

A THEORETICAL ANALYSIS OF CLUB DENSITY, SUBSCRIPTION FEE AND SERVICE*

Amnon Levy¹

Abstract – This theoretical analysis of clubs considers the case of heterogeneous population. Club density, subscription fees and service are analytically related to factors such as the size of the target population, the distribution of ability to enjoy the club service within the target population, the tax-rebate rate on membership payment, the service production costs, which are increased by congestion or reduced by agglomeration, and the club organizational costs. The analysis is conducted for a profit-maximizing club and a budget-balancing and density-targeting club and extended to the special case of networks.

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Corresponding author: Amnon Levy, Department of Economics, University of Wollongong, Northfields Avenue, Wollongong, NSW 2522, Australia. Tel: 61-2-42213658 or 61-2-42258244, Fax: 61-2-42213725, E-mail: amnon_levy@uow.edu.au

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¹ School of Economics and Information Systems, University of Wollongong

INTRODUCTION

The purpose of this paper is to incorporate the assumption of heterogeneous population of members into the analysis of clubs. Theoretical analyses of the economics of clubs have focused on profit-maximizing clubs and assumed for convenience that the population of the potential club members is homogenous. Notable examples are Buchanan (1965), Ng (1974), Berglas (1976), Wooders (1978,1980) who considered the case of many homogeneous 'utility-taking' clubs for which the efficient size is determined by the equality between the benefit of sharing the facility costs with the marginal member is equal to the marginal cost of congestion. Scotchmer (1985) argued that clubs should not be taken as infinitely small and utility-takers and analyzed the number of clubs servicing a homogenous population and their facility size in a Nash equilibrium setting. Recent summaries of the economic literature of clubs are given by Cornes and Sandler (1996) and Sandler and Tschirhart (1997).

The contribution of the present paper to the analysis of clubs is the derivation of possible relationships between club size (or density), subscription fees and service for profit-maximizing clubs and, in particular, for nonprofit clubs under the assumption that the population of the potential club member is heterogeneous. Subscription fees and the ability to enjoy the club service are incorporated into the individual decision of joining a club. A critical ingredient in modeling this decision, and consequently club density, is that the ability to enjoy a club service varies across individuals and, despite being simultaneously and equally offered, a club service appeals to some people more than to others. The distribution of the ability to enjoy the club service in a given population, and factors such as subscription fees, tax concessions and market structure, determine the club density. In turn, club density affects both the club revenues, organizational and congestion costs and agglomeration benefits.

Due to the added complexity introduced by the assumption of heterogeneous population the issue of the efficient number of clubs is not considered by the present paper and the interdependencies between club density, membership fees and service are analyzed for the case of monopolistic clubs. An attempt to extend the analysis to the case of two rival clubs is outlined in the Appendix. Monopolistic clubs can be found in some large places of employment and also in some small and isolated communities. They can take the form of a co-op, a swimming pool, a gymnasium, a golf club, a tennis club, a place of worship, a trade union and a political meeting center. Monopolistic clubs may also be found in large places of residence. There they may take the form of ethnic social and cultural centers and places of worship which are unlikely to be competing with one another on attracting members as they are targeting exclusive ethnic and religious segments of the population.

Many of the aforementioned monopolistic clubs are nonprofit organizations. Some of them may be concerned with the relative size, i.e. density, of the group of members and provide a service under a balanced budget. Hence, in addition to profit-maximizing clubs the analysis is focused on budget-balancing and density-targeting clubs. Examples of budget-balancing and density-targeting clubs are co-ops, trade unions and professional, social, cultural, ethnic and political clubs. Budget-balancing represents a possible financial policy for nonprofit clubs.

(Hilman, 1978) Density-targeting may serve clubs for increasing their bargaining power. (McDonald and Suen, 1992)

The paper is organized as follows. The membership decision and the minimum ability to enjoy the club service required for joining the club are described in section II. The minimum ability to enjoy the club service is subsequently used for analyzing the density of a monopolistic club in section III. Profit-maximizing subscription fees and service and their properties are derived in section IV for the monopolistic club case. Subscription fees and service are also generated for a budget-balancing and density-targeting monopolistic club in section V. The analysis is extended to the special case of networks in section VI. Conclusions are summarized in section VII.

