

CRITICALITY*

Sergio da Silva¹

Resumo: Neste artigo passamos em revista a descoberta de “leis de potência” em diversas disciplinas.

Palavras-chave: leis de potência, criticalidade, escalonamento, complexidade.

Abstract: This article reviews some current interdisciplinary work on so-called ‘power laws’.

Keywords: power laws, criticality, scaling, complexity theory.

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Introduction

Certain complex systems, under certain circumstances, have been discovered to behave in similar, mathematically simple ways. This has been dubbed ‘criticality’. In a critical state there is no reason to look for specific causes of great events. The smallest force can have gigantic effects and sudden upheavals can occur seemingly out of nowhere. Fluctuations of something in a critical state are neither truly random nor easily predicted. The approximate frequency of such upheavals can be predicted, but not when they will happen or what size they will be. A recent book (Buchanan 2000b) reviews the current work on the subject to highlight a deep similarity between the upheavals that affect our lives in ecosystems, economies, and even political systems. This article draws on the book. One of the discoveries in modern physics is that criticality often arises on its own in non-equilibrium systems, and in things in which history matters. The workings of the systems in which upheavals occur may sometimes reflect simple and universal underlying processes that can be captured by simple mathematical ‘games’ that share a common logical framework. Scientists in other fields have already taken advantage of this breakthrough. As observed, in the 13 years since its discovery, the idea of criticality has spread like wildfire into scientific disciplines ranging from geophysics to biology, economics, and history (Bak 2000).

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¹ University of Brasilia. E-mail: sergiodasilva@angelfire.com.

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Critical State Universality

The theoretical work on criticality is based on simple models that can be studied by either mathematical analysis or computer simulations. Understanding the critical state of one class in any system is to immediately understand all systems in that class. The models are more metaphorical than those used in theoretical physics, for instance. Although like all theoretical modeling it must stand the test of comparison with nature. The test comes by adding back a few of the neglected details and observing if they change the results in any essential way.

The principle of universality or ubiquity means that one should focus on the simplest mathematical game belonging to an equivalent universal class. Details are not important in deciding the outcome. Even crude models can work exactly like the real thing. Details have no influence because things in a critical state have no inherent typical scale in either time or space.

Power Laws

In a critical state, something known as a ‘power law’ comes into play to reveal a hidden order and simplicity behind complexity (see box). A power law means that there is no such thing as a normal or typical event, and that there is no qualitative difference between the larger and smaller fluctuations. Upheavals are not unusual. A big event need not have a cause. The causes that trigger a small change on one occasion may initiate a devastating change on another, and no analysis of the conditions at the initial point will suffice to predict the event.

Power laws are incompatible with bell-shaped normal curves. A bell-shaped curve establishes that an average gives someone a good idea of what to expect. A normal distribution is supposedly the norm in nature. Its widespread applicability is a consequence of the central limit theorem - in any case in which a large number of independent influences contribute to the outcome of some event, that outcome will result in a bell curve. Although this is not to say that everything follows a bell curve. An equally large number of things do not. Some are governed by power laws.

If, for instance, one throws frozen potatoes at a wall, they will break into fragments of varying size. If one collects all the pieces up, from the microscopic ones to the large, and puts them into different piles according to weight, a power law for fracture emerges - each time the weight of the fragments is reduced by two, there will be six times as many. Power laws have been discovered for events ranging from forest fires and earthquakes to mass extinction and stock market crashes.

Self-Similarity

Mandelbrot (1963) discovered that if he took a small section of the record of the price of cotton on the Chicago mercantile exchange, and stretched it out so that it became as long as the entire record, it looked much like the whole. He had discovered self-similarity. Following that, he invented the geometry of fractals (Mandelbrot 1982). Self-similarity or scale-invariance (every part is a tiny image of the whole) is reflected in a power law.

The geometric regularity of any power law implies a lack of any typical scale, a feature that shows up clearly in the critical point image, which is a fractal. What counts in the critical state is the simple underlying features of geometry that control how influences can propagate. Fractals and power laws are at work in settings where the critical state underlies their dynamics. Fractals can be produced by chaos, but they also arise in processes of growth or evolution. To understand fractals and power laws, one needs a historical physics, not an equilibrium physics. This is so because the notion of history has no meaning in equilibrium.

