P_H^B}\right)\left(N_H + \frac{R_M^B}{2}\right),$$
$$C_A^W(e_M, e_H, e_I, e_Q) = \left(\frac{e_Q}{E}\right)\left(\frac{\alpha_M e_M}{R_M}\right)\left(\frac{R_M^W - P_M}{P_H^W}\right)\left(\frac{R_M^W - P_M}{2}\right),$$
$$C_I(e_M, e_H, e_I, e_Q) = \left(\frac{\kappa e_Q}{E}\right)\left(\frac{\alpha_I e_I}{N}\right)\left(\frac{N_H}{2}\right)\left(\frac{N_H}{P_H^B}\right).$$

Note that each of the functions above decreases in e_H. This is easy to explain. Auditing high income reporters has no direct deterrent effect on underreporting or nonfiling behavior, but it competes resources away from auditing of medium income reporters and inspecting of nonfilers. Hence, the TA does not find it desirable to exert effort in this activity:

$$e_H = 0. \tag{3}$$

Let

$$V(e_M, e_I, e_Q) \equiv t_H [\rho_A C_A (e_M, e_I, e_Q) + \rho_I C_I (e_M, e_I, e_Q)] - \Psi(e_M, e_I, e_Q; \theta). \tag{4}$$

The TA chooses nonnegative $\{e_M, e_I, e_Q\}$ to maximize (4). To illustrate the circumstances under which both auditing of medium income reporters and inspecting of nonfilers are strategic complements to enforcement quality, it is instructive to solve this maximization problem in two steps. We shall first keep e_Q fixed and maximize (4) with respect to e_M and e_I. This procedure will enable us to define e_M and e_I as implicit functions of e_Q and then characterize the circumstances under which both functions increase in e_Q. In the second step, we place the implicit functions into (4) and then maximize the implied function with respect to e_Q.

Assuming an interior solution, the first order conditions with respect to e_M and e_I are as follows:

$$V_M \equiv \frac{\partial V}{\partial e_M} = t_H\left[\frac{\rho_A C_A\left(e_I + e_Q\right)}{E e_M} - \frac{\rho_I C_I}{E}\right] - (1-\theta)e_M = 0, \tag{5a}$$

$$V_I \equiv \frac{\partial V}{\partial e_I} = t_H\left[\frac{\rho_I C_I\left(e_M + e_Q\right)}{E e_I} - \frac{\rho_A C_A}{E}\right] - (1-\theta)e_I = 0. \tag{5b}$$

Equations (5) indicate that auditing and inspecting efforts are chosen at levels that equate marginal revenues to marginal costs. Given e_Q, these conditions are both necessary and sufficient for a unique local maximum, since

$$V_{MM} = -\frac{(1-\theta)(2e_M + E)}{E} < 0, \tag{6a}$$

$$\left|H_{MI}\right| \equiv V_{MM}V_{II} - V_{MI}^2 = \frac{(1-\theta)^2\left[2E\left(e_I + e_M\right) + \left(2e_M + e_Q\right)\left(2e_I + e_Q\right)\right]}{E^2} > 0. \tag{6b}$$

Conditions (6) tell us that the second order leading principal minor of the Hessian matrix of V, $\left|H_{MI}\right|$, is locally negative definite. Hence, we know that equations (5) can be used to implicitly define the functions

$$e_j = e_j\left(e_Q, \rho_A, \rho_I, t_H, \theta, \alpha_M, \alpha_I, \kappa, R_M^B, R_M^W, N_H, P_L, P_M, P_H^B, P_H^W\right),\ j = I, M.$$

Differentiating equations (5) with respect to e_Q yields:

$$(1-\theta)^2 E^2 e_Q^2 \begin{pmatrix} 2e_M + E & e_M + e_I \\ e_M + e_I & 2e_I + E \end{pmatrix} \begin{pmatrix} \dfrac{\partial e_M}{\partial e_Q} \\ \dfrac{\partial e_I}{\partial e_Q} \end{pmatrix} = \begin{pmatrix} (1-\theta)(e_M + e_I)Ee_M + t_H\left(\rho_A C_A + \rho_I C_I\right)e_Q \\ (1-\theta)(e_M + e_I)Ee_I + t_H\left(\rho_A C_A + \rho_I C_I\right)e_Q \end{pmatrix}. \tag{7}$$