II. MEMBERSHIP DECISION AND THE MINIMUM ABILITY TO ENJOY

A CLUB SERVICE

Following Buchanan (1965) the term club is used to indicate the affiliation of people with a common interest in a service to an organization which makes that service equally accessible to members. The clubs considered hereby practice coarse exclusion: they generate revenues by collecting a uniform subscription fee for accessing their service but do not charge a per use price. (Cf. Helsley, 1991) In addition to fixed costs, they incur the variable costs of producing their service, organizational costs of billing and communicating with members and congestion costs and enjoy agglomeration benefits. The quality of the service depends on the clubs' subscription fees and objective. It is also affected by size externalities that can be positive in the case of a dominant agglomeration effect or negative in the case of a dominant congestion effect.

It is assumed that people make decisions on membership in a club by comparing the periodical subscription fee set by the club to their evaluation of the periodical service provided by the club. Recalling that the population of potential club members is heterogeneous, the modeling of the membership decision incorporates people's ability to enjoy the club service on a zero-one scale and the upper-bound on people's willingness to pay for that service. This approach to modeling the membership decision is chosen so as to facilitate the subsequent sections' analysis of club density.

Let S denote an index of the quality of the periodical service offered by a club to its members. The willingness to pay of any individual i for S is considered to be a fraction $\theta_i \in (0,1)$ of the highest willingness to pay for that service within the population

$$p_i = \theta_i p_{\max}(S) \quad (1)$$

and the upper-bound on individuals' willingness to pay for the periodical service offered by the club is assumed to rise with S (i.e., $p'_{\max} > 0$). Equation 1 suggests that, despite being simultaneously and equally offered to all members,

willingness to pay may not be uniform. θ may vary across individuals because of differences in preferences, incomes and familiarity with the service and skills needed for using the club service, and $\theta_i p'_{\max}(S)$ reflects the marginal utility of the club's service for the i -th person.

Let m denote the periodical club subscription fee and t a flat tax-rebate rate on membership in some clubs, professional associations and trade unions in particular. We may expect i to be a member of a club as long as his, or her, willingness to pay for S is not lower than the tax-rebate adjusted subscription fee:

$$\theta_i p_{\max}(S) \geq (1-t)m. \quad (2)$$

This membership rule implies that the minimum level of ability to enjoy the club service within the group of the club members rises linearly with the subscription fee

$$\theta_{\min} = \left[\frac{1-t}{p_{\max}(S)} \right] m \quad (3)$$

where $p_{\max}(S)$ is calibrated so that $0 \leq \theta_{\min} \leq 1$.

III. CLUB DENSITY AND THE FEASIBLE SUBSCRIPTION FEE-SERVICE SET

As indicated in the introduction, the case of a monopolistic club is considered. An extension to two-rival clubs is provided in Appendix B. The density of the club is defined as the ratio of the number of the club's members (N) to the target population (L). Recalling equation 3, the density of the club can be rendered as

$$\frac{N}{L} = \frac{\int_0^1 f(\theta_i) d\theta_i}{\left[\frac{1-t}{p_{\max}(S)} \right] m} \quad (4)$$

where $f(\theta_i)$ is the density function of the ability to enjoy the club service within the population. While the use of a truncated normal distribution as an approximation of f introduces interesting factors such as the mean and variance of the ability to enjoy the club service within the population, the costs of doing so, in terms of forgone tractability, is enormous. Suppose, instead, that f is uniform. Then the club density can be expressed as

$$\frac{N}{L} = 1 - \left(\frac{1-t}{p_{\max}(S)} \right) m. \quad (5)$$

A hundred per cent club density is reached when the subscription fee goes to zero, when the upper-bound on members' willingness to pay for the club service goes to infinity and when the tax-rebate rate is equal to one. The positive scalar $(1-t)/p_{\max}(S)$ can be interpreted as the aggregate propensity to withdraw membership in the club as the membership-fee rises. This membership withdrawal inclination is moderated by the tax-rebate rate and the upper-bound on the members' willingness to pay for the service provided by the club.

The club-density equation implies that the *zero-density curve* is the locus of all the combinations of subscription fees and service satisfying

$$S = p_{\max}^{-1}((1-t)m). \quad (6)$$

Recalling the assumption that $p'_{\max} > 0$, the corresponding zero-density curve is positively sloped in the first orthant spanned by m and S .

