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MACROECONOMIC INTERDEPENDENCE BETWEEN NATIONS IN A WORLD WITHOUT STRUCTURAL ASYMMETRIES*

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Abstract: *This paper presents a multinational model, fairly simple, but including many of the macroeconomic ingredients required to investigate the diffusion of the effects of shocks between nations. It includes three countries, Germany, France and the United States. Germany and France are linked by a fixed exchange rate and share a common central bank. However, they are in a situation of flexible exchange rates with the United States. Interpreting the results of multinational models is complex task. When the different countries share the property of structural symmetry, the dynamic multipliers of the model can be decomposed locally into three parts, determined by as many independent sub-models. The first sub-model bears on world aggregated variables. It has the structure of the model of a closed economy. The second sub-model bears on variables measuring the difference between Europe and the United States. It has the structure of the model of a small open economy working under a flexible exchange rate system. The third sub-model bears on variables measuring the difference between Germany and the rest of the world. It has the structure of the model of a small open economy working under an exogenous exchange rate. This decomposition is used to get a simple economic interpretation of the effects of two unanticipated shocks on the dynamics of the full model.*

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Introduction

This paper presents a multinational model, fairly simple, but including many of the macroeconomic ingredients required to investigate the diffusion of the effects of shocks between nations. The model can be considered as a representative maquette of much bigger multinational models with rational expectations such as Multimod Mark 3 built by the IMF (Laxton Douglas, Peter Isard, Hamid Faruqee, Eswar Prasad and Bart Turtelboom (1998)), Quest 2 built by the European Commission (Roeger Werner and Jan in't Veld (1997)) and Marmotte built by Cepii and Cepremap (Cadiou Loïc, Stéphane Dées, Stéphanie Guichard, Arjan Kadareja, Jean-Pierre Laffargue and Bronka Rzepkowski (2001)). This model includes three countries, Germany, France and the United States. Germany and France are linked by a fixed exchange rate and share a common central bank, the European Central Bank (ECB). However, they are in a situation of flexible exchange rates with the United States.

Interpreting the results of multinational models is complex task. A shock originating in the United States for instance will affect Germany and France at different extents, then feed back to the US. Aoki (1981) developed an original method to analyse the properties of a multinational model with n countries. When countries hold identical structures and if the model is linear, it can be decomposed into n independent models. The first represents the whole world and works as the model of a closed economy. Each of the $n-1$ other models represents the difference between a country and the rest of the world and works as the model of a small open economy. Moreover, these $n-1$ models are identical to one another although the definition of their variables in terms of those of the full model differs. Laffargue (1985 and 1986) extended this decomposition procedure to the case where the n countries differ by their sizes, but otherwise share identical structures. He also assumed that floating and fixed exchange rate systems co-exist in its world. The assumptions of Laffargue define a world without structural asymmetries where distance or preferential trade agreements play no role. However, this world, as Aoki's world, is not homogenous. Countries produce different goods, households show a preference for domestic commodities and countries are hit by different shocks.

This paper will extend the results by Laffargue to a more comprehensive model, including recent developments in macroeconomics. The first section gives the equations of the model. The second section investigates its long run properties, that is its balanced growth path, with special emphasis on the hysteresis properties of nominal variables. The calibration of the model is given in section 3. Section 4 investigates technical questions related to the simulation of the model. It also gives its rewriting in reduced variables, which is especially appropriate to simulation.

The calibration of section 3 assumes structural symmetries between nations (the precise definition of this concept is given in the section). The decomposition of the model (rather of its linear approximation in the neighbourhood of the economy reference state) is given in section 5. The first sub-model bears on world aggregated variables. It has the structure of the model of a closed economy. The second sub-model bears on variables measuring the difference between Europe and the United States. It has the structure of the model of a small open economy working under a flexible exchange rate system. The third sub-model bears on variables measuring the difference between Germany and the rest of the world. It has the structure of the model of a small open economy working under an exogenous exchange rate[2]. The responses of these three sub-models to shocks are easy to interpret. Section 6 gives and comments the results of several simulations of the full model hit by unanticipated shocks.

The model

The model includes 46 endogenous variables defined in the following table. The indices of Germany, France and the US respectively are 1, 2 and 3. The 7 last endogenous variables will play a technical role in the simulation method, but presently they have no simple economic meaning.

Endogenous variables

e	Exchange rate of the euro relative to the dollar
i_j	Nominal interest rate in country j
π	Average inflation rate of consumption prices in Europe
π_3	Inflation rate of consumption prices in the US
pc_j	Consumption price in country j
p_j	Production price in country j
C_j	Households' consumption in country j
C_{ij}	Households' consumption in country i in goods produced by country j
D_j	Total demand by country j
L_j	Employment in country j
Q_j	Potential production by country j
ω_j	Real labour cost in country j
F_j	Foreign indebtedness in dollars by country j
$\inf_j$	Reduced inflation rate in country j
Def	Reduced depreciation rate of the euro
gc_j	Reduced growth rate of households' consumption in country j

Exogenous variables

N_j	Labour supply in country j
G_j	Autonomous demand by country j
$\overline{\pi}$	Target inflation rate for consumption prices in Europe
$\overline{\pi}_3$	Target inflation rate for consumption prices in the US
A_j	Productivity index of country j

Variables in the reference steady state used in the writing of the model

π^s Inflation rate in Europe

π_3^s Inflation rate in the US

$\overline{Q}_1$ Production in Germany

$\overline{Q}_2$ Production in France

Parameters

$\bar{r}$ Target real interest rate for the ECB

$\bar{r}_3$ Target real interest rate for the Fed

n Growth rate of the labour supply and of the number of consumers (identical across countries)

Equations

(1), (2) $$\left(1+i_j\right)e=\left[1+i_3+0.01\left(\frac{eF_j}{p_jQ_j}-\frac{F_3}{p_3Q_3}\right)\right]e_{+1},\ \ j=1,2$$

(3) $$(1+i_1)^{\overline{Q}_1/(\overline{Q}_1+\overline{Q}_2)}(1+i_2)^{\overline{Q}_2/(\overline{Q}_1+\overline{Q}_2)}=\left(1+\bar{r}\right)\ \left(1+\bar{\pi}\right)+1.5\ \ (\pi-\bar{\pi})$$

(4) $$1+i_3=\left(1+\bar{r}_3\right)\ \left(1+\bar{\pi}_3\right)+1.5\ \ (\pi_3-\bar{\pi}_3)$$

(5) $$1+\pi_3=pc_3/pc_{3,-1}$$

(6) $$1+\pi=\left(\frac{pc_1}{pc_{1,-1}}\right)^{\overline{Q}_1/(\overline{Q}_1+\overline{Q}_2)}\left(\frac{pc_2}{pc_{2,-1}}\right)^{\overline{Q}_2/(\overline{Q}_1+\overline{Q}_2)}$$

(7) to (9) $$\left(\frac{C_{j,+1}}{(1+n)\,C_j}\right)^{\rho}=\left(\frac{1+i_j}{1+\beta}\right)\frac{pc_j}{pc_{j,+1}}\ \ j=1,2,3$$

(10), (11) $$pc_j^{-\sigma}=b_{j1}p_1^{-\sigma}+b_{j2}p_2^{-\sigma}+b_{j3}\left(ep_3\right)^{-\sigma},\ \ j=1,2$$

(12) to (15) $$p_jC_{ij}=b_{ij}\left(p_j/pc_i\right)^{-\sigma}pc_iC_i,\ i=1,2\ \ \ j=1,2$$

(16), (17) $$p_3eC_{i3}=b_{i3}\left(p_3e/pc_i\right)^{-\sigma}pc_iC_i\ ,\ i=1,2$$

(18) $$pc_3^{-\sigma}=b_{31}\left(\frac{p_1}{e}\right)^{-\sigma}+b_{32}\left(\frac{p_2}{e}\right)^{-\sigma}+b_{33}p_3^{-\sigma}$$

(19), (20) $$\left(p_j/e\right)\ C_{3j}=b_{3j}\left(p_j/epc_3\right)^{-\sigma}pc_3C_3\ ,\ \ j=1,2$$

(21) $$p_3C_{33}=b_{33}\left(p_3/pc_3\right)^{-\sigma}pc_3C_3$$

(22) to (24) $$D_j=G_j+C_{1j}+C_{2j}+C_{3j}\ ,\ j=1,2,3$$

(25) to (27) $$Q_j=A_jL_j^{\alpha},\ \ j=1,2,3$$

(28) to (30) $$\alpha A_j L_j^{\alpha-1} = \omega_j,\ j = 1,2,3$$

(31) to (33) $$\ln\left(\frac{\omega_j L_j}{Q_j}\right) = \varphi_{0i} + \varphi_1 \ln\left(\frac{pc_j}{p_j}\right) + \varphi_2 \ln\left(\frac{L_j}{N_j}\right) - \varphi_3\left[(1-\varphi_4)\ln\left(\frac{p_j}{p_{j,-1}}\right) + \varphi_4 \ln\left(\frac{p_j}{pc_{j,-1}}\right)\right]$$
$$j = 1,2,3$$

(34), (35) $$\ln(D_j / Q_j) = \frac{1}{\lambda}\ln\left[\frac{p_j}{p_{j,-1}(1+\pi^s)}\right],\ j = 1,2$$

(36) $$\ln(D_3 / Q_3) = \frac{1}{\lambda}\ln\left[\frac{p_3}{p_{3,-1}(1+\pi_3^s)}\right]$$

(37), (38) $$p_i D_i + e(F_i - F_{i,-1}) = pc_i C_i + p_i G_i + \left[i_3 + 0.01\left(\frac{eF_i}{p_i Q_i} - \frac{F_3}{p_3 Q_3}\right)\right].eF_{i,-1},\ i = 1,2$$

(39) $$F_1 + F_2 + F_3 = 0$$

(40) to (42)
$$\inf_j = (1+\bar{\pi})\frac{pc_j}{pc_{j,-1}},\ j = 1,2$$
$$\inf_3 = (1+\bar{\pi}_3)\frac{pc_3}{pc_{3,-1}}$$

(43) $$def = \frac{1+\bar{\pi}}{1+\bar{\pi}_3}\frac{e}{e_{-1}}$$

(44) to (46) $$gc_j = (1+g)\frac{C_j}{C_{j,-1}},\ j = 1,2,3$$

Comments

Germany and France share a common currency, the euro. So, there is a single exchange rate in the model, between the euro and the dollar. There are two uncovered interest rate parity equations between Germany and the US on one hand (equation (1)) and between France and the US on the other hand (equation (2)). A variable risk premium is included in each equation. For instance, in the case of France, the difference between the interest rate in this country and in the US is increased by a premium. It is proportional to the difference between the rates of the foreign indebtedness of France and the US. In these equations the future value of the exchange rate is rationally anticipated. Index “+1” identifies a variable with a lead of 1 period.

The function of the central banks of the United States and the Euro zone is to stabilise inflation in the neighbourhood of targets, which can differ between the two banks. They react to excess in inflation by raising the interest rate (equations 3 and 4). Inflation considered by central banks is the most recent observed inflation rate, not expected inflation. This backward looking character of monetary rules is important for reasons that will be given later. The interest rates can differ between France and Germany, with difference in the rates of foreign indebtedness in both countries. The interest rate controlled by the European Central Bank is the geometric average of both European rates, with weights proportional to the output of the two countries in the reference state. The same geometric average is applied to the inflation rates observed in both countries (equation 6). The coefficient 1.5 in the reaction function of both central banks means that an increase in inflation will be corrected by a higher increase in nominal interest rate

The consumption function in each country ((7), (8) and (9)) results from the inter-temporal optimisation of their representative households, under an inter-temporal budget constraint. ρ is the inverse of the inter-temporal rate of substitution and β represents the discount rate of households.

Equations (10) to (21) determine consumption prices in each country, and the consumption by country i of commodities produced in country j. Equations (10) to (17) are for Germany and France and equations (18) to (25) are for the United States. These equations result from the maximisation of a utility function with constant elasticity of substitution, according to a procedure developed by Armington (1969) and which has become classical for multinational models. $1+\sigma$ is the elasticity of substitution between the goods produced by the three countries. We norm the parameters of the equations by the relationships: $b_{i1}+b_{i2}+b_{i3}=1$, $i=1,2,3$. So b_{ij} can be interpreted as the market share of country j in country i when all production prices expressed in the same currency are equal.

Equations (22), (23) and (24) define the demand in good produced by each country, which is equal to the effective production of this good. It includes an autonomous demand, which sums Government consumption, investment and net exports to non-modelled countries. So, Government and firms are assumed to use only domestic good.

Equations (25) to (27) are Cobb-Douglas production functions with exogenous capital. α represents the share of labour income in the value added of firms. Equations (27) to (30) determine the demand for labour as resulting from the equality between the labour cost and the marginal productivity of labour. Equations (31) to (33) are wages setting equations and can be interpreted as "quasi supply" functions of labour. Increases in productivity are fully integrated in wages. The first term of the right-hand side represents the effects of wage bargaining when the consumption and the production price do not grow by the same amount. If the consumption price increases faster, workers will ask for as high a rise in wages to protect the purchasing power of their earnings, but firms will oppose because the price of their output increases by less and they will not want to reduce their profit. The last right-hand term represents the effects from the partial indexation of nominal wages. So, an increase in prices will reduce the real cost of labour. Indexation rests for proportion φ_4 on consumption price and for proportion $1-\varphi_4$ on production price.