Financial markets demonstrate several of the properties that characterize complex systems. What is more, they are highly complex, open systems in which many sub-units interact nonlinearly in the presence of feedback and stable governing rules. Earlier attempts to find chaos in financial data, for instance, have been disappointing (e.g. Da Silva 2001) exactly because the phenomenon is likely to emerge in systems that are only moderately complex. Although it cannot be ruled out that financial markets follow chaotic dynamics (fake randomness), the work on econophysics (discussed below) assumes that asset price dynamics are stochastic processes, which may be governed by power laws.

Self-Organized Criticality

Tuning is crucial to reach a critical point in which any tiny event can trigger a huge upheaval. Sometimes criticality can be tuned by nature on its own, though. This is dubbed 'self-organized criticality'. Self-organized criticality seems to show up in things that are driven slowly away from equilibrium, and in which the actions of any individual piece are dominated by its interactions with other elements.

Forest Fires

Forests are a good example of self-organized criticality. The network of trees on a grid seems naturally to tune itself to a critical state in which the next match might spark a fire of any size, even one that would destroy the entire forest. A power law for forest fires has been found: when the area covered by a fire is doubled, it becomes about 2.48 times as rare (Malamud *et al.* 1998). The model invented to account for the spread of fires in forests may also be able to capture the essence of how diseases spread through human populations. In the toy model of forest fires, people have been plugged in place of trees and measles in place of fires (Rhodes and Anderson 1996). The result explained the distribution of measles epidemics on the Faroe Islands in the North Atlantic. Even if the trees are people, and the fire is a disease, disturbances still spread in the same way.

The Role of Free Will

Individual freewill offers no escape from the inevitability of criticality. Even though people interact with one another by virtue of their own personal choices, there nevertheless, exist definite

mathematical patterns in the activity of a group. These patterns cannot help predict what any one person will do, but they may be able to say what is likely to emerge in the aggregate. What is more, the mathematics is not complicated. The way people aggregate into cities, for instance, does not seem to depend on the fact that they are people. The pattern of population within any city may be described by using simple games from the theory of phase transitions (Zanette and Manrubia 1997). Cities are possibly fractals. So there is no typical size for any city, and no reason to see special historical or geographical situations behind the emergence of the very largest.

Avalanches

If one takes a handful of rice (or sand) and drops the grains one by one on to a tabletop, a pile of rice is soon built. However, the pile will not grow taller forever. Eventually the addition of one more grain will cause an avalanche. Such a grain is only special because it happened to fall in the right place at the right time. The addition of a single grain may have no effect, precipitate a small avalanche, or collapse the whole structure. One can predict the likely frequency of the avalanches, but not when they will happen or what size each will be. It may come as no surprise that big avalanches occur less frequently than small ones. What is surprising is that there is a power law - each time the size of an avalanche of rice grains is doubled, it becomes twice as rare (Bak *et al.* 1987). The apparent complexity of the pile collapses to a hidden order and simplicity.

Earthquakes

If fault systems in the earth's crust are in a critical state as well, then predicting when earthquakes will strike, and how destructive they will be, should prove impossible. Actually earthquake prediction research has been conducted for over 100 years with no obvious successes (Geller 1997). An earthquake occurs when the slow movement of the earth's continental plates does not directly bring about any reorganizations of the earth's crust, since friction holds the rocks in place. The movements of the plates simply put the rocks under stress. Only when the stress builds up past some threshold do the rocks move and reorganize themselves, suddenly and violently. Earthquakes thus come from the build up and release of stress and seem to be distributed in energy according to a power law. If the size of an earthquake is doubled, the Gutenberg-Richter power law says those quakes become four times less frequent. The bigger the quake, the rarer it is. Such a number corresponds to a particular type of self-similar, fractal pattern. The distribution is scale invariant, that is, what triggers small and large quakes is precisely the same.