To simplify the following sections' analyses, let the upper-bound on members' willingness to pay be linear in the quality of the service provided by the club

$$p_{\max}(S) = vS \quad (7)$$

where v is a positive scalar indicating the upper-bound on members' marginal willingness to pay for the service provided by the club. Then, the *zero-density line* (ZDL) is given by

$$S = \left(\frac{1-t}{v} \right) m. \quad (8)$$

Consequently, the club's feasible set of subscription fees and service is the region above the ZDL as depicted by Figure 1.

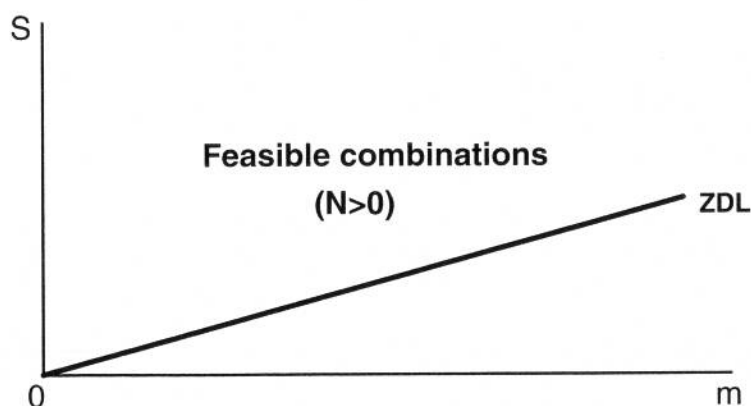


Figure 1. The zero-density line and feasible combinations

IV. SUBSCRIPTION FEES AND SERVICE OF A PROFIT-MAXIMIZING CLUB

The club considered in this section is economic, has a monopoly on providing a service, and sets its subscription fee and quality of service so as to maximize profit: the difference between the sum of the members' periodical membership payments and the club's periodical operational costs. In addition to a fixed cost, C_0 , the club has two types of variable costs: the service-production costs, C_1 , and the organizational costs (e.g., billing and communicating with members), C_2 .

Suppose that the service-production costs are quadratic in the level of service and affected by the club size (i.e., club externalities) as displayed by the second term on the right-hand side of the following equation

$$C_1 = c_1 S^2 + c_2 N \quad (9)$$

where C_1 is a positive scalar, and C_2 is a scalar indicating the marginal club externalities, which is positive when the quality of the service is adversely affected by the club membership size because of a dominant congestion effect, but negative when the quality of the service is enhanced by a dominant agglomeration effect. (Alternatively, C_2 can be expressed as the difference between a constant marginal congestion and a constant marginal agglomeration.) Suppose also that the organizational costs are proportional to the number of members

$$C_2 = c_3 N \quad (10)$$

where C_3 is a positive scalar. Then, the monopolistic club's profit equation is

$$\Pi = (m - c_2 - c_3)N - c_1 S^2 - C_0. \quad (11)$$

By substituting the club's density equation 5 for N and equation 7 for $P_{\max}$ into equation 11 the profit equation can be rendered as

$$\Pi = (m - c_2 - c_3) \left[1 - \left(\frac{1-t}{vS} \right) m \right] L - c_1 S^2 - C_0. \quad (12)$$

In order to gain insight and facilitate the geometrical presentation the maximization of Π with respect to m and S is conducted in two stages. By differentiating Π with respect to m and setting the derivative to be equal to zero, the monopolistic profit-maximizing subscription fee for a given quality of service is

$$m = 0.5 \left(\frac{v}{1-t} \right) S + 0.5(c_2 + c_3). \quad (13)$$

That is, for any given quality of service the profit-maximizing subscription fee is equal to half the marginal club's externalities and organizational costs plus half the upper-bound on members' willingness to pay for the service deflated by the effective cost of a dollar spent as membership payment (i.e., one minus the tax-rebate rate). This relationship is portrayed in Figure 2 by the *profit-maximizing membership-fee line* (PMMFL).