Close inspection of the linear system of equations (7) reveals that at least one of its variables, $\frac{\partial e_M}{\partial e_Q}$ and $\frac{\partial e_I}{\partial e_Q}$, must be positive. In other words, at least one of the enforcement quantities must be strategically complement to enforcement quality. Indeed, it is quite reasonable to expect that both auditing and inspecting are strategic complements to quality, since

$$\frac{\partial e_M}{\partial e_Q} = \frac{(1-\theta)E\left(e_M + e_I\right)\left[e_M\left(E + e_I\right) - e_I^2\right] + t_H\left(\rho_A C_A + \rho_I C_I\right)\left(2e_I + e_Q\right)e_Q}{(1-\theta)Ee_Q\left[E^2 + \left(E + e_Q\right)\left(e_M + e_I\right) + 4e_M e_I\right]}, \tag{8a}$$

$$\frac{\partial e_I}{\partial e_Q} = \frac{(1-\theta)E\left(e_M + e_I\right)\left[e_I\left(E + e_M\right) - e_M^2\right] + t_H\left(\rho_A C_A + \rho_I C_I\right)\left(2e_M + e_Q\right)e_Q}{(1-\theta)Ee_Q\left[E^2 + \left(E + e_Q\right)\left(e_M + e_I\right) + 4e_M e_I\right]}. \tag{8b}$$

Partial derivatives (8) are both positive if $e_M = e_I$. They are also positive with unequal auditing and inspecting efforts provided these quantities are not strongly dissimilar. It is possible that one effort level is so large relative to the other that one of the partial derivatives turns out to be negative. For example, (8a) could be negative if e_I is very large relative to e_M. The other partial derivative, however, would necessarily be positive. To minimize ambiguities in what follows, we shall assume that, for any $e_Q \geq 0$, the optimal auditing and inspecting efforts chosen by the TA satisfy the following inequalities:

$$e_M\left(E + e_I\right) > e_I^2, \tag{9a}$$

$$e_I\left(E + e_M\right) > e_M^2. \tag{9b}$$

Although inequalities (9) are apparently quite reasonable, in the sense that they hold as long as auditing and inspecting efforts are not too strongly dissimilar, it should be clear that they are only sufficient, not necessary, conditions for the partial derivatives (8) to be positive. Inequalities (9) are quite useful because they enable us to sign some partial derivatives in our comparative statics analysis below.

To examine the TA's choice of quality effort, let us define $\Gamma(e_Q) \equiv V(e_M(e_Q,\bullet), e_I(e_Q,\bullet), e_Q)$. Maximizing $\Gamma(e_Q)$ with respect to e_Q and assuming an interior solution we obtain:

$$\Gamma_Q \equiv \frac{d\Gamma}{de_Q} = \frac{t_H(\rho_A C_A + \rho_I C_I)(e_M + e_I)}{Ee_Q} - (1-\theta)e_Q = 0. \tag{10}$$

Equation (10) tells us that the TA exerts quality effort at a level that equates the marginal revenue and marginal cost from such an activity. It is important to alert the reader that, due to the strategic complementarities, Γ is not necessarily strictly concave in the neighborhood of a maximum. Fortunately, the (omitted) condition required for Γ to be locally strictly concave does not appear to be very restrictive. Assuming that this condition is satisfied, we can use (10) to implicitly define the following function:

$$e_Q = e_Q(\rho_A, \rho_I, t_H, \theta, \alpha_M, \alpha_I, \kappa, R_M^B, R_M^W, N_H, P_L, P_M, P_H^B, P_H^W).$$