Mass Extinctions

Life on earth suffers sporadic and catastrophic episodes of collapse. There were at least five mass extinctions of species in the earth's history (a brief account is provided by Vines 1999). Most biologists do not believe that evolution is itself capable of causing upheavals. There is evolution, and there are the upheavals caused by exogenous shocks.

However, mass extinctions are possibly intrinsic outcomes of the dynamics of evolution. Indeed a power law for the distribution of extinction sizes that fits the fossil record well has been found (Solé and Manrubia 1996). Such a power law happens to be identical to that for earthquakes. Every time the size of an extinction (as measured by the number of families of species that become extinct) is doubled, it becomes four times as rare. Thus, mass extinctions need not have big, exogenous causes; they might be provoked by small, endogenous events.

The Physics of History

The concept of criticality might be useful in understanding history, which is punctuated by inexplicable and unforeseeable upheavals. The First World War, for instance, is the archetypal example of an unanticipated upheaval in world history (Buchanan 2000b). Figuratively, Hegel and Marx thought of history as the growth of a tree that follows a simple progression toward some mature, stable end-point. Some writers (e.g. Fukayama 1992) have speculated that we are approaching the end of history, as the world seems to be settling into some ultimate equilibrium of global democracy and capitalism. Toynbee saw regular cycles in the rise and fall of civilizations. History may instead be completely random with no perceptible pattern. Historian H. A. L. Fisher has commented that there is only one safe rule for the historian; that he should recognize in the development of human destinies the play of the contingent and the unforeseen (cited by Buchanan 2000c). Nevertheless there seem to be undeniable trends in history, one of the most obvious being the increase in our scientific understanding and technology. Perhaps history is chaotic, it looks random in its workings, and yet it is not random at all. Ferguson (1997) has pointed that historical chaos reconciles the notions of causality and contingency. Or maybe history lives in a critical state.

In a critical history, contingency would become powerful beyond measure. Contingency takes place when immediate events control a supposed destiny. As observed, contingency is the hallmark of the critical state (Bak 2000). Kennedy (1987) has suggested that the large-scale historical rhythm in the interactions between great powers is largely a consequence of the natural build up and release of stress driven by national interests. Usually the stress finds its release through armed conflict, after which the influence of each nation is brought back into balance with its true economic strength.

Wars seem to strike with the same statistical pattern as do earthquakes or avalanches in the rice-pile game. Statistics over five centuries have uncovered a power law for wars (Levy 1983). Every time the number of deaths is doubled, wars of that size become 2.62 times less common. Such a power law implies that when a war starts out no one knows how big it will become. There seem to be no special conditions to trigger a great conflict. Interestingly the forest-fire game seems to capture the crucial elements of the way that conflicts spread (Turcotte 1999). A war may begin in a manner similar to the ignition of a forest.

On the History of Science

Kuhn (1962)'s major achievement was to show that science works even though scientists are like everyone else, that is, they labor under the burden of being human. It could be argued that he discovered a pattern of universal change that runs deeper than he suspected. He identified science as one setting in which the universal build up and release of stress influences history. 'Normal' scientific work turns up inconsistencies, and leads to the growth of stress within the existing fabric of ideas. When this maladjustment reaches some threshold, normal science breaks down. Science cannot go further by accumulation and extension, and has to rebuild some portion of the existing network. The system shifts in a 'revolution'. Normal scientific work thus seems to be analogous to the drifting of continental plates, and scientific revolutions are akin to earthquakes.

Every new idea of science is something like a grain falling on the pile of knowledge. Each scientific research paper is a package of ideas which, when it settles down in the pre-existing network of ideas, triggers some rearrangement. To measure the overall size of the intellectual earthquake triggered by a paper, one may look to the total number of times it is cited by other papers. Here again there seems to be no typical number of citations for a paper. The distribution of citations follows a scale-invariant power law (Redner 1998). Every time the number of citations is doubled, the number of papers receiving them falls by about eight times.

If the history of science were in a critical state, the ultimate consequences of any new idea would not depend on its own inherent profundity so much as on where it happens to fall within the network of ideas. The mark of the great scientist would lie not in having deep ideas that revolutionize science but in taking insights that have the potential to do so and in making that potential real. Even if scientists were all genetically identical clones, such revolutionary achievements would still be done by a selected few.