By substituting equation 13 for m into equation 12 and rearranging terms, the club's profit function can be concentrated on S

$$\Pi = 0.25 \left(\frac{vL}{1-t} \right) S + 0.25 \left(\frac{(1-t)L(c_2 + c_3)^2}{v} \right) \frac{1}{S} - c_1 S^2 - 0.5(c_2 + c_3)L - C_0. \quad (14)$$

The monopolistic profit-maximizing quality of service should obey the first-order condition

$$\frac{d\Pi}{dS} = \frac{vL}{1-t} - \frac{(1-t)L(c_2 + c_3)^2}{vS^2} - 8c_1 S^* = 0. \quad (15)$$

The second-order condition for maximum profit is satisfied as

$$\frac{d^2\Pi}{dS^2} = \frac{-2(1-t)(c_2 + c_3)^2}{vS^3} - 8c_1 < 0. \quad (16)$$

The monopolistic profit-maximizing quality of service, S^* , is obtained by solving the polynomial 15. The corresponding subscription fee is obtained by substituting S^* into equation 13. This profit-maximizing combination of the monopolistic club's quality of service and subscription fee is indicated in Figure 2 by E.

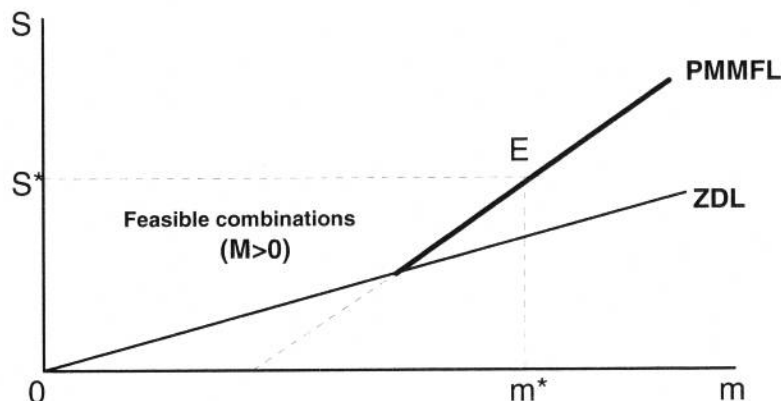


Figure 2. The monopolistic profit-maximizing service and subscription fee

The total differentiation of the first-order condition 15 and the negative sign of the second-order condition lead to the following claims about the properties of the monopolistic profit-maximizing service. (See Appendix A for proofs.)

Claim 1: The effects of the maximum marginal willingness to pay (v) and the tax-rebate rate (t) on the service offered by the profit-maximizing club are positive.

The underlying rationale is that the higher the upper-bound on the marginal willingness to pay and the tax-rebate rate on membership payment, the lower the overall membership withdrawal coefficient and hence the greater the club's revenues and profit and its incentive to provide a higher quality service.

Claim 2: The effect of the target population size on the monopolistic profit-maximizing service depends on the product of the marginal club externalities plus the marginal organizational costs and the membership withdrawal coefficient. If initially this product is larger (smaller) than the profit-maximizing service, an increase in the target population size would raise (lower) the level of the monopolistic profit-maximizing service.

Note that in the case of a dominant agglomeration effect (i.e., $c_2 < 0$) it is less likely that an increase in the target client population (L) will raise the profit-maximizing quality of service level than in the case of a dominant congestion effect.

Claim 3: The higher the service-production costs' coefficient, the lower the monopolistic profit-maximizing quality of service.

Claim 4: The higher the marginal costs of the club's externalities and organization, the lower the monopolistic profit-maximizing quality of service.

Claim 5: The effects of changes in the model's parameters on the monopolistic profit-maximizing subscription fee have the same direction as the claimed effects on the monopolistic profit-maximizing service, and are amplified by the upper-bound on the marginal willingness to pay and the tax-rebate rate.

V. SUBSCRIPTION FEES AND SERVICE OF A BUDGET-BALANCING AND DENSITY-TARGETING CLUB

Let us now consider the subscription fee and quality of service set by a nonprofit monopolistic club so as to cover the costs of servicing its members and ensure a predetermined membership percentage (i.e., density) target. This case is interesting as it is likely to be the policy of some social clubs to generate positive externalities and also of professional associations and trade unions to increase their bargaining power.