At stage 1 of the game, income earners anticipate how the TA will react to their decisions and behave accordingly. Given the results above, conditions (1) and (2) may be rewritten as follows:

$$\begin{aligned} &U((1-t_H)Y_H) \geq U(Y_H - t_M Y_M) \\ &-\left(\frac{\alpha_M e_M(e_Q(\bullet),\bullet)}{R_M}\right)\left(\frac{e_Q(\bullet)}{E(\bullet)}\right)\left(\frac{P_H^W - w}{P_H^W}\right)[U(Y_H - t_M Y_M) - U([1-(1+f)t_H]Y_H + ft_M Y_M)], \end{aligned} \tag{11}$$

$$\begin{aligned} &\left[1-\left(\frac{\alpha_I e_I(e_Q(\bullet),\bullet)}{N}\right)\left(\frac{\kappa e_Q(\bullet)}{E(\bullet)}\right)\left(\frac{P_H^B - b}{P_H^B}\right)\right]U(Y_H) \leq \max\{U((1-t_H)Y_H), U(Y_H - t_M Y_M) \\ &-\left(\frac{\alpha_M e_M(e_Q(\bullet),\bullet)}{R_M}\right)\left(\frac{\kappa e_Q(\bullet)}{E(\bullet)}\right)\left(\frac{P_H^B - b}{P_H^B}\right)[U(Y_H - t_M Y_M) - U([1-(1+f)t_H]Y_H + ft_M Y_M)]\Big\}, \end{aligned} \tag{12}$$

where $E(\bullet) \equiv e_M(e_Q(\bullet),\bullet) + e_I(e_Q(\bullet),\bullet) + e_Q(\bullet)$.

We now investigate the properties of an equilibrium for an economy characterized by a low level of development. We define such an economy as one where we observe a low enough value for the parameter κ so that condition (12) is violated for all $b \in [0, P_H^B]$ This, in turn, implies that $N_H = P_H^B$ and $R_M^W = R_M$. In words, all high blacks are nonfilers and thus those who report medium income are all whites, either medium or high whites. Furthermore, we assume that condition (11) is satisfied binding for some type $w^* \in (0, P_H^W]$ when $N_H = P_H^B$ and $R_M^W = R_M$. Hence, all high whites whose types are in the interval $[0, w^*]$ report truthfully and all high whites whose types are in the interval $[w^*, P_H^W]$ underreport their incomes. That is to say, $w^* = R_H^W = P_H^W - (R_M^W - P_M)$ and

$$U((1-t_H)Y_H) = U(Y_H - t_M Y_M) - \left(\frac{\alpha_M e_M(e_Q(\bullet),\bullet)}{R_M^W}\right)\left(\frac{e_Q(\bullet)}{E(\bullet)}\right)\left(\frac{R_M^W - P_M}{P_H^W}\right)\left[U(Y_H - t_M Y_M) - U([1-(1+f)t_H]Y_H + ft_M Y_M)\right] \quad (13)$$

It is important to note that the quantity $R_M^W - P_M$ gives us the number of high whites who underreport income; i.e.,. Since P_M is fixed, changes in R_M^W represent changes in the number of underreporters. Equation (13) defines R_M^W as an implicit function of the relevant exogenous variables:

$$R_M^W = R_M^W(t_M, t_H, Y_M, Y_H, f, \rho_A, \rho_I, \theta, \alpha_M, \alpha_I, \kappa, P_H^B, P_L, P_M, P_H^W).$$

It is useful to summarize the results that we have obtained so far. The subgame perfect equilibrium for the income tax game played by income earners and the TA is characterized by equations (3), (5a), (5b), (10), (13), $N_H = P_H^B$ and $R_M^W = R_M$. Equations (5a), (5b) and (10) inform us that the efforts exerted by the TA in auditing medium-income reports, inspecting nonfilers and improving quality are set at levels that equate the TA's marginal revenues to its marginal costs. Equation (3) tell us that the TA finds it unprofitable to exert any effort at all in audits of high-income reports. Equations (13), $N_H = P_H^B$ and $R_M^W = R_M$ describe the equilibrium for income earners in an economy characterized by low development, an economy where the degree of information about the black sector is so low relative to the degree of information available about the white sector that no high black finds it advantageous to file a tax return. Equation (13), however, demonstrates that the TA's efforts are not totally in vain, since they deter some high whites from underreporting their incomes.