Equations (34) to (36) represent the adjustment mechanism between supply and demand on the goods markets. In each country a movement in prices will correct an imbalance on this market. In the steady state this market is in equilibrium. Parameter λ represents the speed of adjustment between supply and demand. Q_i represents potential output. It differs from effective production equal to the total demand for the domestic good, and is the reference for the demand of labour and for the computation of the productivity term which appears in the wage equation. As these three equations relates the current output gap to last observed inflation, the reaction functions of the central banks, which are interpreted in this paper in terms of targeting inflation, could also be interpreted as Taylor's rules[3].

If international borrowing were impossible, the nominal income of country *i* would be equal to the sum of its consumption $pc_i C_i$ and of its autonomous demand $p_i G_i$. The availability of foreign finance breaks, at least temporarily, the link between national production and spending (equations (37) and (38)). To simplify, we will assume that foreign indebtedness is wholly libelled in dollars. France and Germany are charged the US interest rate plus their risk premium for their foreign debt. It is sufficient to write these accounting identities for Germany and France. The equation for the US would be redundant because of Walras identity. (39) represents the equilibrium of the capital market: the debts by some countries are balanced by the assets of the other countries. The last equations define technical variables, which will be useful to implement the simulations of the model.

Two comments can be given on the homogeneity of the model relatively to nominal values. First, if the same positive number multiplied the six prices in the model, the equations would still be satisfied. We would get the same result if we multiplied the exchange rate by a positive number and if we divided the two American prices by the same number.

The balanced growth path

The model allows for the definition of a balanced growth path. Let the long run growth rate of technical progress be γ. It is assumed to be the same in the three countries. Then their natural growth rate is: $g = (1+\gamma)\ (1+n)^{\alpha} - 1$. The growth rates of the variables of the model along the balanced growth path are given in the following table:

Variables	Growth rates along the balanced growth path
p_1, p_2, pc_1, pc_2	$\overline{\pi}$
p_3, pc_3	$\overline{\pi_3}$
D_j, C_j, C_{ij}, Q_j, for $j = 1,2,3$ and $i = 1,2,3$	g

Variables	Growth rates along the balanced growth path
L_j for $j=1,2,3$	n
ω_j for $j=1,2,3$	$\dfrac{1+g}{1+n}$
e	$\dfrac{1+\overline{\pi}}{1+\overline{\pi}_3}-1$
i_j for $j=1,2,3$	0

The equation system, which determines the initial values of the balanced growth path, is given in ***Appendix 1***. This system will be solved in the rest of this section.

The consumption equations in the steady state are:

(7s), (8s) $$\left(\frac{1+g}{1+n}\right)^{\rho}=\left(\frac{1+i_j}{1+\beta}\right)\frac{1}{\left(1+\pi^s\right)},\ j=1,2$$

(9s) $$\left(\frac{1+g}{1+n}\right)^{\rho}=\left(\frac{1+i_3}{1+\beta}\right)\frac{1}{\left(1+\pi_3^s\right)}$$

The real interest rates in the three countries can easily be deduced from the previous equations:

$$1+r_j=\frac{1+i_j}{1+\pi^s}=\left(1+\beta\right)\left(\frac{1+g}{1+n}\right)^{\rho}=\frac{1+i_3}{1+\pi_3^s}=1+r_3\ ,\ j=1,2$$

Real interest rates are the same in the three countries. As under fixed exchange rates inflation rates cannot permanently differ, nominal interest rates are equal between France and Germany. Now, look at the reaction functions of the two central banks, in the steady state, (3s) and (4s):

(3s) $$(1+i_1)^{0.6}(1+i_2)^{0.4}=(1+\overline{r})(1+\overline{\pi})+1.5\ \left(\pi^s-\overline{\pi}\right)$$

(4s) $$1+i_3=\left(1+\overline{r}_3\right)\left(1+\overline{\pi}_3\right)+1.5\ \left(\pi_3^s-\overline{\pi}_3\right)$$

By using previous results this system becomes:

$$\begin{cases}\dfrac{1+i_j}{1+\pi^s}=1+r=\dfrac{\left(1+\overline{r}\right)\ \left(1+\overline{\pi}\right)}{1+\pi^s}+1.5\ \left(\dfrac{\pi^s-\overline{\pi}}{1+\pi^s}\right)\\ \dfrac{1+i_3}{1+\pi_3^s}=1+r=\dfrac{\left(1+\overline{r}_3\right)\ \left(1+\overline{\pi}_3\right)}{1+\pi_3^s}+1.5\ \left(\dfrac{\pi_3^s-\overline{\pi}_3}{1+\pi_3^s}\right)\end{cases};\ j=1,2$$

Actually, central banks can freely choose only one parameter: $(1+\bar{r})(1+\bar{\pi})-1.5\ \bar{\pi}$ for the ECB and: $(1+\bar{r}_3)(1+\bar{\pi}_3)-1.5\ \bar{\pi}_3$ for the Fed. This parameter was decomposed just to reach some interesting economic interpretation. With this decomposition, the target inflation rate and the steady state one are equal. The target real interest rate is equal to the balanced growth real interest rate. Thus: $\pi^s = \bar{\pi}$, $\pi_3^s = \bar{\pi}_3$ and $r = \bar{r} = \bar{r}_3$. Then, nominal interest rates are given by equations:

$$i_1 = i_2 = (1+r)\ (1+\bar{\pi}) \text{ and } i_3 = (1+r)\ (1+\bar{\pi}_3)$$

Now introduce the equations (1s) and (2s):

$$(1s) \qquad (1+i_1)(1+\bar{\pi}_3) = \left[1+i_3+0.01\ \left(\frac{F_1}{p_1Q_1}-\frac{F_3}{p_3Q_3}\right)\right](1+\bar{\pi})$$

$$(2s) \qquad (1+i_2)(1+\bar{\pi}_3) = \left[1+i_3+0.01\ \left(\frac{F_2}{p_2Q_2}-\frac{F_3}{p_3Q_3}\right)\right](1+\bar{\pi})$$

The equality of the three real interest rates to one another and the equality of the steady state inflation rates to their targets, imply that the rates of foreign indebtedness in the three countries are equal. Moreover, as the sum of the foreign debts in the three countries is zero (equation 39s), we get that the three foreign debts F_j, $j=1,2,3$, are zero.

Let p_1, p_2 and ep_3 be fixed at arbitrary levels. Then equations (10s), (11s) and (18s) determine pc_1, pc_2 and epc_3. The six equations (28s) to (33s) determine L_j and ω_j for $j=1,2,3$. The three equations (25s), (26s) and (27s) determine Q_j for $j=1,2,3$. Equations (34s), (35s) and (36s) determine D_j for $j=1,2,3$. The twelve equations (12s) to (17s) and (19s) to (24s) determine C_i and C_{ij} for $i, j=1,2,3$. Finally, equations (37s) and (38s) constraint the choices of the three variables, which initially were arbitrarily chosen. For example, ep_3 can be chosen freely, and these two equations determine p_1 and p_2.

Thus, there is a double nominal indeterminacy in the steady state (McCallum (1986)): the production price in the United States p_3 and the exchange rate of the euro relative to the dollar e. This indeterminacy does not extend to real variables, the steady state values of which are perfectly determined. Let us try to understand the intuition of nominal indeterminacy[4].

A reaction function of the central bank, which is wholly forward looking, will leave undetermined the level of the path of prices

Consider a real neo-classical growth model of a closed economy, with a representative household who optimises inter-temporally. This model determines the path of the real interest rate r. Assume that the real part of the economy is stabilised on its long run path and so that r stays unchanged over time. Now let us add a nominal part to the model. To do that we introduce a price level p and a nominal interest rate i. So, we have two new variables. We also have the accounting identity:

$$1+i=(1+r)\frac{p_{+1}}{p}.$$

Introduce the reaction function of the central bank, which targets the expected inflation rate:

$$(1+i)=(1+r)(1+\overline{\pi})+1.5\left(\frac{p_{+1}}{p}-1-\overline{\pi}\right)$$

$\overline{\pi}$ is the target inflation rate. Denote π^s the steady state inflation rate and i^s the steady state nominal interest rate. We deduce from the three last equations: $\pi^s=\overline{\pi}$, $1+i^s=(1+r)(1+\overline{\pi})$, and: $1+i=(1+r)\frac{p_{+1}}{p}=(1+r)(1+\pi^s)+1.5(\frac{p_{+1}}{p}-1-\pi^s)$.

Thus, if: $r\neq 0.5$, then: $p=p_{+1}/(1+\pi^s)$

So, the model determines inflation and leaves undetermined the level of the path of prices.

Now, assume that the central bank partly targets the observed inflation rate:

$$(1+i)=(1+r)(1+\overline{\pi})+1.5\left(\zeta\frac{p}{p_{-1}}+(1-\zeta)\frac{p_{+1}}{p}-1-\overline{\pi}\right)., \text{ with: } 0<\zeta\leq 1$$

We have the same steady state as before for the inflation and the nominal interest rates. The price dynamics is given by:

$$\frac{p}{(1+\pi^s)p_{-1}}-1=\frac{1+r-1.5(1-\zeta)}{1.5\zeta}\left(\frac{p_{+1}}{(1+\pi^s)p}-1\right)$$

If $1.5>1+\overline{r}$, this *forward* equation has a unique solution: $p=(1+\pi^s)p_{-1}$

If the simulation starts at time 0, the price at time -1 is given and determined the level of the path of prices. So, if the central bank is partly *backward looking*, there will be an hysteresis in the price level: its steady state value is undetermined, but its dynamic path is determined by the price level inherited from the past and which plays the role of the nominal anchor.

In the model of this paper, the degree of indeterminacy in the steady state and of the hysteresis, is equal to 2, which is the number of central banks following independent inflation targets. This result depends on the fact that central banks do not put any nominal anchor in their reaction function such as price levels, money supply levels or nominal exchange rates targets. The model of this paper does not assume the decomposability of the previous example from the real side of the economy to its nominal part. However, simulations show that if central banks are not backward looking enough in their targets, or if they do not react sufficiently strongly to changes in inflation, the model will have infinity of path solutions for its nominal variables.

Calibration

The calibration of the model determines the values of its parameters and of its variables in the reference steady state. The values of some parameters are selected *a priori*, according to the usual results of econometric studies. The values of some variables in the reference state are put equal to their observed values in 1999. The values of the other variables and parameters are determined by the equations of the model in the steady state and by the assumptions which define a symmetrical world. First give the list of the parameters, the values of which were *a priori* fixed:

Parameters	**Values**
g	2.5%
n	1%
ρ	0.5
α	0.7
φ_1	0.2
φ_2	0.3
φ_3	0.1
φ_4	0
σ	0.7
λ_{adj}	0.41

The substitution elasticity between national and foreign goods is put equal to: $\sigma + 1 = 1.7$. It implies a price elasticity of imports with the same value. It is higher than those given by macro-econometric studies of international trade, which are often less than 1. However, it is consistent with the results of micro-econometric studies, especially when they correct the endogeneity bias by using instrumental variables.

2.5% and 1% per year are likely growth rates of GDP and active population in the long run. The share of labour income in output α is put to 0.7. The parameters of the wage curve φ_1, φ_2 and φ_3 were taken from the econometric study by Guichard and Laffargue (2001). Finally, λ has the same value as in the model of the European Commission Quest II.

Recently observed inflation rates in Germany and France on one hand and in the United States on the other hand, determined the target inflation rates in the model: $\bar{\pi} = \pi = 2\%$, $\bar{\pi}_3 = \pi_3 = 3\%$. The real interest rate is assumed to be the same in the three countries and equal to 5%. Equations (7s), (8s) and (9s) determine the discount rate of households. Nominal interest rates are given by equations (3s) and (4s). The values of production in the three countries and of employment, active population and consumption in Germany are taken from OECD national accounts for 1999. The exchange rate between the euro and the dollar was put to 1, and the six prices of the model are equal to 1 in the reference steady state.

In this paper countries are assumed to possess the property of structural symmetry. Under this property, the linear approximation of the model can be decomposed into three independent sub-models. Loosely speaking structural symmetry means that countries only differ by their sizes. The *first* component of the precise definition is that in the reference steady state, the shares of the autonomous demand and of households' consumption are the same in the three countries: $G_i = \varphi Q_i$, $C_i = (1-\varphi)Q_i$, $i = 1,2,3$. Moreover the reference real costs of labour ω_j are the same in the three countries. We also assume that the reference ratios of employment in the three countries are proportional to the ratios of their outputs. $L_j = lQ_j$, $j = 1,2,3$. The reference unemployment rates are the same in the three countries.

The *second* component of the definition is that in the reference steady state bilateral trade flows are balanced and proportional to the products of the outputs of both partners, with a universal coefficient κ.

$$C_{12} = C_{21} = \kappa Q_1 Q_2\ ,\ C_{13} = C_{31} = \kappa Q_1 Q_3,\ C_{23} = C_{32} = \kappa Q_2 Q_3.$$

$(C_{12} + C_{13})/Q_1$ represents the ratio between imports and production in country 1, that is the rate of openness of this country. It is equal to κ time the size of the rest of the world $Q_2 + Q_3$. So, the larger a country is, the smaller the rest of the world and its rate of openness are. In the same way the rate of openness of country i to country j is proportional to the size of country j. It is independent of the size of country i: the US are as open to France as Germany, that is both countries import from France the same proportion of their domestic production. The second component of the definition of structural symmetry is especially unrealistic. Bilateral trade flows do not only depend on the size of both partners, but also of their distance, of their membership or not to a common free trade area, etc.[5].