Recalling the fixed costs, service production costs, club externalities' costs and organizational costs specified in the previous section, the balanced-budget constraint can be displayed as:

$$C_0 + c_1 S^2 + c_2 N + c_3 N = mN. \quad (17)$$

Consequently, the relationship between the quality of the service provided by the club and the budget-balancing subscription fee is

$$S = \sqrt{\frac{(m - c_2 - c_3)N - C_0}{c_1}} \quad (18)$$

Moreover, in recalling the club-density equation 5 and equation 7, this relationship can be rendered as

$$S = \sqrt{\frac{(m - c_2 - c_3)[1 - \left(\frac{1-t}{vS}\right)m]L - C_0}{c_1}} \quad (19)$$

Consequently, the budget-balancing subscription fee for any service-quality level is given by

$$m = \frac{\frac{c_1}{L} S^3 + (c_2 + c_3 + C_0)S - (c_2 + c_3) \frac{1-t}{v}}{S - \frac{1-t}{v}}. \quad (20)$$

By differentiating equation 20 it can be shown that the budget-balancing combinations of m and S are located on a U-shaped *balanced-budget curve* as depicted in Figure 3. That is, up to a critical subscription fee the quality of the budget-balancing service decreases but then rises.

If in addition to balancing its budget the club sets the quality of its service so as to achieve a membership rate x , and if the ability to enjoy its service is uniformly distributed within the population on a scale of 0 to 1, then the critical member has an ability of $1-x$. Recalling equations 2 and 7, the club ensures that a share of x of the target population joins its ranks by setting the subscription fee to be

$$m = \left(\frac{(1-x)v}{1-t} \right) S. \quad (21)$$

The set of all combinations of m and S ensuring x -membership rate is depicted by the *x-membership-rate line* in Figure 3.

The combinations of m and S satisfying simultaneously both the balanced-budget constraint and the x -membership-rate target can be found by equating the terms on the right-hand sides of equations 20 and 21. These combinations are found in the intersection between the *balanced-budget curve* and the *x-membership-rate line* as indicated by A and B in Figure 3. From the members' perspective, the positive service-quality differential between B and A is a compensation for the membership-fee differential between these combinations.

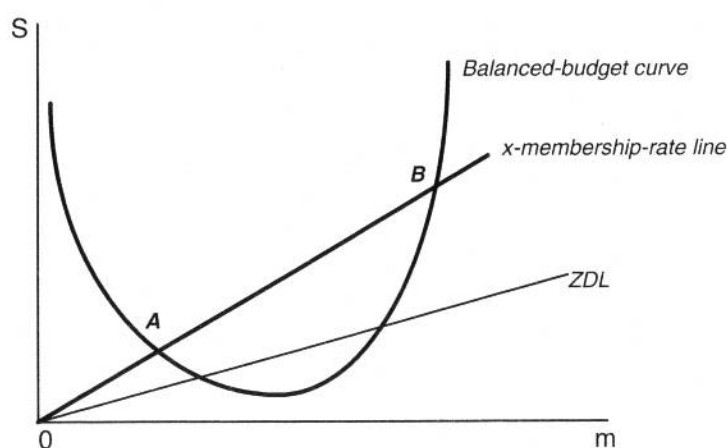


Figure 3. Budget-balancing and x -membership-rate combinations

VI. EXTENSION TO NETWORKS

Networks can be viewed as clubs in which members' utility increases with the density of the network. These special clubs can be generally classified as social networks or economic networks. Much of the literature on networks deals with social networks and is written from a sociological perspective. A survey of that strand of literature is provided by Wellman and Berkowitz (1988). A brief survey of the economic literature on networks is provided by Jackson and Wolinsky (1996). In terms of the model presented in section II, the network service to its members is a function of the network density and the individual willingness to pay for his, or her, membership can be expressed as

$$p_i = \theta_i p_{\max} (N / L) \quad (22)$$

where $p'_{\max} > 0$.

Consider for convenience the following linear specification

$$p_{\max} = \beta \left(\frac{N}{L} \right) \quad (23)$$

where β is a positive scalar indicating the limit of the maximum willingness to pay, within the target population, for a membership in the network when the network density converges to one. If, as assumed in section II, θ is uniformly distributed within the unit interval, the network density is given by

$$\frac{N}{L} = 0.5 \left[1 + \sqrt{1 - 4 \left(\frac{1-t}{\beta} \right) m} \right]. \quad (24)$$

The implications of this density formula for the subscription fee are indicated in the following for two types of networks: 1. a density-targeting network and 2. a profit-maximizing network.