The strategic complementarity there necessarily exists between at least one of the effective enforcement quantities and enforcement quality is, perhaps, the most important message of our analysis up to this point. Knowing that improvements in enforcement quality lead to improvements in audits and inspections, the TA does not have any incentive to sacrifice enforcement quality in order to save resources that can be allocated in the other activities. Conditions (8a) and (8b) demonstrate that at least one of the effective enforcement quantities, audits of medium-income reports and inspections of nonfilers, must be strategically complement to enforcement quality. Conditions (9a) and (9b), which rule out strongly dissimilar effort levels, provide sufficient conditions for both effective enforcement quantities to be strategic complements to enforcement quality.

Comparative Statics

It is important to know which refutable hypotheses, if any, are implied by our model. As we shall discuss below, our comparative statics analysis is plagued with ambiguities. Whenever possible, we will attempt to shed some light on the sources of ambiguities. In a few cases, the relationships become unambiguous by further placing some apparently reasonable restrictions.

To better understand the sources of ambiguities, we provide a list of the relevant partial derivatives for each stage of the income tax game. To save space, we do not present the partial derivatives per se, but only indicate if they are positive, negative or ambiguous. Consistent with the way we solved the game, we start by listing the relevant partial derivatives for the last stage of the game. Given e_Q, we obtain the following partial derivatives for the enforcement quantity efforts:

$$\frac{\partial e_M}{\partial \rho_A} > 0; \qquad \frac{\partial e_I}{\partial \rho_A} < 0; \tag{14a}$$

$$\frac{\partial e_M}{\partial \rho_I} < 0; \qquad \frac{\partial e_I}{\partial \rho_I} > 0; \tag{14b}$$

$$\frac{\partial e_M}{\partial t_H} > 0; \qquad \frac{\partial e_I}{\partial t_H} > 0; \tag{14c}$$

$$\frac{\partial e_M}{\partial \theta} > 0; \qquad \frac{\partial e_I}{\partial \theta} > 0; \tag{14d}$$

$$\frac{\partial e_M}{\partial \alpha_M} > 0; \qquad \frac{\partial e_I}{\partial \alpha_M} < 0; \tag{14e}$$

$$\frac{\partial e_M}{\partial \alpha_I} < 0; \qquad \frac{\partial e_I}{\partial \alpha_I} > 0; \tag{14f}$$

$$\frac{\partial e_M}{\partial \kappa} < 0; \qquad \frac{\partial e_I}{\partial \kappa} > 0; \tag{14g}$$

$$\frac{\partial e_M}{\partial R_M^W} > 0; \qquad \frac{\partial e_I}{\partial R_M^W} < 0; \tag{14h}$$

$$\frac{\partial e_M}{\partial P_L} > 0; \qquad \frac{\partial e_I}{\partial P_L} < 0; \tag{14i}$$

$$\frac{\partial e_M}{\partial P_M} > 0; \qquad \frac{\partial e_I}{\partial P_M} < 0; \tag{14j}$$

$$\frac{\partial e_M}{\partial P_H^B} > 0; \qquad \frac{\partial e_I}{\partial P_H^B} < 0; \tag{14k}$$

$$\frac{\partial e_M}{\partial P_H^W} < 0; \qquad \frac{\partial e_I}{\partial P_H^W} > 0. \tag{14l}$$

Partial derivatives (14a) tell us that auditing effort increases and inspecting effort decreases as the "auditing performance reward rate," ρ_A, increases. Auditing and inspecting activities are strategic substitutes, even though each activity is strategically complement to enforcement quality. The rationale for partial derivatives (14b) is identical to the rationale for partial derivatives (14a). The signs of partial derivatives (14c) and (14d) follow from our assumption that enforcement quantity efforts are not strongly dissimilar, conditions (9). Audits of medium reports and inspections of nonfilers become more lucrative as the highest tax rate increases. Provided the TA does not strongly favor one of these activities over the other, effort levels increase in both activities. By a similar reasoning, the profitability of both activities rises when the overall cost of production lowers as a result of an improvement in the degree of computerization.