Finally, the 9 b_{ij} must verify the system:

$$
\begin{aligned}
&1 = b_{j1} + b_{j2} + b_{j3} \qquad j = 1,2,3 \\
&C_{i1} = b_{i1}C_i,\ i = 1,2,3 \\
&C_{i2} = b_{i2}C_i,\ i = 1,23 \\
&C_{i3} = b_{i3}C_i\ ,\ i = 1,2,3
\end{aligned}
$$

Let be:

$$(1-\varphi)b_{21} = \alpha Q_1,\ (1-\varphi)b_{31} = \alpha Q_1,\ (1-\varphi)b_{12} = \alpha Q_2,\ (1-\varphi)b_{32} = \alpha Q_2,$$
$$(1-\varphi)b_{13} = \alpha Q_3,\ (1-\varphi)b_{23} = \alpha Q_3$$

After the elimination of the two proportionality coefficients α and φ, we get 5 relationships between the b_{ij} relative to two different countries:

$$b_{12} = b_{32},\ b_{13} = b_{23},\ b_{31} = b_{21},\ b_{21}Q_2 = b_{12}Q_1,\ b_{31}Q_3 = b_{13}Q_1.$$

From these 5 relationships we can deduce the redundant equation: $b_{32}Q_3 = b_{23}Q_2$.

So, we can freely choose one of the b_{ij}. The five previous equations, and the normality conditions, which put to 1 the sums of some of these coefficients, determine the eight others b_{ij}. We put: $b_{12} = 0.05$.

The *third* and last component of the assumption of structural symmetry is that all the elasticity parameters in the equations have the same values across countries. As inflation is higher in the US than in Europe, we deduce from the wage equation that φ_{03} is smaller than φ_{01} and φ_{02} which are equal to each other. We also have: $A_j = l^{-\alpha}Q_j^{1-\alpha}$ and: $\omega_j = \alpha / l$.

The main values of the reference steady state are in the following table:

Year 1999	Germany	France	United States
GDP in billions dollars	1760,3	1168,1	4953,9
Consumption in billions dollars	1010	1010*1168,1/1760,3	1010*4953,9/1760,3
Active population	38,5	38,5*1168,1/1760,3	38.5*4953,9/1760,3
Employment	34,9	34,9*1168,1/1760,3	34.9*4953,9/1760,3

Simulation problems and writing of the model in reduced variables

To simulate the model, the initial values of the lagged endogenous variables are fixed to their reference values. Then we introduce a series of shocks on the exogenous variables. These shocks were unanticipated before the time when the simulation starts. Finally, we fix the terminal values of the endogenous variables with a lead at their steady state values. These are computed by using the steady state model after the introduction of the permanent shocks on the exogenous variables. Unfortunately, nominal variables are undetermined in the steady state model because of its double hysteresis, and some of these variables appear with a lead in the dynamic model so their terminal values have to be known

There is a solution to this problem. Assume X to be a variable appearing with a lead of 1 period. We do not know its terminal value, but we know the terminal value of its growth rate: $x = X / X_{-1}$. So, we substitute the leads of X by: $X_{+1} = Xx_{+1}$. Then, variable X will only appear in a contemporaneous or lagged form. So it is useless to know its terminal value. x will appear with a lead. However, its terminal value is known. Actually, we applied this method to all the lead variables of the model (nominal and real) which made useless the computation of its terminal steady state[6]. To do that we had to introduce the 7 variables of type x, which were presented above gc_j, $\inf_j$ and def.

Blanchard and Kahn (1980) established the conditions for the existence and the uniqueness of a solution path for a model that has a steady state. Laffargue (2000) extended these conditions and discussed the problem of hysteresis. *Dynare* software checks for these conditions then can simulate a model having this feature. The model presented here has no steady state, but has balanced growth paths, with growth rates, which can differ between variables. So, the model was rewritten in reduced variables, that is with variables corrected of their long run trend. Then it has a steady state and we can directly apply Blanchard and Kahn's conditions. There are 46 equations.

(1r, 2r) $$\left(1+i_j\right)\left(1+\bar{\pi}_3\right)=\left[1+i_3+0.01\left(\frac{eF_j}{p_jQ_j}-\frac{F_3}{p_3Q_3}\right)\right]def_{+1}\left(1+\bar{\pi}\right),\ j=1,2$$

(3r) $$(1+i_1)^{\bar{Q}_1/(\bar{Q}_1+\bar{Q}_2)}\left(1+i_2\right)^{\bar{Q}_2/(\bar{Q}_1+\bar{Q}_2)}=(1+\bar{r})(1+\bar{\pi})+1.5\ (\pi-\bar{\pi})$$

(4r) $$1+i_3=(1+\bar{r})(1+\bar{\pi}_3)+1.5\ (\pi_3-\bar{\pi}_3)$$

(5r)*(11)* $$1+\pi_3=\left(1+\bar{\pi}_3\right)pc_3 / pc_{3,-1}$$

(6r) $$\frac{1+\pi}{1+\bar{\pi}}=\left(pc_1 / pc_{1,-1}\right)^{\bar{Q}_1/(\bar{Q}_1+\bar{Q}_2)}\left(pc_2 / pc_{2,-1}\right)^{\bar{Q}_2/(\bar{Q}_1+\bar{Q}_2)}$$

(7r), (8r) $$\left(\frac{(1+g)\ gc_{j,+1}}{(1+n)}\right)^{\rho}=\left(\frac{1+i_j}{1+\beta}\right)\frac{1}{\inf_{j,+1}(1+\bar{\pi})},\ j=1,2$$

(9r) $$\left(\frac{(1+g)\ gc_{3,+1}}{(1+n)}\right)^{\rho}=\left(\frac{1+i_3}{1+\beta}\right)\frac{1}{\inf_{3,+1}(1+\bar{\pi}_3)}$$

(10r), (11r) $$pc_j^{-\sigma}=b_{j1}p_1^{-\sigma}+b_{j2}p_2^{-\sigma}+b_{j3}\left(ep_3\right)^{-\sigma},\ j=1,2$$

(12r) to (15r) $$p_jC_{ij}=b_{ij}\left(p_j / pc_i\right)^{-\sigma}pc_iC_i,\ i=1,2,\ j=1,2$$

(16r), (17r) $$p_3eC_{i3}=b_{i3}\left(p_3e / pc_i\right)^{-\sigma}pc_iC_i,\ i=1,2$$

(18r) $$pc_3^{-\sigma}=b_{31}\left(\frac{p_1}{e}\right)^{-\sigma}+b_{32}\left(\frac{p_2}{e}\right)^{-\sigma}+b_{33}p_3{}^{-\sigma}$$

(19r, 20r) $$\left(p_j / e\right)\ C_{31}=b_{3j}\left(p_j / epc_3\right)^{-\sigma}pc_3C_3,\ j=1,2$$

(21r) $$p_3 C_{33} = b_{33}\left(p_3 / pc_3\right)^{-\sigma} pc_3 C_3$$

(22r) to (24r) $$D_j = G_j + C_{1j} + C_{2j} + C_{3j}\,,\ j = 1,2,3$$

(25r) to (27r) $$Q_j = A_j L_j^{\alpha},\ j = 1,2,3$$

(28r) to (30r) $$\alpha A_j L_j^{\alpha-1} = \omega_j\,,\ j = 1,2,3$$

(31r), (32r) $$\ln(\omega_j L_j / Q_j) = \varphi_{0j} + \varphi_1 \ln\left(\frac{pc_j}{p_j}\right) + \varphi_2 \ln\left(\frac{L_j}{N_j}\right) - \varphi_3 \ln\left(\frac{p_j\left(1+\overline{\pi}\right)}{p_{j,-1}}\right),\ j = 1,2$$

(33r) $$\ln(\omega_3 L_3 / Q_3) = \varphi_{03} + \varphi_1 \ln\left(\frac{pc_3}{p_3}\right) + \varphi_2 \ln\left(\frac{L_3}{N_3}\right) - \varphi_3 \ln\left(\frac{p_3\left(1+\overline{\pi}_3\right)}{p_{3,-1}}\right)$$

(34r) to (36r) $$\ln(D_j / Q_j) = \frac{1}{\lambda}\ln\left[\frac{p_j}{p_{j,-1}}\right],\ j = 1,2,3$$

(37r), (38r) $$p_i D_i + e\left(F_i - \frac{F_{i,-1}}{\left(1+\overline{\pi}_3\right)\left(1+g\right)}\right) = pc_i C_i + p_i G_i + \left[i_3 + 0.01\left(\frac{eF_i}{p_i Q_i} - \frac{F_3}{p_3 Q_3}\right)\right]\frac{eF_{i,-1}}{\left(1+\overline{\pi}_3\right)\left(1+g\right)},\ i = 1,2$$

(39r) $$F_1 + F_2 + F_3 = 0$$

(40r) to (42r) $$\inf_j = pc_j / pc_{j,-1},\ j = 1,2,3$$

(43r) $$def = e / e_{-1}$$

(44r) to (46r) $$gc_j = C_j / C_{j,-1},\ j = 1,2,3$$

Blanchard and Kahn's conditions are satisfied. More precisely there are 10 nonzero eigenvalues. They are equal to: 0.2801, 0.6894, 0.9474, 0.9478, 1, 1, 1.0770, 1.0796, 1.1288 et 1.1897. The two eigenvalues equal to 1 come from the double hysteresis commented above. There are 4 eigenvalues larger than 1 and 4 non-redundant lead variables.

Decomposition of the model

Under the assumption of structural symmetry, the dynamics of the model can be locally decomposed into three dynamics determined by as many independent models. These models can be given simple economic interpretation. To make this decomposition, first we will write the equations of the linear approximation of the model in the neighbourhood of its reference state. Then, we will identify the three models: the world model, the model of the difference between Europe and the United States and the model of the difference between Germany and the rest of the world.

Linear approximation of the model

This approximation is computed in the neighbourhood of the reference steady state of the model. Variables with a line above are put at their values in the reference steady state. Variables with no line above are put at the difference between their current value and their reference value.

(1r, 2r) $$\left(1+\bar{\pi}_3\right)i_j = \left(1+\bar{\pi}\right)\left[i_3 + 0.01\left(\frac{F_j}{\bar{Q}_1} - \frac{F_3}{\bar{Q}_3}\right) + (1+\bar{i}_3)def_{+1}\right],\ j=1,2$$

(3r) $$\frac{\bar{Q}_1}{\bar{Q}_1+\bar{Q}_2}i_1 + \frac{\bar{Q}_2}{\bar{Q}_1+\bar{Q}_2}i_2 = 1.5\pi$$

(4r) $$i_3 = 1.5\ \pi_3$$

(5r) $$\pi_3 = \left(1+\bar{\pi}_3\right)(pc_3 - pc_{3,-1})$$

(6r) $$\frac{\pi}{1+\bar{\pi}} = \frac{\bar{Q}_1}{\bar{Q}_1+\bar{Q}_2}\left(pc_1 - pc_{1,-1}\right) + \frac{\bar{Q}_2}{\bar{Q}_1+\bar{Q}_2}\left(pc_2 - pc_{2,-1}\right)$$

(7r) to (9r) $$\rho gc_{j,+1} = i_j / \left(1+\bar{i}_j\right) - \inf_{j,+1},\ j=1,2,3$$

(10r) (11r), $$pc_j = b_{j1}p_1 + b_{j2}p_2 + b_{j3}\left(e+p_3\right),\ j=1,2,$$

(12r) to (15r) $$p_j + C_i / b_{ij}\bar{C}_i = -\sigma\left(p_j - pc_i\right) + pc_i + C_i/\bar{C}_i,\ i=1,2,\ j=1,2$$

(16r), (17r) $$p_3 + e + C_{i3}/b_{i3}\bar{C}_i = -\sigma\left(p_3 + e - pc_i\right) + pc_i + C_i/\bar{C}_i,\ i=1,2$$

(18r) $$pc_3 = b_{31}\left(p_1 - e\right) + b_{32}\left(p_2 - e\right) + b_{33}p_3$$

(19r, 20r) $$p_j - e + C_{3j}/b_{3j}\bar{C}_3 = -\sigma\left(p_j - e - pc_3\right) + pc_3 + C_3/\bar{C}_3,\ j=1,2$$

(21r) $$p_3 + C_{33}/b_{33}\bar{C}_3 = -\sigma\left(p_3 - pc_3\right) + pc_3 + C_3/\bar{C}_3$$

(22r) to (24r), $$D_j = G_j + C_{1j} + C_{2j} + C_{3j},\ j=1,2,3$$

(25r) to (27r) $$Q_j/\bar{Q}_j = A_j/\bar{A}_j + \alpha L_j/\bar{L}_j,\ j=1,2,3$$

(28r) to (30r) $$A_j/\bar{A}_j + (\alpha-1)L_j/\bar{L}_j = \omega_j/\bar{\omega}_j,\ j=1,2,3$$

(31r) to (33r) $$\omega_j/\bar{\omega}_j + L_j/\bar{L}_j - Q_j/\bar{Q}_j = \varphi_1\left(pc_j - p_j\right) + \varphi_2\left(L_j/\bar{L}_j\right) - \varphi_3\left(p_j - p_{j,-1}\right),\ j=1,2,3$$

(34r) to (36r) $$D_j - Q_j = \frac{\bar{Q}_j}{\lambda}(p_j - p_{j,-1}),\ j=1,2,3$$

(37r), (38r) $$D_i + \bar{Q}_i p_i + \left(F_i - \frac{F_{i,-1}}{\left(1+\bar{\pi}_3\right)(1+g)}\right) = \bar{C}_i pc_i + C_i + \bar{G}_i p_i + G_i + \bar{i}_3 \frac{F_{i,-1}}{\left(1+\bar{\pi}_3\right)(1+g)},\ i=1,2$$

(39r) $$F_1 + F_2 + F_3 = 0$$

(40r) to (42r) $$\inf_j = pc_j - pc_{j,-1},\ j=1,2,3$$

(43r) $$def = e - e_{-1}$$

(44r) to (46r) $$gc_j = (C_j - C_{j,-1})/\bar{C}_j,\ j=1,2,3$$

This linear approximation can be decomposed into three independent dynamic systems, which can receive economic interpretation.