By setting the left-hand side of equation 24 at a desired density level x the subscription fee for a density-targeting network is

$$m = \frac{x(1-x)\beta}{1-t}. \quad (25)$$

As displayed by Figure 5, up to a 50 per cent network density the subscription fee increases with the target density and then declines. So long as the network density is less than half there is a sufficient portion of the target population willing to be affiliated to the network and ready to pay a higher subscription fee for

the improved service associated with an increased network density. However, in order to exceed a 50 per cent membership rate the subscription fee should be lowered so as to attract individuals with relatively low ability to enjoy the network service. The subscription fee peaks at the 50 per cent density level because of the assumption that the ability to enjoy the network service is uniformly distributed within the target population.

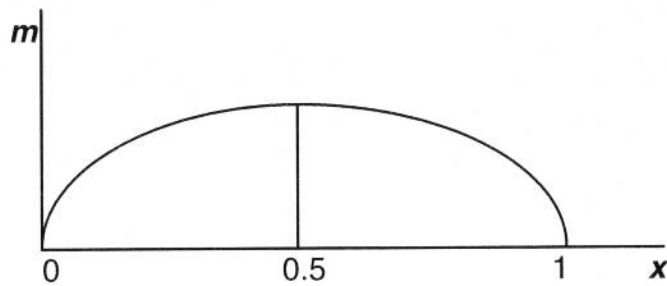


Figure 4. *Density-targeting-network's subscription fee*

Assuming that in addition to the fixed costs of establishment, C_0 , the network's operational costs are proportional to the number of members (cN) and recalling equation 24, the network's profit is given by

$$\Pi = (m - c)0.5L \left[1 + \sqrt{1 - 4 \left(\frac{1-t}{\beta} \right) m} \right] - C_0 \quad (26)$$

and the profit-maximizing subscription fee can be shown to be equal to the marginal operational costs plus half the tax-adjusted limit of the maximum willingness to pay for a membership in the network

$$m^* = c + 0.5 \left(\frac{\beta}{1-t} \right). \quad (27)$$

CONCLUSION

This paper offered a theoretical analysis of clubs for the case of heterogeneous population of members. Subscription fees and ability to enjoy a club service are incorporated into the individual decision to join a club. Club density was related to the distribution of the ability to enjoy the club service, subscription fees and tax concessions. It was shown that the club density declines linearly with the subscription fee. The decline in the club density stemming from an increase in the subscription fee is moderated by the upper bound on individuals' willingness to pay for the club service and the tax-rebate rate.

As the club density affects both the club revenues and organizational costs and generates negative externalities due to congestion and positive ones due to greater opportunities for interaction among members, its determination was integrated into the analysis of the choice of subscription fee and quality of service by a profit-maximizing monopolistic club and, alternatively, by a non-profit monopolistic club which balances its budget and seeks the affiliation of a desired share of the target population.

The substitution of the aforementioned linear relationship between club density and subscription fee into a profit-maximizing objective function implied that for any given quality of service the profit-maximizing subscription fee is equal to half the marginal club externalities and organizational costs plus half the upper-bound on members' willingness to pay for the club service deflated by the effective cost of a dollar spent on membership payment. The effect of the overall membership withdrawal coefficient on the quality of service offered by a profit-maximizing club was found to be negative. Moreover, the higher the members' ability to enjoy the club service and the higher the tax-rebate rate on membership payment, the higher the quality of service of the profit-maximizing club. The effect of the target population size on the quality of service of the profit-maximizing club was found to depend on the product of the marginal organizational costs and the membership withdrawal coefficient. It was argued that if initially the aforementioned product is larger (smaller) than the profit-maximizing service-quality an increase in the target population size would raise (lower) the profit-maximizing service-quality. It was shown that the higher the marginal service-production costs and the marginal organizational costs the lower the profit-maximizing service-quality. It was also argued that the effects of changes in the model's parameters on the profit-maximizing subscription fee have the same directions as the aforementioned effects on the profit-maximizing service-quality and are amplified by the upper-bound willingness to pay coefficient and the tax-rebate rate.