Econophysics and Economics

If markets were critically organized, even the great stock market crashes would be simply ordinary (although infrequent) events. In an efficient market, when supply matches demand, prices have their proper values, that is, values corresponding to the underlying 'fundamentals'. Market prices could bounce up and down erratically still, but huge fluctuations could not be accounted for. Price changes would behave like the bell curve according to most of mainstream economics. Greater than some typical size, price changes ought to be extremely rare. Prices would follow a gentle random walk. Black Monday, 19 October 1987 was the largest single-day free fall in market history. The crash was nearly twice as severe as the stock market collapse of 1929, although this time it did not trigger a depression. What made the Dow Jones industrial average lose more than 22 per cent of its value in just one day? It is difficult to believe that there could be a sudden change in the fundamentals which would lead agents simultaneously within half a day to the view that returns in the future had gone down by over 20 per cent. A dubious explanation has been that the crash was caused by portfolio insurance computer programs, which sold stocks as the market went lower.

When Mandelbrot (1963) looked at how random changes in prices were distributed by size, he did not find a bell curve. Instead, he discovered that price changes are governed by a power law: price changes do not have a typical size. This allows one to see large fluctuations in market prices as a result of the natural, internal workings of markets; they can strike from time to time even if there are no sudden alterations of the fundamentals. One reason might be that real-world markets are not in equilibrium. Movements of markets appear to be unpredictable in the direction up or down. (Other references are Mandelbrot 1997 and Bak *et. al* 1993.)

Other power laws have been discovered. Price fluctuations in the Standard & Poor 500 stock index were found to become about sixteen times less likely each time the size is doubled (Gopikrishnan *et al.* 1998). Scaling behavior for the Standard & Poor 500 has also been detected by Mantegna and Stanley (1995). A similar power-law holds for the prices of the stocks of individual companies (Plerou *et al.* 1999). Power-laws were found too in the Milan stock exchange (Mantegna 1991) and in foreign exchange markets (Müller *et al.* 1990). A scale-invariant power law has also been found for financial market volatility: the market does not have a typical wildness in its fluctuations (Liu *et al.* 1999).

In a simple game of the stock market, Lux and Marchesi (1999) have found self-similarity, structure on all time scales, and a distribution of price changes that follows a power law. Interestingly Ghashghaie *et al.* (1996) have suggested that foreign exchange market dynamics correspond to hydrodynamic turbulence. (Mantegna and Stanley 2000 provide comprehensive references to the work on econophysics.)

A power-law for distribution of wealth according to Pareto law has also been found (Bouchaud and Mezard 2000). If one counts how many people in America have a net worth of a billion dollars, one will find that about four times as many have a net worth of about half a billion. Four times as many again are worth a quarter of a billion, and so on. (A non-technical discussion of this paper is presented by Buchanan 2000a.)

Power laws are expected to coexist uneasily with mainstream finance theory, for instance, which is built on the efficient market hypothesis. Mandelbrot's earlier findings were not absorbed into mainstream finance, which has since been relying (although not completely) on Gaussian distributions (the debate can be appreciated in Cootner 1964). However, econophysicists adopt a more conciliatory gesture. They see the efficient market as an idealized system and real markets as only approximately efficient; they think the concept of efficient markets is still useful in any attempt to model financial markets. But rather than simply assuming normality, they try to fully characterize the statistical properties of the random processes observed in financial markets (Mantegna and Stanley 2000, pp. 12-13).

International finance economists, by contrast, are likely to welcome the concept of criticality. After all, as Krugman (1989, p. 61) once remarked, most economists today believe that foreign exchange markets behave more like the unstable and irrational asset markets described by Keynes than the efficient markets described by modern finance theory. Krugman himself has made an attempt to incorporate the notion of criticality into economics (Krugman 1996).