The world model

Define the reference world output as: $\overline{Q}_M = \overline{Q}_1 + \overline{Q}_2 + \overline{Q}_3$. $\overline{Q}_i / \overline{Q}_M$ represents the relative size of country i in the world economy. The reference world consumption and employment $\overline{C}_M$ and $\overline{L}_M$ are the sums of the reference values of these variables over the three countries. The reference world labour cost is: $\overline{\omega}_M = \alpha \overline{Q}_M / \overline{L}_M$. The reference world productivity is: $\overline{A}_M = \overline{Q}_M / \overline{L}_M^{\alpha} = \overline{Q}_M^{1-\alpha} / l^{\alpha}$.

Endogenous variables (10)

C, D, L and Q are the sums of the variables with the same names over the three countries. gc, inf, ω, p et pc are the averages of the variables with the same names over the three countries, weighted by the relative sizes of the countries. Finally:

$$i = \frac{(\overline{Q}_1 i_1 + \overline{Q}_2 i_2)/(1+\overline{\pi}) + \overline{Q}_3 i_3 /(1+\overline{\pi}_3)}{\overline{Q}_M}.$$

Exogenous variables

G is the sum of the variables with the same name over the three countries.

$$A = (\overline{Q}_1 A_1 / \overline{A}_1 + \overline{Q}_2 A_2 / \overline{A}_2 + \overline{Q}_3 A_3 / \overline{A}_3)\overline{A}_M / \overline{Q}_M = (\overline{Q}_1^{\alpha} A_1 + \overline{Q}_2^{\alpha} A_2 + \overline{Q}_3^{\alpha} A_3)/\overline{Q}_M^{\alpha}$$

Equations (10)

(1) $i = 1.5(pc - pc_{-1})$

(2) $\rho gc_{+1} = i/(1+\overline{r}) - \inf_{+1}$

(3) $pc = p$

(4) $D = G + C$

(5) $Q/\vec{Q}_M = A/\overline{A}_M + \alpha L/\overline{L}_M$

(6) $A/\overline{A}_M + (\alpha - 1)L/\overline{L}_M = \omega/\overline{\omega}_M$

(7) $\omega/\overline{\omega}_M + L/\overline{L}_M - Q/\overline{Q}_M = \varphi_2 L/\overline{L}_M - \varphi_3(p - p_{-1})$

(8) $\dfrac{D-Q}{\overline{Q}_M} = (p - p_{-1})/\lambda$

(9) $\inf = pc - pc_{-1}$

(10) $gc = (C - C_{-1})/\overline{C}_M$

This system of equations can be interpreted as representing the linear approximation of a neo-classical model of a closed economy. (5) is the production function which determines potential output. (6) is the demand for labour (the marginal productivity of labour computed for potential output, is equal to labour cost). (7) is the "quasi-supply" of labour (wage equation). (4) defines effective demand. (8) is the equilibrium of the goods market (the stickiness of price allows for some transitory disequilibrium between supply and demand). (2) and (10) represent consumption behaviour which rests upon an inter-temporal optimisation of households over an infinite horizon. (1) is the reaction function of the central bank.

The nonzero eigenvalues are equal to 1 and 1.1288. The first one results from the hysteresis of the price level and has no effect on real variables nor on the rates of inflation and interest. The second eigenvalue is equal to one of the eigenvalues of the full model. As the model has no eigenvalue smaller than one, the adjustment of the world economy to an unanticipated shock takes place instantaneously. The wage equation and the equilibrium between effective demand and potential output, include lagged price terms. Although these terms represent nominal rigidities in the model, it has no Keynesian properties. An unanticipated permanent increase in government consumption, that is in autonomous demand G by 1 thousand million dollars, implies an immediate and permanent decrease in households' consumption, and has no effect on activity nor prices. Indeed, consumption is equal to the permanent income of this agent. An increase in Government consumption will require increases in taxes, which, by Ricardian equivalence, will reduce permanent income by the same amount. However, a raise in G at a future date anticipated in the present time will affect activity between this time and the date of the shock.

A permanent increase in the productivity of firms A by 0.001, will imply an immediate increase in households' consumption, demand and production by 34.5 thousand million dollars. Employment does not move, but the cost of labour increases by 0.44%. Price and interest rate do not move.

The difference model between Europe and the US

The reference European production is defined as the sum of the reference productions in Germany and in France: $\overline{Q}_E = \overline{Q}_1 + \overline{Q}_2$. The reference European consumption and employment $\overline{C}_E$ and $\overline{L}_E$ are defined in the same way. $\overline{Q}_E / \overline{Q}_3$ represents the ratio of the sizes of Europe to the US. $\overline{Q}_1 / \overline{Q}_E$ and $\overline{Q}_2 / \overline{Q}_E$ are the relative sizes of Germany and France in Europe. The reference European cost of labour is: $\overline{\varpi}_E = \alpha \overline{Q}_E / \overline{L}_E$. The reference European productivity is: $\overline{A}_E = \overline{Q}_E / \overline{L}_E^{\alpha} = \overline{Q}_E^{1-\alpha} / l^{\alpha}$.

Endogenous variables (14)

C, D, F, L and Q are the sums of the variables with the same names in the two European countries, minus the variable with the same name in the US time the ratio of the sizes between the US and Europe. DN is total households' consumption in European goods minus the demand by households of American goods time the ratio of the sizes between the US and Europe. gc, inf, ω, p and pc are the averages of the variables with the same names in the two European countries weighted by the relative sizes in Europe of these countries, minus the variable with the same name in the United States. def and e are equal to their values in the full model. Finally: $i = \frac{1}{1+\bar{\pi}} \frac{\bar{Q}_1 i_1 + \bar{Q}_2 i_2}{\bar{Q}_E} - \frac{1}{1+\bar{\pi}_3} i_3$

Exogenous variables

G is the sum of the variables with the same names in the two European countries, minus the variable with the same name in the US time the ratio of the size of the US and Europe.

$$A = \left(\frac{\bar{Q}_1 A_1 / \bar{A}_1 + \bar{Q}_2 A_2 / \bar{A}_2}{\bar{Q}_E} - A_3 / \bar{A}_3 \right) \bar{A}_E = (\bar{Q}_1^{\alpha} A_1 + \bar{Q}_2^{\alpha} A_2 - \frac{\bar{Q}_E}{\bar{Q}_3} \bar{Q}_3^{\alpha} A_3) / \bar{Q}_E^{\alpha}$$

Define: $b = b_{13} + b_{31} + b_{32}$

Equations (14)

(1) $i = \frac{1}{1+\bar{\pi}_3}[0.01F / \overline{Q_E} + (1+\bar{i}_3) def_{+1}]$

(2) $i=1.5\left(pc-pc_{-1}\right)$

(3) $\rho gc_{+1} = i/(1+\bar{r}) - \inf_{+1}$

(4) $pc = (1-b)p + be$

(5) $DN = (1-b)C + \bar{C}_E (1+\sigma)(2-b)(pc-p)$

(6) $D = G + DN$

(7) $Q/\bar{Q}_E = A/\bar{A}_E + \alpha L/\bar{L}_E$

(8) $A/\bar{A}_E + (\alpha-1)L/L_E = \omega/\overline{\omega_E}$

(9) $\omega/\overline{\omega_E} + L/\bar{L}_E - Q/\bar{Q}_E = \varphi_1(pc-p) + \varphi_2 L/\bar{L}_E - \varphi_3(p-p_{-1})$

(10) $\frac{D-Q}{\overline{Q_E}} = (p-p_{-1})/\lambda$

(11) $D + \vec{C}_E(p-pc) + F - \frac{(1+\bar{i}_3)F_{-1}}{(1+\bar{\pi}_3)(1+g)} = C + G$

(12) $\inf = pc - pc_{-1}$

(13) $def = e - e_{-1}$

(14) $gc = (C - C_{-1})/\bar{C}_E$

This system of equations can be interpreted as representing the linear approximation of a model of a small open economy under a flexible exchange rate regime. (7) is the production function which determines potential output. (8) is the demand for labour. (9) is the quasi -supply of labour. (6) defines the effective demand of domestic goods as the sum of autonomous demand and of the households' demand for these goods. Autonomous demand wholly bears on these goods. Total household's consumption C is allocated between demand for domestic goods DN and net imports (equation (5)). The consumption price is the weighted average of the production price and the exchange rate (equation (4)). (3) (12) and (14) represent households' behaviour. (2) is the reaction function of the central bank. (1) is the uncovered interest rate parity equation, with a risk premium depending on foreign indebtedness. (11) is an accounting identity which gives the dynamic of foreign indebtedness.

The nonzero eigenvalues are equal to 0.2801, 0.9564, 1, 1.0926 and 1.1906. The unitary eigenvalue is associated with the hysteresis of the price level and the exchange rate. The other eigenvalues are near those of the full model (0.2801, 0.9474, 1.0770, 1.1897). The presence of eigenvalues smaller than 1 implies a dynamics less trivial than for the world model, presenting some sluggishness after an unanticipated permanent shock. To give this dynamics an interpretation, we will start by investigating the long run steady state of the model.

Long term equilibrium

In the long run the nominal interest rate is equal to the real interest rate (determined by the discount rate of households) plus the inflation target. If this target does not change, the nominal interest rate remains unchanged across all simulations. Its difference with the interest rate in the rest of the world is an increasing function of foreign debt. As both these rates are fixed, foreign indebtedness will stay at its zero reference level for all the shocks that will be investigated[7].

The long run (steady sate) model includes a first bloc of supply equations, which are the production function (7), the demand for labour (8) and the "quasi-supply" of labour (9). If there is no effect of the real exchange rate on the determination of labour cost ($\varphi_1 = 0$), this bloc determines the supply of national good Q, employment L and the real cost of labour ω.

In the long run the current accounts balance is in equilibrium. As foreign debt F and its interest costs are zero in the steady state, the trade balance is also in equilibrium. That is the production of domestic goods is equal to national income and to national spending. The later is the sum of autonomous demand G and of households' consumption C. The first component only bears on domestic goods of price p. The second component bears on a mix of domestic and rest of the world goods, and its price is the consumption price index pc. The real exchange rate $pc-p$ appears in the equality between nominal income and spending:

(11) $Q=\overline{C}_E(pc-p)+C+G$

Moreover, the output of domestic goods is equal to the demand for it. This demand is partly autonomous and partly comes from households. Their consumption is allocated between domestic and rest of the world goods, in relation with the value of the real exchange rate:

(5) $Q=G+(1-b)C+\overline{C}_E(1+\sigma)(2-b)(pc-p)$

The two last equations determine households' consumption and the real exchange rate. The hysteresis property of the model implies that its steady state leaves the price level undetermined. So, for an arbitrary output price p, two accounting equations give the nominal exchange rate e and the consumption price pc: equation (4) and relationship: $pc=p+(pc-p)$.

Thus, a permanent increase in autonomous demand by 1 thousand million dollars has no consequences for production, employment and the real cost of labour. As the demand for domestic goods increases, the real exchange rate appreciates. This raises the real income of households, which reduces the crowding out effects of households' consumption: it decreases by less than 1 thousand million dollars this time.

If there is some effect of the real exchange rate on labour costs, the appreciation of the first variable, will induce a decrease in labour costs, so an increase in output and employment.

Simulations show that in the long run, a permanent increase in autonomous demand by 1 thousand million dollars, will induce an increase in output by 113 million dollars, in employment by 3200 workers, an appreciation of the real exchange rate by 0.085%. Households' consumption will decrease by 750 million dollars. The production price increases by 0.042%, the consumption price decreases by 0.0042% and the nominal exchange rate appreciates by 0.0205%.

What will the effects of an increase in productivity A by 0.001, be? The supply bloc shows that production and labour costs will increase, and that employment will remain unchanged. The equality between income and spending, and the equilibrium of the domestic good market, determine the real exchange rate and households' consumption. It can easily be shown that the highest supply of the domestic goods will depreciate the real exchange rate and increase national consumption. Simulations give an increase in production by 15.3 thousand million dollars. Consumption increases by 12.7 thousand million dollars, consumption price increases by 0,070%, production price decreases by 0,072% (so the real exchange rate depreciates by 0,142%), the nominal exchange rate depreciates by 0,34%. The depreciation of the real exchange rate will induce a raise in labour costs, and employment will decrease by 54000 workers.