An improvement in auditing productivity leads to an increase in auditing effort and a decrease in inspecting effort, as partial derivatives (14e) indicate. The rationale for partial derivatives (14f) is identical. As for the effects on enforcement quantity efforts of an improvement in the TA's degree of information about black sector activities, we find that inspecting effort definitely rises because inspections of nonfilers becomes more lucrative. In principle, it is not clear whether such an improvement would increase or decrease the profitability of audits of medium reports. If some high blacks were to file returns, auditing of medium reports would become more profitable, holding everything else constant. But, the opportunity cost of increasing audits may be substantial. Since auditing and inspecting are strategic substitutes, an increase in audits would necessarily compete resources away from inspections. Hence, provided some high blacks filed returns, the information improvement would produce positive and negative effects on auditing of medium reports, and the implied relationship between these quantities would be ambiguous. In the equilibrium examined in this paper, however, no high black files a return. This implies that the TA finds it desirable to reduce auditing effort as the degree of information about the black sector improves.

To understand partial derivatives (14h) - (14l), it is instructive to remember how the expected numbers of people convicted for underreporting and nonfiling are calculated, functions C_A and C_I. Since, in equilibrium, $R_M^B = 0,$ we have $C_A^B = 0$ and $C_A = C_A^W$. Now, note that an increase in the number of medium reports by high whites leads to an increase in C_A and to no change in C_I. Therefore, auditing of medium reports becomes more profitable relative to inspecting of nonfilers as the number of underreporters increases, effects that partial derivatives (14h) illustrate. Since the logic for results (14i) - (14l) is similar, we will not comment on them.

The results concerning the comparative statics for enforcement quality are murky. Because both enforcement quantity efforts depend positively on enforcement quality, the effects on enforcement quality effort of changes in the parameters are both direct and indirect. As the indirect effects influence the overall effects as a sum, the overall effects on enforcement quality effort cannot be unambiguously determined unless both indirect effects have the same sign. For example, the effect on enforcement quality effort of an increase in the auditing performance reward rate is ambiguous because such an increase leads to a simultaneous expansion in auditing effort and contraction in inspecting effort, and the overall effect on enforcement quality effort depends, among other things, on the sum of these two effects. Therefore, there are only two partial derivatives for the enforcement quality effort function that can be unambiguously signed:

$$\frac{\partial e_Q}{\partial t_H} > 0 \qquad \text{and} \qquad \frac{\partial e_Q}{\partial \theta} > 0. \tag{15}$$

The first partial derivative in (15) tells us that the TA increases quality effort as the highest tax rate rises because such a change in highest tax rate raises the profitability of enforcement quality. This result is implied by the positive impact on the profitability of both enforcement quantities following an increase in the highest tax rate. By a similar token, an improvement in the degree of computerization leads to a decrease in the overall cost of production. This motivates the TA to exert more effort on both enforcement quantity activities and, consequently, to increase enforcement quality effort.

Let us now turn to the relevant comparative statics for stage 1 of the game. We shall examine the effects on the variable R_M^W of changes in the set of parameters $\{\theta, t_H, t_M, Y_H, Y_M\}$. As we observed above, changes in the other exogenous variables lead to ambiguous results. Unfortunately, none of the signs of the partial derivatives for the R_M^W function can be determined unambiguously. The sources of ambiguities in this case are basically two: (i) the concavity of U (i.e., risk aversion); and (ii) the enforcement quality rate, $q \equiv e_Q / E.$

Risk aversion is a source of ambiguity for all partial derivatives except for the partial derivative with respect to 2. The only source of ambiguity for this partial derivative concerns the effect on the enforcement quality rate of changes in 2:

$$\frac{\partial q}{\partial \theta} = \frac{1}{E}\left(\frac{\partial e_Q}{\partial \theta} - \left(\frac{1}{E}\right)\frac{\partial E}{\partial \theta}\right). \tag{16}$$

Since $\frac{\partial e_Q}{\partial \theta} > 0$ and $\frac{\partial E}{\partial \theta} > 0$, the sign of partial derivative (16) cannot be determined unambiguously. This is problematic because equation (13) clearly demonstrates that the variable of interest, R_M^W, depends on the enforcement quality rate.