If a macroeconomy is critically organized, a given (monetary or real) shock is not to blame by itself for destabilizing the economy. How an economy is organized and prepared to respond to shocks turns to be more important. As observed, what survives today of Keynesian economics is not the scientific demonstration that underemployment equilibrium is possible, but Keynes's intuition that a market economy is inherently unstable, and that the source of instability

lies in the logic of financial markets. Market capitalism should be neither left alone nor abolished, but stabilized (Skidelsky 2000, p. 85). From a theoretical perspective, absorption of the concept of criticality into macroeconomics would favor a replacement of the current new neoclassical synthesis (e.g. Goodfriend and King 1997) by an approach based on Keynes's intuition as referred to above. Nonetheless one might speculate that policymakers would have a harder job to do in a critically organized macroeconomy.

Concluding Remarks

In what sense can it be true that an earthquake, a forest fire, a mass extinction and a stock market crash are events of the very same type? How could an event as tumultuous as the 1987 stock market crash arrive without any warning? Great earthquakes, forest fires, mass extinctions, and stock market crashes may be merely the expected large fluctuations that arise universally in non-equilibrium systems in a critical state. Here the organization of a system depends in no way on the precise nature of the things involved, but only on the way that influences can propagate from one thing to the next. Upheavals result from the natural build up and release of stress.

Just as it is tempting to seek great causes behind great earthquakes or mass extinctions, it is also tempting to see great persons behind the great events in history. However the only general cause for such an event may be the underlying organizations of the critical state, which make upheavals not only possible but also inevitable. Criticality may also be present in the ways ideas evolve and change. The character of the critical state is reflected in remarkably simple, statistical laws - the scale-free power laws that reveal a lack of any expected size for the next event.

Example of a Power Law

To illustrate what a power law is about, let us take some data from the experiment of dropping an object from different levels of a tall tower (e.g. Peitgen *et al.* 1992, p.195-197). The law governing free fall can be thought of as a power law. The first column in Table 1 lists the height of free fall in meters. The second column lists the drop time of the object in seconds. The last two columns display their respective logarithms to the base 10. Figures 1 and 2 show the data graphically. The relation between height and drop time is not linear (Figure 1), but there is a power law (Figure 2) because the drop time (t) changes as if it were a power of the height (h).

Table 1

height	time	log (height)	log (time)
4	0.89	0.602059991	-0.050609993
8	1.26	0.903089987	0.100370545
12	1.55	1.079181246	0.190331698
16	1.79	1.204119983	0.252853031
20	2	1.301029996	0.301029996
24	2.19	1.380211242	0.340444115
28	2.37	1.447158031	0.374748346
32	2.53	1.505149978	0.403120521

The power law takes the form

$$t = c \cdot h^d.$$

The problem is to verify such a conjecture and determine c and d . Taking the logarithm on both sides obtains

$$\log t = d \cdot \log h + \log c.$$

Figure 2 shows that a log-log plot of t versus h generates a straight line with slope d and y-intercept $b = \log c$. So $c = 10^b$. From Figure 2 we can get $d = 0.5$ and

$$\log c = -0.34. \text{ Then } c = 10^{-0.34} = 0.45.$$

The power law is thus

$$t = 0.45 \cdot h^{0.5}.$$

This agrees with the Newtonian law of motion, which implies that the distance fallen is proportional to the square of the drop time:

$$h = \frac{g}{2} t^2$$

(where g 9.81 m/sec² is the gravitational acceleration). Solving for t yields

$$t = \sqrt{\frac{2h}{g}} \approx 0.452 \cdot h^{0.5}.$$

Figure 1 – Drop time versus height of free fall

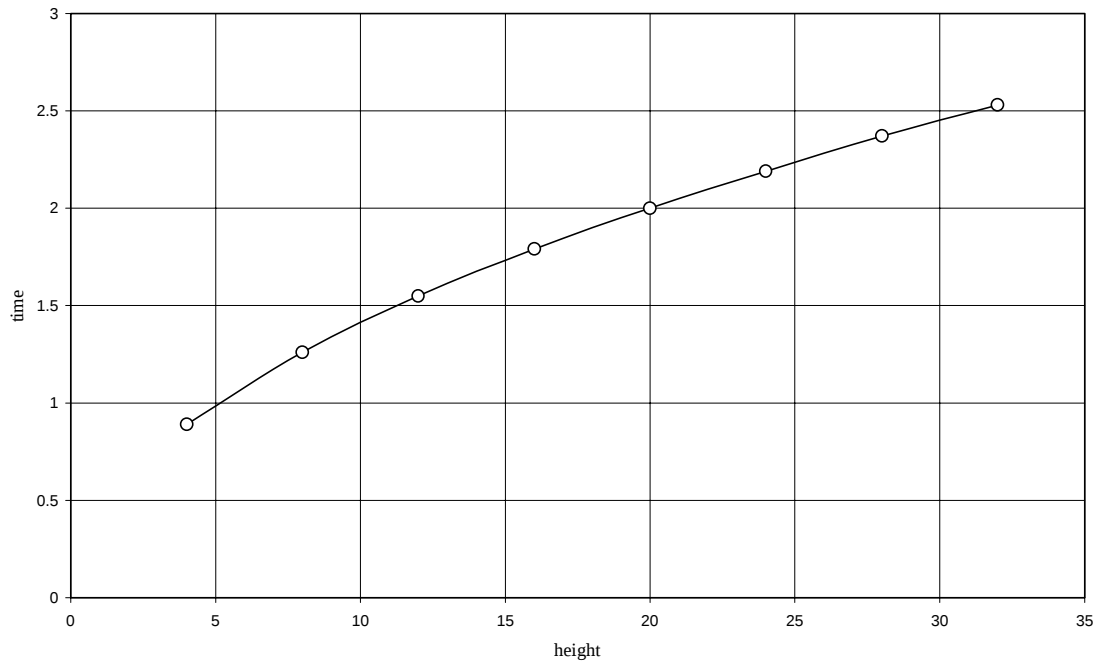
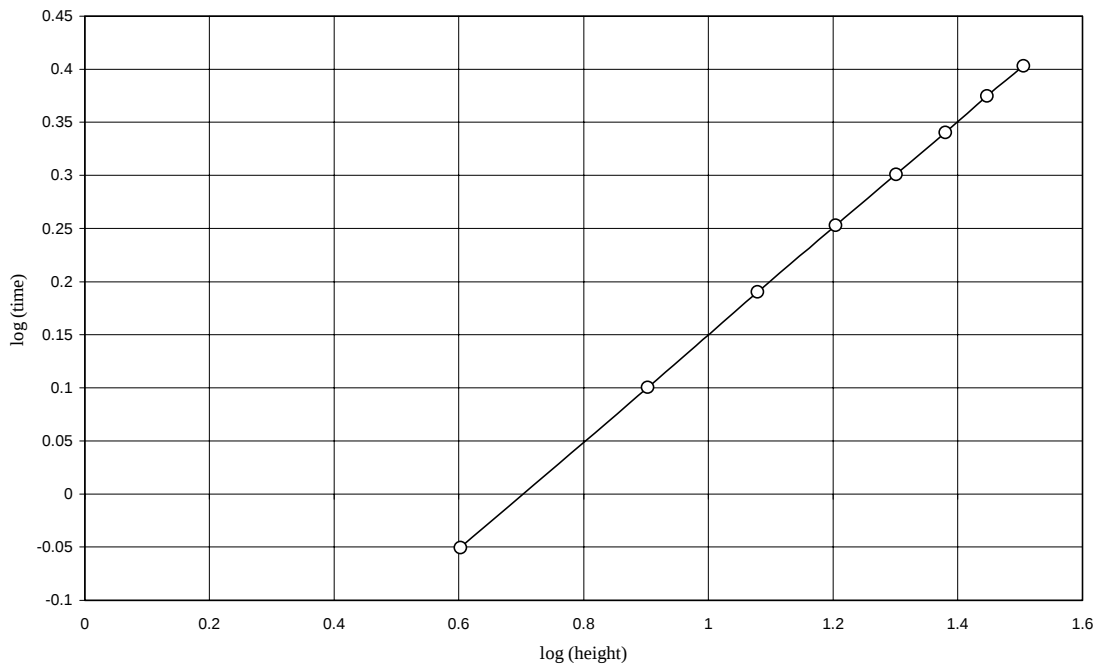


Figure 2 – Log of drop time versus log of height of fall



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