Short run equilibrium

To understand the nature of this equilibrium we will start by considering the extreme situation when production price is fixed, that is when the disequilibrium between potential output and effective demand does not disappear over time, and when labour cost is insensitive to changes in the real exchange rate ($\varphi_1 = 0$). Under these assumptions, the dynamic model can be decomposed into two independent blocs. The first is the supply bloc which includes, as before, equations (7), (8) and (9) and which determines potential domestic output Q, employment L and real labour cost ω. The second bloc represents demand and includes all the other equations, but equation (6). As demand is not any more connected to potential output, its long run level is now undetermined, as the level of its dynamic path. This implies a new hysteresis, one of the eigenvalues larger than 1 having become equal to 1. But what is more important is that in this block of equations, autonomous demand G does not appear. So, we can accept that a change in this variable will have no effect on the variables of the demand bloc such as households' consumption, the exchange rate, the interest rate and foreign debt. Autonomous demand appears in the last equation:

(6) $D = G + DN$

Thus, an increase in autonomous demand implies an increase by the same amount of effective production, without any other effect on the economy. As employment and wages are related to potential output which does not move, this means that the increase in effective production is reached without having to hire more labour or to bear more costs, and this in a permanent way.

On the other hand, an increase in productivity affects potential output and labour costs, which reach almost immediately their new long run values. However, there are no effects on effective demand, interest rate, foreign debt and exchange rate.

The model does not make such extreme assumption as the perfect rigidity of prices: the disequilibrium between potential output and effective demand disappears progressively, actually quickly, by a smooth change in the output price. Thus, simulations give in the short run, results intermediate between those got in the long run and the previous extreme Keynesian results. Actually, with the calibration made here, the results of the first periods are nearer the long run results than the Keynesian results. When autonomous demand increases, the short-run Keynesian multiplier is smaller than 1, (it is around 0.33), and it quickly decreases to its long run value of 0.11. More sticky prices will increase the value of the short run Keynesian multiplier (but it will remain under 1) and will slow the speed of its adjustment to its long run value.

In a similar way, the increase in productivity has a short run effect near enough its long term effect, and the convergence to this last value is fast. The dynamics of most variables is dominated by eigenvalue 0.2801, so adjustments are quick and monotonous. However, the nominal interest rate decreases immediately when autonomous demand increases, and increases immediately when productivity increases. However, the nominal interest rate, decreases immediately when autonomous demand increases, and increases immediately when productivity increases, before coming back to its reference value. The other transitory dynamics, associated with eigenvalue 0.9564, can be noticed clearly only for the dynamics of foreign debt.

The difference model between Germany and the rest of the world

The reference non-German production is the sum of the reference outputs in the United States and in France: $\overline{Q}_R = \overline{Q}_2 + \overline{Q}_3$. The non-German reference consumption and employment $\overline{C}_R$ and $\overline{L}_R$ are defined in the same way. $\overline{Q}_1 / \overline{Q}_R$ represents the ratio of size between Germany and the rest of the world. $\overline{Q}_2 / \overline{Q}_R$ and $\overline{Q}_3 / \overline{Q}_R$ represent the relative sizes of France and the US in the rest of the world.

Endogenous variables (13)

C, D, F, L and Q are the differences of the variables with the same names in Germany and in the rest of the world, these last multiplied by the ratio of the sizes of Germany and the rest of the world. DN is total households' consumption in German goods less this consumption in American and French goods time the ratio of sizes between Germany and the rest of the world. gc, $\inf$, ω, p and pc are the variables with the same names in Germany less their averages over France and the US weighted by the relative sizes of these two countries in the rest of the world. def is equal to its value in the full model time the ratio of the sizes of the US and the rest of the world. Finally: $i = \dfrac{i_1}{1+\pi} - \dfrac{\overline{Q}_2 \dfrac{i_2}{1+\pi} + \overline{Q}_3 \dfrac{i_3}{1+\pi_3}}{\overline{Q}_R}$

Exogenous variables

G is the difference between this variable for Germany and for the rest of the world, the latter multiplied by the ratio of the sizes of Germany and the rest of the world. e is equal to its value in the full model time the ratio of the sizes of the US and the rest of the world. In simulations, the model of the difference between Europe and the United States gives the changes in this variable. Finally:

$$A = \left(A_1 / \overline{A}_1 - \frac{\overline{Q}_2 A_2 / \overline{A}_2 + \overline{Q}_3 A_3 / \overline{A}_3}{\overline{Q}_R} \right) \overline{A}_1 = A_1 - \frac{\overline{Q}_2^{\alpha} A_2 + \overline{Q}_3^{\alpha} A_3}{\overline{Q}_1^{\alpha}} \frac{\overline{Q}_1}{\overline{Q}_R}$$

Equations (13) and comment

The equations of this model are the same as for the model of the difference between Europe and the United States, with two exceptions. First, equation (2), which represents the reaction of the central bank to changes in inflation, is no more present in the model: as the exchange rate follows an exogenous path there is no room left for an independent monetary policy. Secondly, index E, which represents Europe and appeared for some variables in the reference state of the previous model, is substituted by index 1 which identifies Germany.

The system of equations can be interpreted as the linear approximation of the model of a small open economy under an exogenous exchange rate. The previous model of the difference between Europe and the United States determines this rate. Thus, the decomposition of the dynamics we reach in this section is hierarchical rather than total.

The nonzero eigenvalues are equal to 0.6893, 0.9568 and 1.0962. They are near the eigenvalues of the full model (0.6894, 0.9478 and 1.0796). There is no more eigenvalue equal to 1: when the exchange rate is exogenous, hysteresis on prices disappears, prices in the rest of the world giving a nominal anchor.

To get an economic interpretation of the simulations of the model we can start by examining its long run equilibrium, that is its steady state. This state is exactly the same as for the model of the difference between Europe and the United States, with an exception relative to prices and the exchange rate. After having determined the real exchange rate as in the previous model, two accounting identities give the two prices of the model: equation (3) and identity: $p = pc - (pc - p)$. There is no more hysteresis and indetermination of the price level in the long run.

Simulations show that the long run effects of an increase in autonomous demand by 1 thousand million dollars on activity variables, are the same as with the previous model. The appreciation of the real exchange rate is higher and occurs through an increase in the production price by 0.0415% and in the consumption price by only 0.0275%.

In the short run the Keynesian multiplier is lower than 1, for the same reasons as for the previous model. However, it is higher than for this model (0.65), which is consistent with the results of Mundell and Fleming that fiscal policy is more efficient when the exchange rate is fixed than when it is floating. The dynamics of the last model is also slower, driven by the eigenvalue 0.6893. A possible explanation is that when the exchange rate is fixed, nominal rigidities bear not only on the production price, but also on the exchange rate.

An increase in productivity by 0.001 has the same long run effects on activity variables as for the model of the difference between Europe and the United States, but multiplied by $(\overline{Q}_1 / \overline{Q}_E)^{\alpha} \approx 0.7$. The dynamics is slower, the decrease in the production price is stronger, the consumption price decreases instead of increasing and the exchange rate does not move.

The consequence of devaluation that is of an increase in the exchange rate by 1% was also simulated. In the long run, the only effects are an increase in the production and the consumption price by 1%. The sluggishness of prices forbids this state to be reached immediately. In the short run consumption price increases more than production price. Then, the inflation rates on both prices are positive and decreasing over time. The movement in the consumption price induces, by inter-temporal arbitrage, an immediate increase in consumption, which dampens afterward before coming back to its reference level. The transitory depreciation of the real exchange rate increases real labour cost, and reduces employment in the short run. As households' consumption will for some time be less intensive in foreign good, which have become more expensive, effective production increases temporarily. The improvement in the trade balance initiates a temporary process of debt reduction.

Expressions of the variables of the full model in terms of the variables of the three sub-models

The results of the simulation of the full model are linear combinations of the results of simulations with the three sub-models of this section, at least for small shocks. The formulae allowing getting the results of the simulations of the full model out of the simulations of the sub-models are given in Appendix 2.

Two simulations with the model

The dynamic multipliers resulting from two unanticipated permanent shocks, will be computed and commented, first an increase in Government consumption in Germany, then an increase in productivity in the United States.

Permanent increase in Government consumption in Germany by 1 thousand million dollars

This policy increases autonomous demand by 1 thousand million dollars in each of the three sub-models of section 5. The exchange rate appreciates nominally by 0.0205% and really by 0.0085% in the model of the difference between Europe and the US. The appreciation of the euro occurs very quickly. The full model gives the same result (Graph 1.1).

Start by investigating the long run. Production and employment remain unchanged in the world model. In the model of the difference between Europe and the United States, the real appreciation of the exchange rate lowers labour cost and increases employment and potential output. So, with the full model, production increases in Europe and decreases in the United States by 77 million dollars (Graph 1.13). Production also increases in the difference model between Germany and the rest of the world. So the two difference models explain why production increases in Germany by 89 million dollars. For France, the result of the second difference model dominates the result of the first one, and production decreases by 12 million dollars. (Graphs 1.11 and 1.12). The same comments can be made on employment. It increases by 2500 in Germany, and decreases by 500 in France and 2000 in the United States (Graphs 1.14, 1.15 and 1.16).

Households' consumption suffers from full crowding out at the world level, and decreases by 1 thousand million dollars. This decrease is first allocated among the three countries proportionally to their sizes. However, this decrease is amplified in Europe and reduced in the US by the result given by the model of the difference between Europe and the US. So, household's consumption decreases by 160 million dollars in the US (Graph 1.7). The decrease in consumption in Germany is still amplified by the result of the model of the difference between Germany and the rest of the world. Finally, household's consumption decreases in Germany by 800 million dollars. In France it decreases by less than 40 million dollars. (Graphs 1.5 and 1.6).

The world price level remains unchanged. However, production price increases in both difference models[8]. This explains that this price decreases in the United States by 0.0015% (Graph 1.4), increases in Germany by 0.019% and decreases in France by 0.022%. (Graphs 1.2 and 1.3).

The same comments can be made for consumption price. This price increases in Germany by 0.008%, decreases in France by 0.019% and increases in the US by 0.0016% (Graphs 1.17, 1.18 and 1.19). In the US, production price decreases and consumption price increases. This apparent contradiction comes from the appreciation of the euro, which increases the price in dollars from goods imported from Europe.

The stickiness of production prices implies that these prices and consumption prices converge to their long run values monotonically in the two difference sub-models and in the full model (Graphs 1.17, 1.18 and 1.19). The decrease in consumption prices in France dominates the raise of this variable in Germany, and inflation is negative and increasing in Europe. Thus, the monetary rules followed by the ECB and the Fed, explain the movements in the nominal interest rate. In France and in Germany it decreases at the time of the shock by 0.0028%, then it progressively increases. In the United States it increases by 0.0017% then decreases smoothly. Both rates will be back to their reference levels in the long run (Graphs 1.20, 1.21 and 1.22). The decrease in the interest rate in Europe is dampened by the fact that the ECB takes German inflation into account. So, real interest rate increases in France and decreases in Germany. It increases in the US.

Households' inter-temporal arbitrage depresses their consumption in the short run in France by 160 million dollars (instead of 40 million in the long run). In the US the short run decrease in households' consumption is by 260 million dollars (instead of 160 million in the long run). However, in Germany, the temporary decrease in the real interest rate in the short run slows down the decrease in consumption to its depressed long run level (Graphs 1.5, 1.6 and 1.7).

The movements in national households' consumption are reflected in domestic effective production (the demand for production goods). The temporarily depressed level of consumption in France and in the US induces a temporary decrease, stronger than the long run one, of effective production in both countries. However, we will have a temporary increase in effective production in Germany (Graphs 1.8, 1.9 and 1.10).

Permanent increase in productivity in the US by 0.5%

This policy increases the global productivity in the United States A_3 by 0.5%. Start by analysing the long run. Euro appreciates by 0.30% (Graph 2.1). This variation results from the highest supply of American goods on the world market. Production increases by 24 thousand million dollars in the United States, 650 million dollars in Germany and 420 million dollars in France (Graphs 2.11, 2.12 and 2.13). The spill over of technological progress in the US to a raise in production in Germany and in France results from the appreciation of the real exchange rate in Europe, coming from a higher supply of American goods. World employment is unchanged. It decreases by 30000 in the US and increases 12000 in France and 18000 Germany (Graphs 2.23, 2.24 and 2.25). These movements result from more expensive labour in the US caused by the real depreciation of the dollar, and cheaper labour in Europe caused by the real appreciation of the euro.

The movements in real exchange rates explain why households' consumption increases less than production in the US and more in both European countries. So, consumption increases by 23 thousand million dollars in the United States, by 1400 million dollars in Germany and 1 thousand million dollars in France. (Graphs 2.5, 2.6 and 2.7). In the three sub-models, interest rates remain unchanged in the long run. So, interest rates and foreign indebtedness stay at their reference levels in the full model in the long run.

The world price level remains unchanged. The production price decreases in the United States by 0.023% and increases in Germany and France by 0.040%. (Graphs 2.2, 2.3 and 2.4). The consumption price increases in the US by 0.022% and decreases in both European countries by 0.038%. (Graphs 2.17, 2.18 and 2.19). These results are consistent with those given by the two difference models.

Production price stickiness explains that these prices, and consumption prices, will converge monotonically to their long run values. Inflation or deflation is relatively high at the time of the shock, then dampens smoothly. Thus, the reaction functions of the ECB and the Fed explain the movements of nominal interest rates. In France an in Germany they decrease at the time of the shock by 0.040%, then increase smoothly and converge to their reference value. In the United States it increases by 0.024% then decreases to its reference value (Graphs 2.20, 2.21 and 2.22).