By substituting the linear relationship between the club density and subscription fee into a budget-balancing objective it was shown that the budget-balancing combinations of subscription fee and service-quality are located on a U-shaped curve. That is, up to a critical level the positive effect of an increase in the budget-balancing subscription fee on the club revenues is outweighed by the negative effect of diminishing club density on the club revenues and hence forcing the club to reduce costs by lowering the service-quality. Beyond that critical level the positive effect of an increase in the budget-balancing subscription fee on the club revenues outweighs the negative effect of declining club density on the club's

revenues and hence enables the budget-balancing club to raise the quality of its service. In turn, an improvement in the club's service is necessary for increasing the target population's willingness to pay. It was argued that if the club sets the quality of its service so as to achieve a target membership rate, the locus of all combinations of subscription fee and service rate will be a positively sloped line. The intersection of this line with the U-shaped balanced-budget curve indicates the combinations of subscription fee and service-quality satisfying both the target density and balanced budget.

The analysis was extended to the special cases of profit-maximizing and density-targeting networks. It was suggested that the density-targeting subscription fee increases with the target density so long that the network density is lower than 50 per cent and then declines. It was argued that the profit-maximizing subscription fee exceeds the marginal operational costs by half the tax-adjusted limit of the maximum willingness to pay for a membership in the network.

APPENDIX A: PROOFS OF CLAIMS 1-5

Proof of claim 1:

$$\frac{dS^*}{d\left(\frac{1-t}{v}\right)} = \frac{[((c_2 + c_3)^2 / S^{*2}) + (v^2 / (1-t)^2)]L}{d^2\Pi / dS^2} < 0. \quad (A1)$$

Proof of claim 2:

$$\frac{dS^*}{dL} = \left\{ \frac{\frac{(c_2 + c_3)^2(1-t)}{vS^2} - \frac{v}{1-t}}{\frac{d^2\Pi}{dS^2}} \right\} \begin{matrix} > \\ = \\ < \end{matrix} 0 \text{ as } S^* \begin{matrix} < \\ = \\ > \end{matrix} \left\{ (c_2 + c_3) \frac{(1-t)}{v} \right\}. \quad (A2)$$

Proof of claim 3:

$$\frac{dS^*}{dc_1} = \frac{8S^*}{d^2\Pi / dS^2} < 0. \quad (A3)$$

Proof of claim 4:

$$\frac{dS^*}{d(c_2 + c_3)} = \frac{\frac{2(1-t)L(c_2 + c_3)}{vS^{*2}}}{\frac{d^2\Pi}{dS^2}} < 0. \quad (A4)$$

Proof of claim 5: By virtue of equation 13 the profit-maximizing subscription fee is positively related to the club's service-quality.

APPENDIX B: EXTENSION TO TWO RIVAL CLUBS

Consider an industry comprising two non-identical clubs and a population of potential customers that for reasons such as habit, loyalty and snobbism might evaluate the service of the clubs in a biased manner. Recalling equations 5 and 7, the number of members affiliated to *club 1* can be expressed as:

$$N_1 = (L - N_2) \left[1 - \frac{1-t}{v_1 S_1} m_1 \right] \quad (\text{B1})$$

and the number of members affiliated to *club 2* is

$$N_2 = (L - N_1) \left[1 - \frac{1-t}{v_2 S_2} m_2 \right] \quad (\text{B2})$$

where v_1 is not necessarily equal to v_2 because of the aforementioned service-evaluation biases.

By substituting equation B2 into equation B1 for N_2 , solving for N_1 and dividing by L , the density of *club 1* is:

$$\frac{N_1}{L} = 1 - \left[\frac{\frac{m_1}{v_1 S_1}}{\frac{m_1}{v_1 S_1} - \frac{(1-t)m_1 m_2}{v_1 v_2 S_1 S_2} + \frac{m_2}{v_2 S_2}} \right] \quad (\text{B3})$$

and, by symmetry, the density of *club 2* is:

$$\frac{N_2}{L} = 1 - \left[\frac{\frac{m_2}{v_2 S_2}}{\frac{m_1}{v_1 S_1} - \frac{(1-t)m_1 m_2}{v_1 v_2 S_1 S_2} + \frac{m_2}{v_2 S_2}} \right]. \quad (\text{B4})$$

As can be intuitively expected, the differentiation of equations B3 and B4 implies that the density of each club increases with its service-quality, its service assessment coefficient, and the subscription fee set by its rival and decreases with its own subscription fee, the service level of its rival, and the assessment coefficient of the service of its rival. The curves CLUB1 and CLUB2 in Figure 5 display the

relationship between club density and subscription fee-service ratio for *club 1* and *club 2*, respectively for the case where $v_1 > v_2$.