If we assume, as it seems reasonable, that the enforcement quality rate increases with an improvement in the degree of computerization, $\frac{\partial q}{\partial \theta} > 0$, we are able to conclusively state that underreporting falls as the TA becomes more computerized, $\frac{\partial R_M^W}{\partial \theta} < 0.$ The TA's optimal responses to an improvement in the degree of computerization are effective in increasing the expected cost of underreporting. Consequently, fewer high whites attempt to evade taxes.

If we assume risk neutrality instead of risk aversion, we obtain conclusive results for three of the remaining four partial derivatives. With risk neutral income earners, equation (13) simplifies to:

$$(1+f)\left(\frac{\alpha_M e_M(e_Q(\bullet),\bullet)}{E(\bullet)}\right)\left(\frac{e_Q(\bullet)}{E(\bullet)}\right)\left(\frac{R_M^W - P_M}{P_H^W}\right) = 1. \tag{17}$$

Unlike before, the implicit function defined by (17) is independent of the parameters $\{t_M, Y_H, Y_M\}$. These parameters do not influence the TA's efforts. It follows that a change in any of these parameters does not affect the expected cost of underreporting. Hence:

$$\frac{\partial R_M^W}{\partial t_M} = \frac{\partial R_M^W}{\partial Y_H} = \frac{\partial R_M^W}{\partial Y_M} \equiv 0. \tag{18}$$

Similar to the case for the 2 parameter, we can only determine the sign of the partial derivative with respect to t_H, $\frac{\partial R_M^W}{\partial t_H}$, if we further assume that:

$$\frac{\partial q}{\partial t_H} = \frac{1}{E}\left(\frac{\partial e_Q}{\partial t_H} - \left(\frac{1}{E}\right)\frac{\partial E}{\partial t_H}\right) > 0. \tag{19}$$

If the enforcement quality rate is positively related to the highest tax rate, underreporting falls as the highest tax rate rises. The TA's optimal responses to an increase in the highest tax rate raise the expected cost of underreporting which, in turn, effectively deters some high whites from underreporting their incomes.

Conclusion

The simple model studied in this paper illustrates some of the positive incentive effects of tax performance outcome indicators. By rewarding total collections, the government induces the autonomous tax administration to exert effort in improving the quality of tax enforcement. Improvements in quality effort, in turn, may increase the efficiency of both auditing of underreporters and inspecting of nonfilers. Although audits of underreporters and inspections of nonfilers are substitutes for each other, at least one of these enforcement quantities is necessarily positively related to enforcement quality.

Holding enforcement quality effort constant and assuming that the enforcement quantity efforts are not strongly dissimilar, we were able to sign most of the relevant partial derivatives in our comparative statics analysis for the quantity effort variables. These partial derivatives had the expected signs. The two most important results turned out to be the impacts on the quantity effort variables of changes in the high tax rate and in the degree of computerization. We showed that quantity efforts rise with increases in the high tax rate and in the degree of computerization. These results were quite significant because they enabled us to conclusively ascertain that enforcement quality also rises with increases in such parameters.

Due to ambiguities introduced by risk aversion and the enforcement quality rate, we were unable to obtain conclusive results in our comparative statics analysis concerning tax evasion incentives. By restricting our attention to situations where the enforcement quality rate rises with increases in the degree of computerization, we were able to state that tax evasion falls with improvements in computerization. Tax evasion also falls with increases in the highest tax rate if income earners are risk neutral and the enforcement quality rate rises with increases in the highest tax rate. Both of these effects on tax evasion incentives are due to the actions undertaken by the TA in response to increases in the degree of computerization and in the highest tax rate. Because each of such changes in the parameters motivates the TA to work harder in all enforcement activities, the expected cost of underreporting increases. Hence, fewer high whites attempt to evade taxes.

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