The overreaction of the central banks implies that real interest rates decrease in Europe and increases in the US. So, the inter-temporal arbitrage of households increases consumption in the short run in Germany and in France, above their new long term values (by 2.3 thousand million dollars instead of 1.5 in the long term for Germany, and 1.5 against 0.9 for France). In the United States, the temporary increase in the real interest rate slows down the raise in consumption toward its higher long term level (Graphs 2.5, 2.6 and 2.7).

The movements in national households' consumption are reflected in domestic effective production (the demand for production goods). The temporarily inflated levels of consumption in Germany and France, drive effective productions above their new long run levels. In the US, the increase in effective production is slowed down (Graphs 2.8, 2.9 and 2.10).

The shock investigated in this paragraph takes place in the US. Both European countries are in a perfectly symmetrical situation relatively to this shock, but for their difference in sizes. Thus, prices, interest rates, labour costs and real exchange rates change by the same amounts in Germany and in France. Changes in potential output, consumption and employment differ in proportion to the sizes of both countries. The increase in German consumption investigated in the previous paragraph is different. France and the US are not in a symmetrical position with Germany and the fiscal shock because they are related to Germany by different exchange rate regimes.

Conclusion

The decomposition of the response of national economies to shocks facilitates a great deal the analysis of economic interdependence in an integrated world. However, this decomposition was made under several strong assumptions. How can it be used to analyse the simulations of multinational models, which were estimated or calibrated without reference to these assumptions?

First, the simulations of a multinational model are computed in the neighbourhood of a reference path of the economy, which can be very different from a balanced growth path (even if it converges to such a path). The reason is that the recent history and the immediate future of the economy can differ a lot from a steady path. If the model were linear, this would not affect its dynamic multipliers. However, models are non-linear, and the dynamic multipliers depend on the reference path around which they are computed. Probably, the biggest difference between the current state of the economy and its balanced growth path bears on the rate of indebtedness of countries and Governments. As indebted agents must pay for interests at every period, a change in the interest rate may have very different effects in the neighbourhood of the reference path and in the neighbourhood of a balanced growth path. So, changing the reference path (and the initial values) of the model into a balanced growth path, and examining how dynamic multipliers are modified, can give useful insights on the property of the models. This can be the first step in the interpretation of the dynamic multipliers.

The second step of the interpretation, starting with the model simulated in the neighbourhood of a balanced growth path, is to assume that all the elasticity parameters are the same across countries. Probably, differences in the values of these parameters are fragile, because they rest upon time series econometrics, which is very sensitive to details of specification and so lacks robustness. It will in general be possible to change the values of these elasticities without changing the balanced growth path of the economy, by adjusting the constant and scale variables of equations. This step will identify the part in dynamic multipliers which is related to differences in elasticities between countries.

The third step of interpretation will introduce new changes, which are that on the balanced growth path, bilateral trade flows will become proportional to the product of outputs of partners countries. This will change the balanced growth path, and will probably have tremendous effects on the multipliers.

In the fourth step, the domestic structures of the countries will be made homothetic to one another. Under this last assumption, the model can be decomposed into simpler and easy to interpret sub-models, as in this paper.

The advantage of this decomposition method is to identify in the responses to shocks what is due to the current imbalance of the economy, to heterogeneity between countries, to asymmetries in international flows and to what is left in a structural symmetric world.

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Appendix 1: Equations determining the initial values of the balanced growth path

(1s), (2s) $$\left(1+i_j\right)\left(1+\bar{\pi}_3\right)=\left[1+i_3+0.01\left(\frac{F_j}{p_jQ_j}-\frac{F_3}{p_3Q_3}\right)\right]\left(1+\bar{\pi}\right),\ j=1,2$$

(3s) $$(1+i_1)^{0.6}\left(1+i_2\right)^{0.4}=(1+\bar{r})(1+\bar{\pi})+1.5\ \left(\pi^s-\bar{\pi}\right)$$

(4s) $$1+i_3=\left(1+\bar{r}_3\right)\left(1+\bar{\pi}_3\right)+1.5\ \left(\pi_3^s-\bar{\pi}_3\right)$$

(5s) $$\pi_3^s=\bar{\pi}_3$$

(6s) $$\pi^s=\bar{\pi}$$

(7s), (8s) $$\left(\frac{1+g}{1+n}\right)^{\rho}=\left(\frac{1+i_j}{1+\beta}\right)\frac{1}{\left(1+\pi^s\right)},\ j=1,2$$

(9s) $$\left(\frac{1+g}{1+n}\right)^{\rho}=\left(\frac{1+i_3}{1+\beta}\right)\frac{1}{\left(1+\pi_3^s\right)}$$

(10s), (11s) $$pc_j^{-\sigma}=b_{j1}p_1^{-\sigma}+b_{j2}p_2^{-\sigma}+b_{j3}\left(ep_3\right)^{-\sigma},\ j=1,2$$

(12s) to (15s) $$p_jC_{ij}=b_{ij}\left(p_j/pc_i\right)^{-\sigma}pc_iC_i,\ i=1,2,\ j=1,2$$

(16s), (17s) $$p_3eC_{i3}=b_{i3}\left(p_3e/pc_i\right)^{-\sigma}pc_iC_i\ ,\ i=1,2$$

(18s) $$pc_3^{-\sigma}=b_{31}\left(\frac{p_1}{e}\right)^{-\sigma}+b_{32}\left(\frac{p_2}{e}\right)^{-\sigma}+b_{33}p_3^{\ -\sigma}$$

(19s), (20s) $$\left(p_j/e\right)\ C_{3j}=b_{3j}\left(p_j/epc_3\right)^{-\sigma}pc_3C_3,\ j=1,2$$

(21s) $$p_3C_{33}=b_{33}\left(p_3/pc_3\right)^{-\sigma}pc_3C_3$$

(22s) to (24s) $$D_j=G_j+C_{1j}+C_{2j}+C_{3j},\ j=1,2,3$$

(25s) to (27s) $$Q_j=A_jL_j^{\alpha},\ j=1,2,3$$

(28s) to (30s) $$\alpha A_jL_j^{\alpha-1}=\omega_j,\ j=1,2,3$$

(31s), (32s) $$\ln\left(\frac{\omega_jL_j}{Q_j}\right)=\varphi_{0i}+\varphi_1\ln\left(\frac{pc_j}{p_j}\right)+\varphi_2\ln\left(\frac{L_j}{N_j}\right)-\varphi_3\ln\left(1+\pi_j^s\right),\ j=1,2$$

(33s) $$\ln\left(\frac{\omega_3L_3}{Q_3}\right)=\varphi_{03}+\varphi_1\ln\left(\frac{pc_3}{p_3}\right)+\varphi_2\ln\left(\frac{L_3}{N_3}\right)-\varphi_3\ln\left(1+\pi_3^s\right)$$

(34s) to (35s) $$D_j=Q_j,\ j=1,2,3$$

(37s), (38s) $$p_iD_i=pc_iC_i+p_iG_i+\left[i_3+0.01\left(\frac{F_i}{p_iQ_i}-\frac{F_3}{p_3Q_3}\right)\right]\frac{eF_i}{\left(1+\bar{\pi}_3\right)\left(1+g\right)},\ i=1,2$$

(39s) $$F_1+F_2+F_3=0$$

(40s) to (42s) $$\inf_j=1,\ j=1,2,3$$

(43s) to (45s) $$def=1$$

(46s) $$gc_j=1,\ j=1,2,3$$

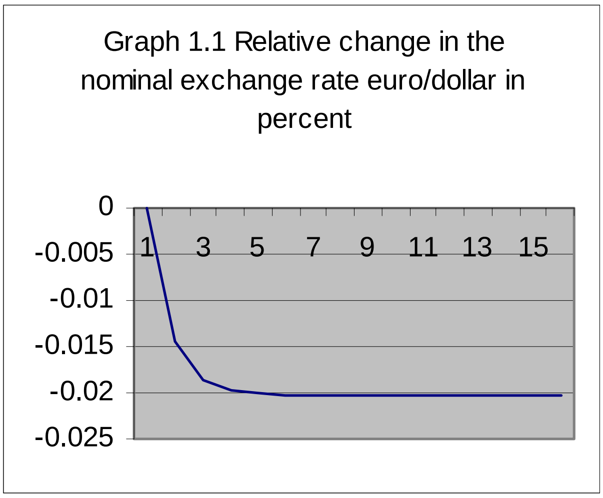
Appendix 2: Expression of the endogenous variables of the complete model in terms of the variables of the three sub-models

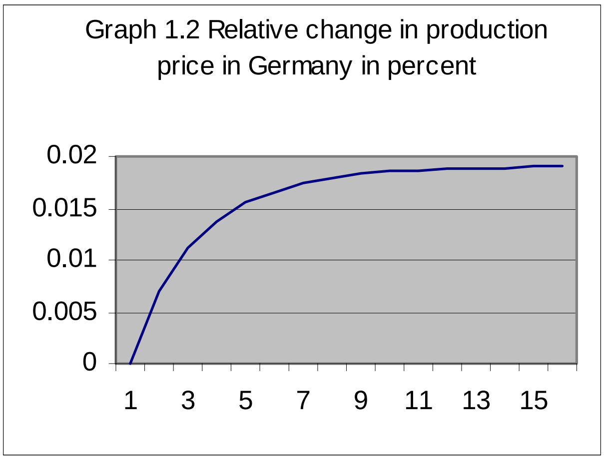
Indices M, E and A identify the variables of the three sub-models, respectively the world model, the model of the difference between Europe and the US and the model of the difference between Germany and the rest of the world. We have:

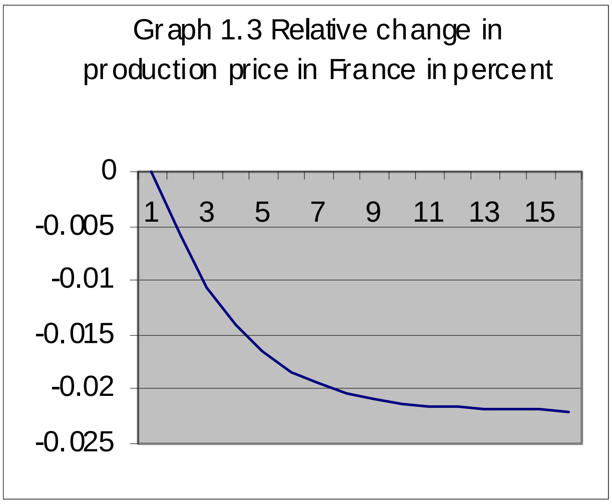
$$def = def_E$$
$$e = e_E$$
For: $X = C, D, F, L, Q$:
$$X_1 = [\overline{Q}_1 X_M + \overline{Q}_R X_A] / \overline{Q}_M$$
$$X_2 = [\overline{Q}_2 X_M + \overline{Q}_3 X_E - \overline{Q}_R X_A] / \overline{Q}_M$$
$$X_3 = [\overline{Q}_3 X_M - \overline{Q}_3 X_E] / \overline{Q}_M$$
For: $x = gc, \mathrm{inf}, \omega, p, pc$:
$$x_1 = x_M + \frac{\overline{Q}_R}{\overline{Q}_M} x_A$$
$$x_2 = x_M + \frac{\overline{Q}_3 \overline{Q}_E x_E - \overline{Q}_1 \overline{Q}_R x_A}{\overline{Q}_2 \overline{Q}_M}$$
$$x_3 = x_M - \frac{\overline{Q}_E}{\overline{Q}_M} x_E$$
$$i_1 = (1+\overline{\pi})(i_M + \frac{\overline{Q}_R}{\overline{Q}_M} i_A)$$
$$i_2 = (1+\overline{\pi})(i_M + \frac{\overline{Q}_3 \overline{Q}_E i_E - \overline{Q}_1 \overline{Q}_R i_A}{\overline{Q}_2 \overline{Q}_M})$$
$$i_3 = \left(1+\overline{\pi}_3\right) i_M - \left(1+\overline{\pi}_3\right) \frac{\overline{Q}_E}{\overline{Q}_M} i_E$$

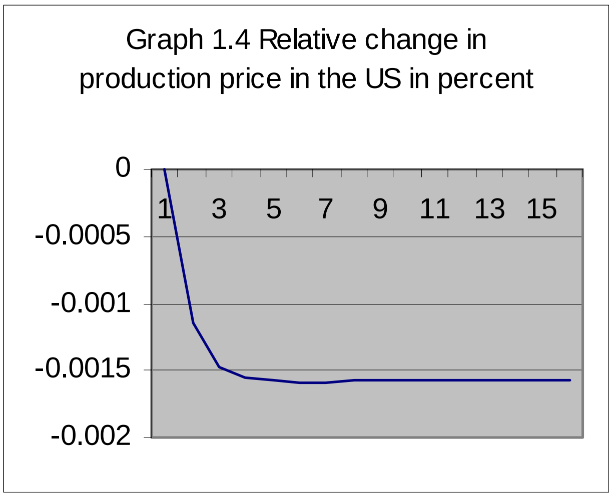
The C_{ij} can be easily computed with the equations of the full model (9r to 11r and 13r to 15r).