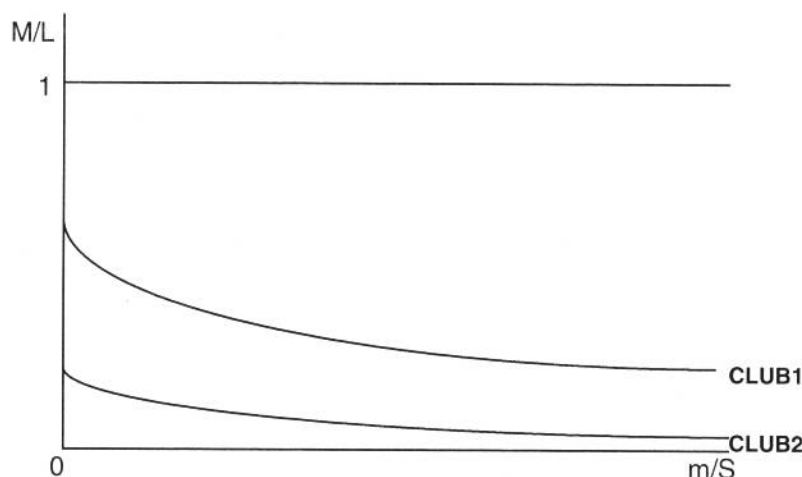


Figure 5. Clubs' density where $v_1 > v_2$

In recalling equations 11 and B3 the profit function of *club 1* is

$$\Pi_1 = (m_1 - c_{21} - c_{31}) \left[1 - \frac{\frac{m_1}{v_1 S_1}}{\frac{m_1}{v_1 S_1} - \frac{(1-t)m_1 m_2^e}{v_1 v_2 S_1 S_2^e} + \frac{m_2^e}{v_2 S_2^e}} \right] L - c_{11} S_1^2 - C_{01} \quad (\text{B5})$$

and in recalling equation B4 the profit function of *club 2* is

$$\Pi_2 = (m_2 - c_{22} - c_{32}) \left[1 - \frac{\frac{m_2}{v_2 S_2}}{\frac{m_1^e}{v_1 S_1^e} - \frac{(1-t)m_1^e m_2}{v_1 v_2 S_1^e S_2} + \frac{m_2}{v_2 S_2}} \right] L - c_{12} S_2^2 - C_{02} \quad (\text{B6})$$

where the superscript e denotes expected values of the rival club's control variables. The Cournot-equilibrium subscription fees and service-quality levels are obtained by solving the combined set of first-order conditions for *club 1* and *club 2* simultaneously under the assumption that the expected values are confirmed in equilibrium. That is, m_1^* , m_2^* , S_1^* and S_2^* satisfying simultaneously

$$\frac{\partial \Pi_1}{\partial m_1} = N_1 + (m_1 - c_{21} - c_{31}) \frac{\partial N_1}{\partial m_1} = 0 \quad (\text{B7})$$

$$\frac{\partial \Pi_1}{\partial S_1} = (m_1 - c_{21} - c_{31}) \frac{\partial N_1}{\partial S_1} - 2c_{11}S_1 = 0 \quad (\text{B8})$$

$$\frac{\partial \Pi_2}{\partial m_2} = N_2 + (m_2 - c_{22} - c_{32}) \frac{\partial N_2}{\partial m_2} = 0 \quad (\text{B9})$$

$$\frac{\partial \Pi_2}{\partial S_2} = (m_2 - c_{22} - c_{32}) \frac{\partial N_2}{\partial S_2} - 2c_{12}S_2 = 0 \quad (\text{B10})$$

where,

$$N_1 = \left[1 - \frac{\frac{m_1}{v_1 S_1}}{\frac{m_1}{v_1 S_1} - \frac{(1-t)m_1 m_2}{v_1 v_2 S_1 S_2} + \frac{m_2}{v_2 S_2}} \right] L \quad (\text{B11})$$

and

$$N_2 = \left[1 - \frac{\frac{m_2}{v_2 S_2}}{\frac{m_1}{v_1 S_1} - \frac{(1-t)m_1 m_2}{v_1 v_2 S_1 S_2} + \frac{m_2}{v_2 S_2}} \right] L. \quad (\text{B12})$$

Possible comparative statics' results can be obtained through simulations.

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