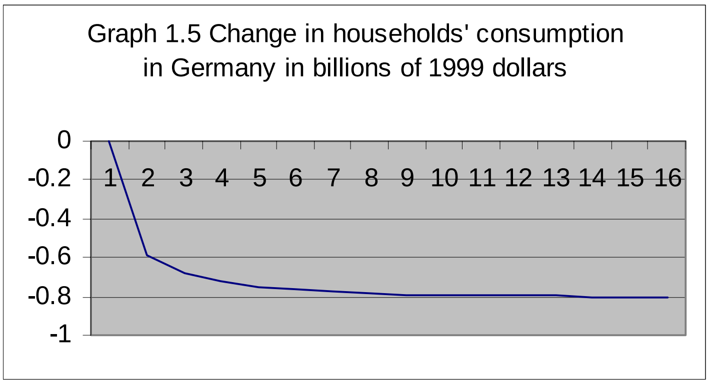
ANNEXES

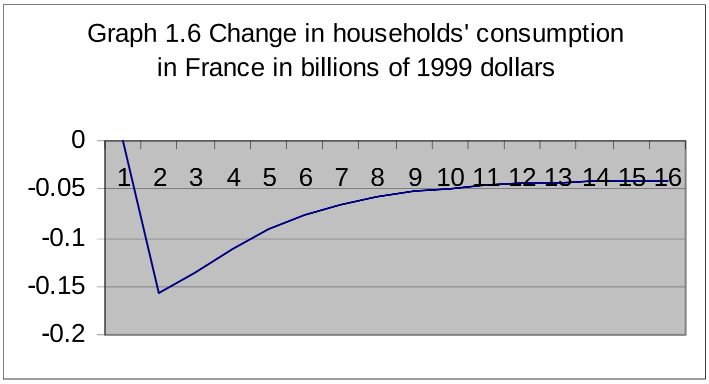

Graph 1.1 Relative change in the nominal exchange rate euro/dollar in percent
0
-0.005
-0.01
-0.015
-0.02
-0.025
1 3 5 7 9 11 13 15


Graph 1.2 Relative change in production price in Germany in percent
0.02
0.015
0.01
0.005
0
1 3 5 7 9 11 13 15

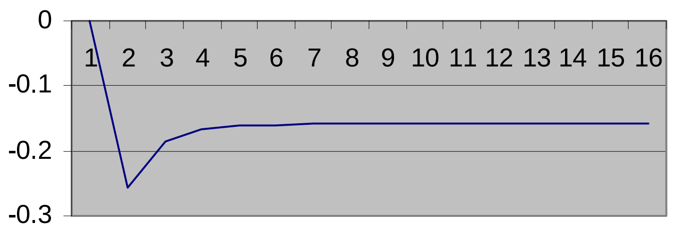

Graph 1.3 Relative change in production price in France in percent
0
-0.005
-0.01
-0.015
-0.02
-0.025
1 3 5 7 9 11 13 15


Graph 1.4 Relative change in production price in the US in percent
0
-0.0005
-0.001
-0.0015
-0.002
1 3 5 7 9 11 13 15

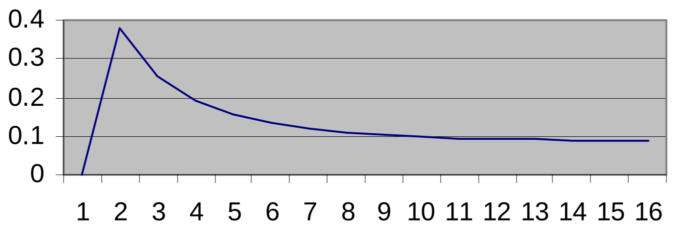

Graph 1.5 Change in households' consumption in Germany in billions of 1999 dollars
0
-0.2
-0.4
-0.6
-0.8
-1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16


Graph 1.6 Change in households' consumption in France in billions of 1999 dollars
0
-0.05
-0.1
-0.15
-0.2
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

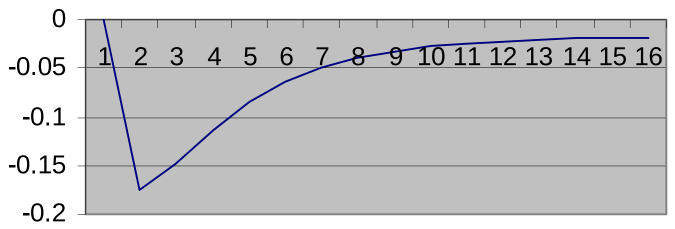
Graph 1.7 Change in households' consumption in the US in billions of 1999 dollar



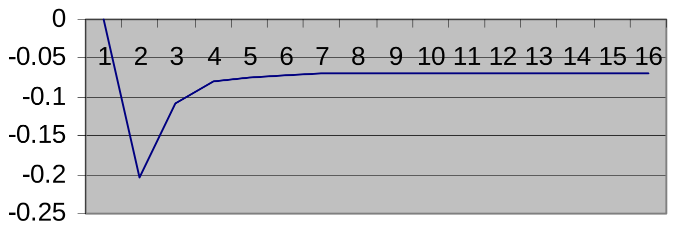
Graph. 1.8 Change in effective production in Germany in billions of 1999 dollars



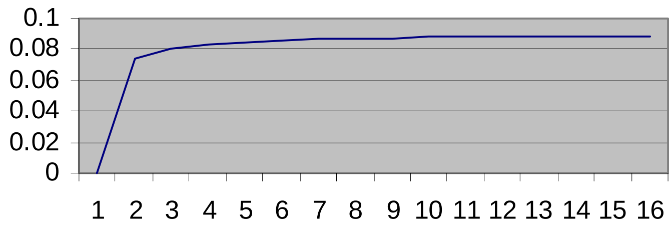
Graph 1.9 Change in effective production in France in billions of 1999 dollars



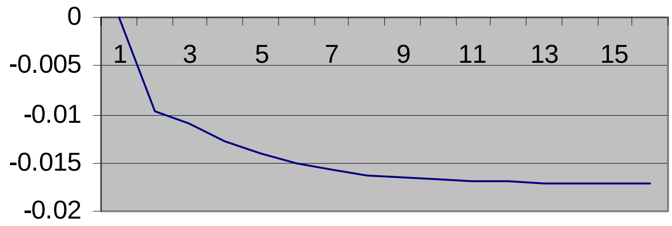
Graph 1.10 Change in effective production in the US in billions of 1999 dollars



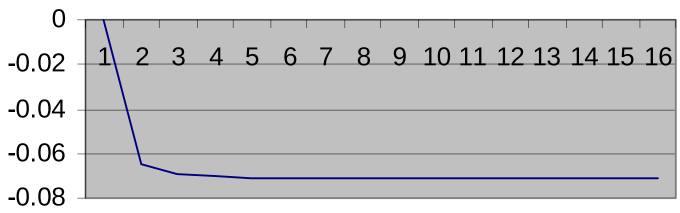
Graph 1.11 Change in potential output in Germany in billions of 1999 dollars



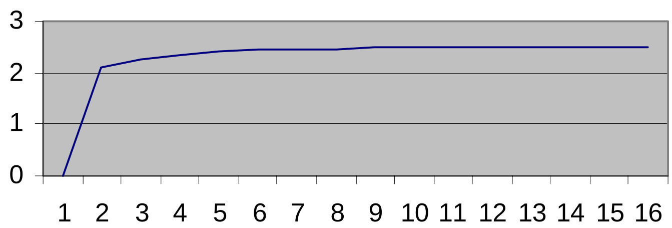
Graph1.12 Change in potential output in France in billions of 1999 dollars



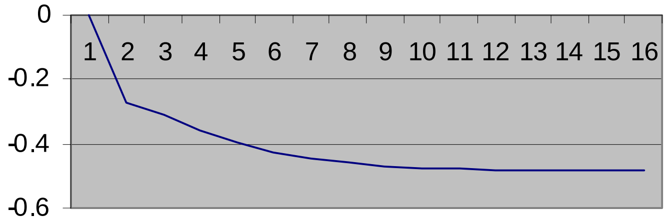
Graph1.13 Change in potential output in the US in billions of 1999 dollars



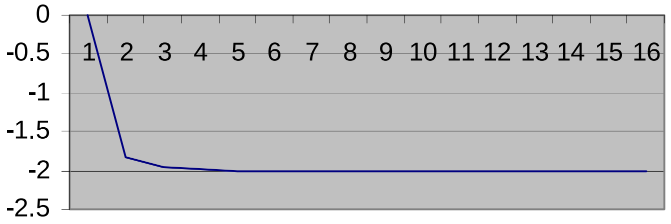
Graph 1.14 Change in employment in Germany in thousands of people



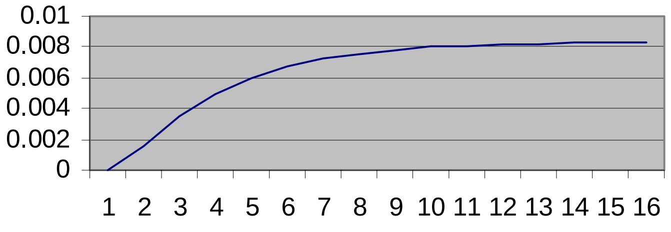
Graph 1.15 Change in employment in France in thousands of people



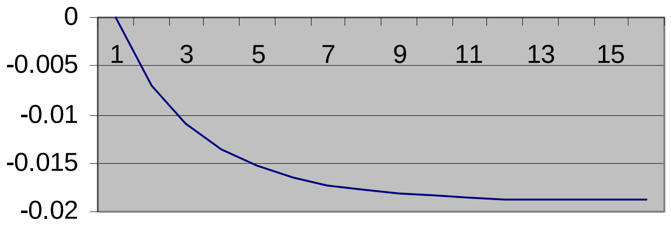
Graph 1.16 Change in employment in the US in thousands of people



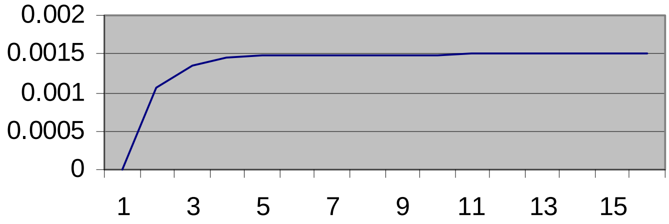
Graph 1.17 Relative change in consumption price in Germany in percent



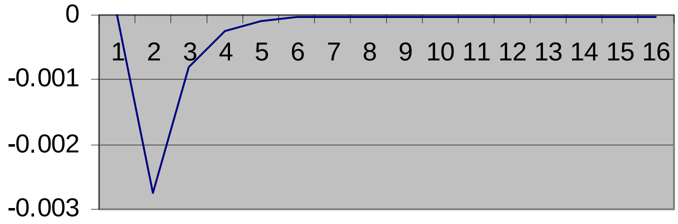
Graph 1.18 Relative change in consumption price in France in percent



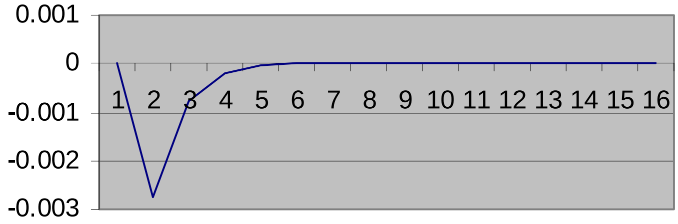
Graph 1.19 Relative change in consumption price in the US in percent



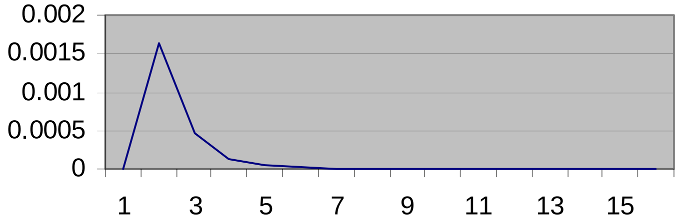
Graph 1.20 Change in the nominal interest rate in Germany in percent



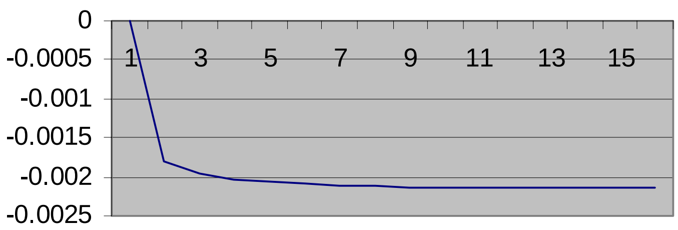
Graph 1.21 Change in the nominal interest rate in France in percent



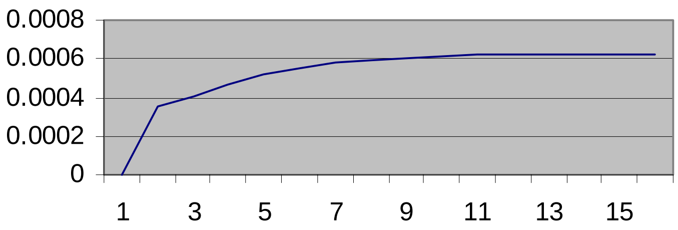
Graph 1.22 Change in the nominal interest rate in the US in percent



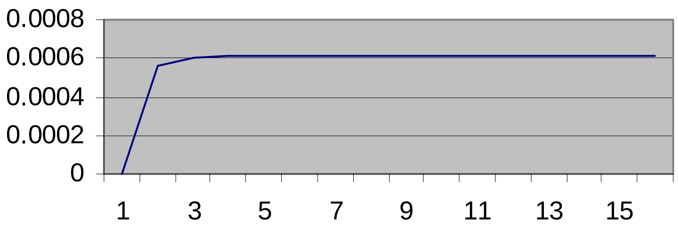
Graph 1.23 Relative change in labour cost in Germany in percent



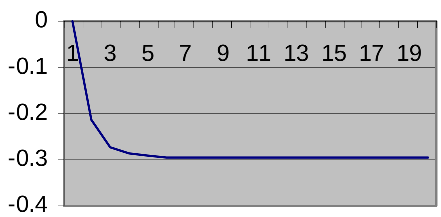
Graph 1.24 Relative change in labour cost in France in percent



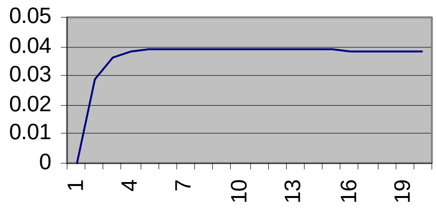
Graph 1.25 Relative change in labour cost in the US in percent



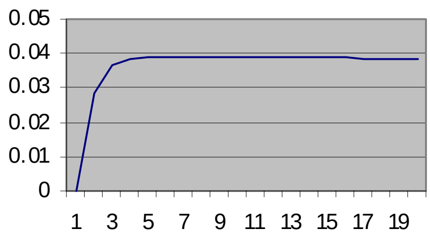
Graph 2.1 Relative change in the nominal exchange rate euro/dollar in percent



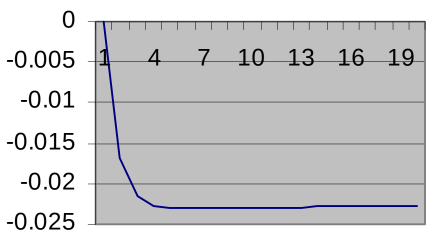
Graph 2.2 Relative change in production price in Germany in percent



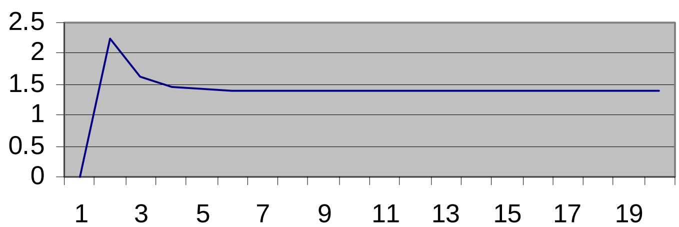
Graph 2.3 Relative change in production price in France in percent



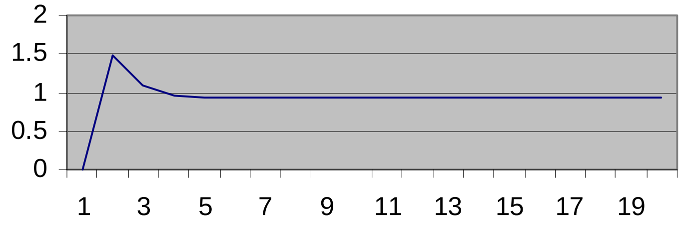
Graph 2.4 Relative change in production price in the US in percent



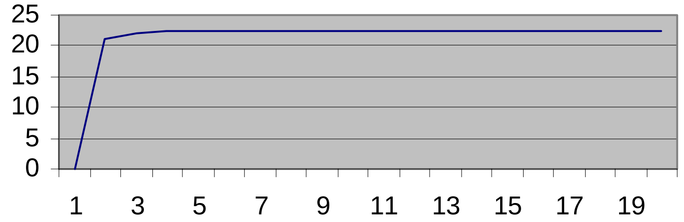
Graph 2.5 Change in households' consumption in Germany in billions of 1999 dollars



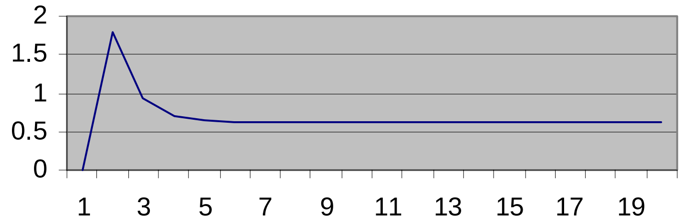
Graph 2.6 Change in households' consumption in France in billions of 1999 dollars



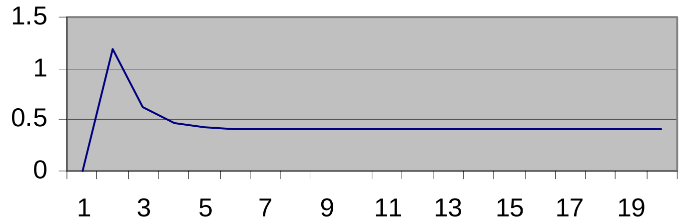
Graph 2.7 Change in households' consumption in the US in billions of 1999 dollars



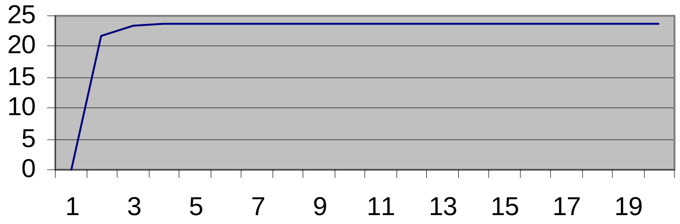
Graph. 2.8 Change in effective production in Germany in billions of 1999 dollars



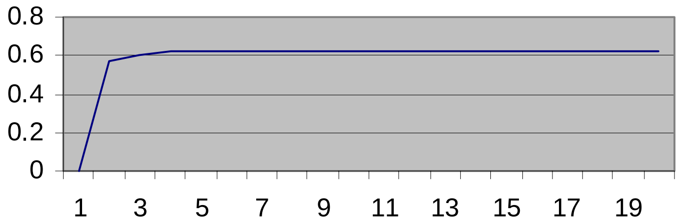
Graph 2.9 Change in effective production in France in billions of 1999 dollars



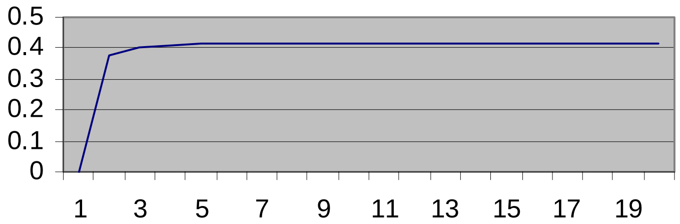
Graph 2.10 Change in effective production in the US in billions of 1999 dollars



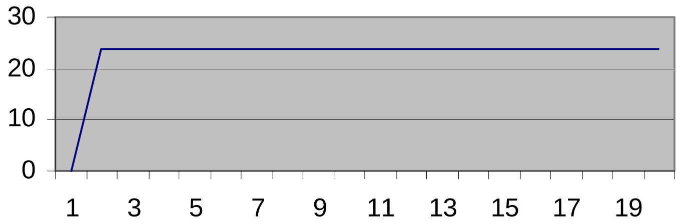
Graph 2.11 Change in potential output in Germany in billions of 1999 dollars



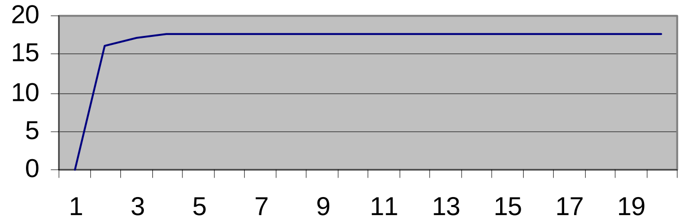
Graph 2.12 Change in potential output in France in billions of 1999 dollars



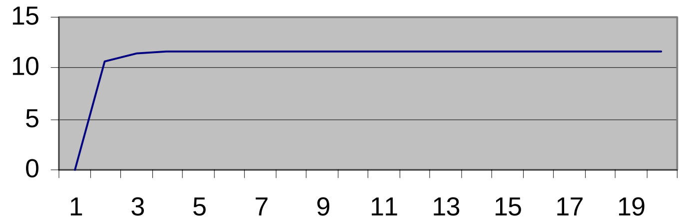
Graph 2.13 Change in potential output in the US in billions of 1999 dollars



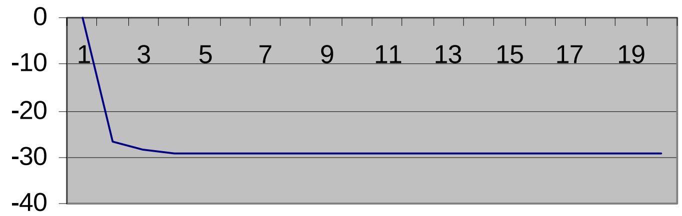
Graph 2.14 Change in employment in Germany in thousands of people



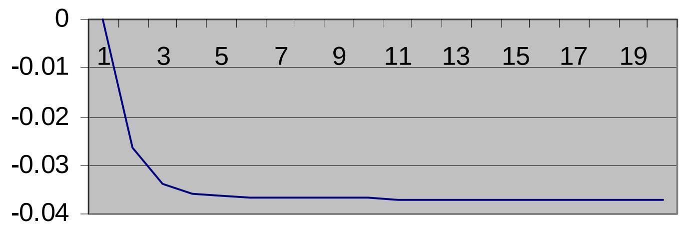
Graph 2.15 Change in employment in France in thousands of people



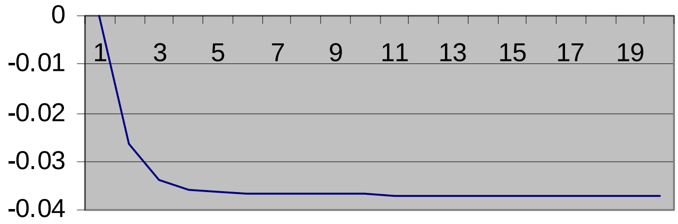
Graph 2.16 Change in employment in the US in thousands of people



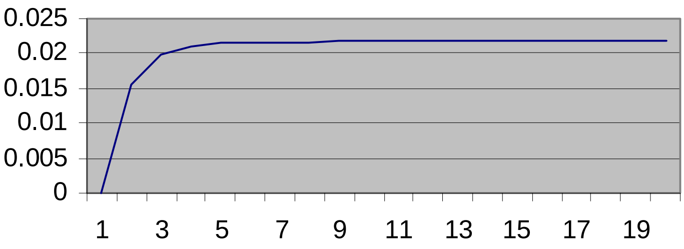
Graph 2.17 Relative change in consumption price in Germany in percent



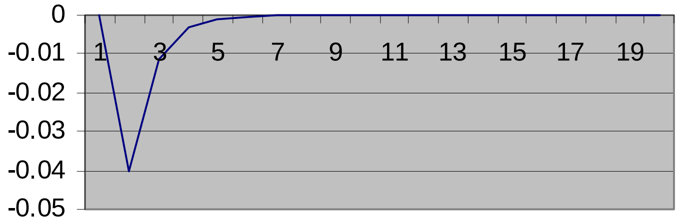
Graph 2.18 Relative change in consumption price in France in percent



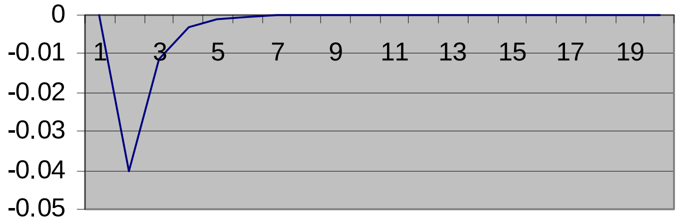
Graph 2.19 Relative change in consumption price in the US in percent



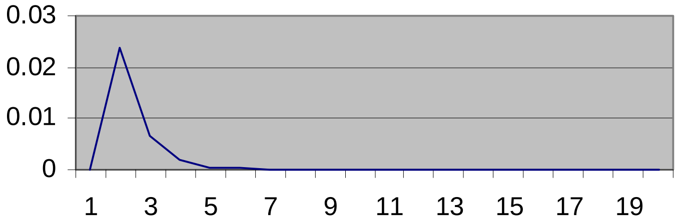
Graph 2.20 Change in the nominal interest rate in Germany in percent



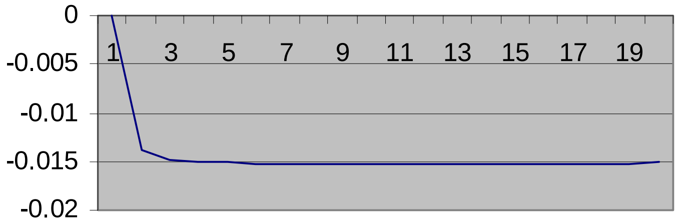
Graph 2.21 Change in the nominal interest rate in France in percent



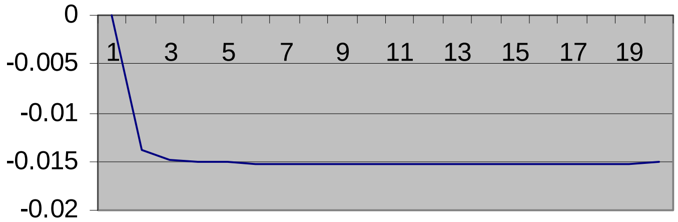
Graph 2.22 Change in the nominal interest rate in the US in percent



Graph 2.23 Relative change in labour cost in Germany in percent



Graph 2.24 Relative change in labour cost in France in percent



Graph 2.25 Relative change in labour cost in the US in percent